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SENIOR

MAGNETISM AND ELECTRICITY

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SENIOR
MAGNETISM AND ELECTRICITY

BY

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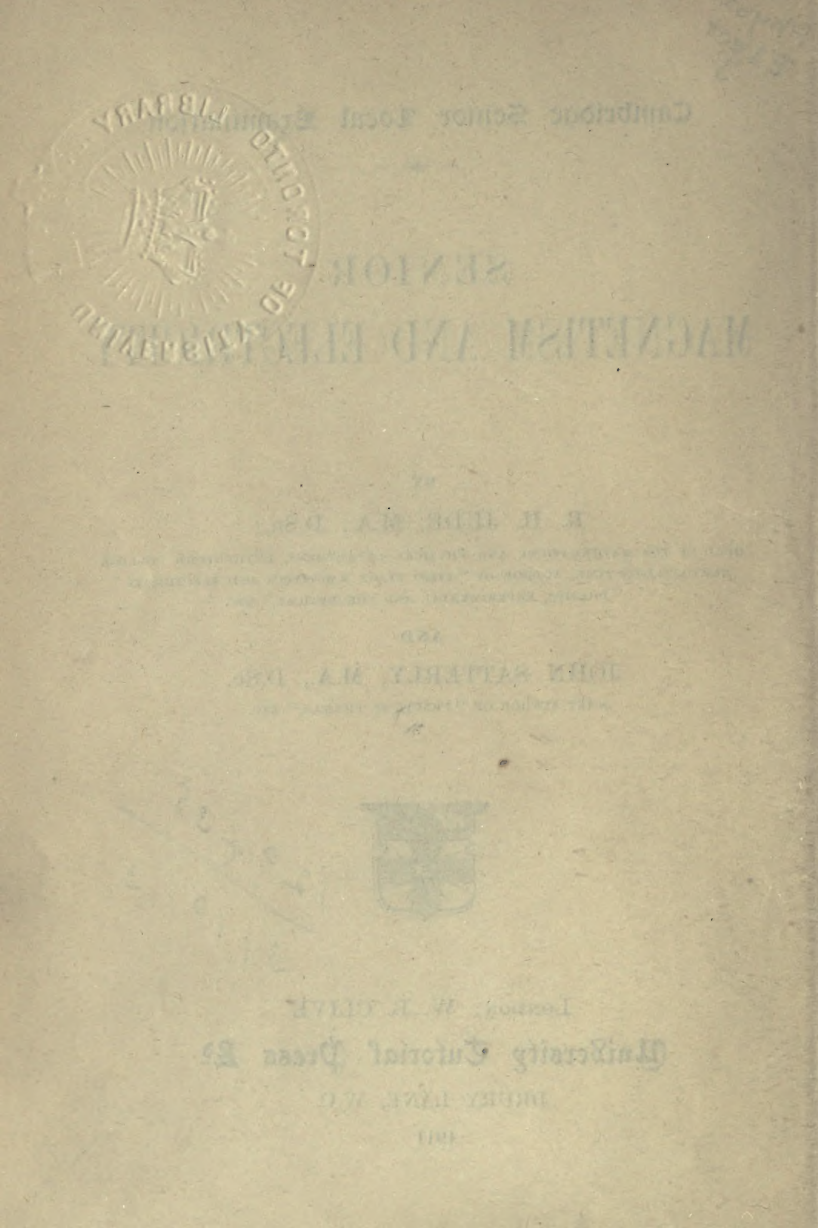

PREFACE.

THE object of this book is to provide a sound course of Elementary Magnetism and Electricity, suitable for use in schools and colleges, containing a lucid account of the theory illustrated by a large collection of simple and instructive experiments. Modern ideas on the nature of electricity have received full attention, so far as this is consistent with a strictly elementary treatment of the subject.

Part I. deals with Magnetism, a subject which is much better fitted for easy experimental work than Electrostatics. For example, the Law of Inverse Squares, and the conception of Lines of Force are far easier to demonstrate by practical methods in Magnetism than in Electricity.

In Part II.—Electrostatics—the conception of Potential is introduced very early, is illustrated practically by means of the gold-leaf electroscope, and is then made the foundation of the whole subject. Numerous diagrams of lines of force and equipotential surfaces are given, in order to impress on the student the importance of the medium surrounding the "charged" bodies.

Part III.—Electrodynamics—opens with a simple treatment of the general principles, which are then treated in detail in the later chapters. The account of the theory of voltaic cells, though strictly elementary, is quite up to date.

Throughout the book the experiments described are specially suitable for school use: elaborate and expensive apparatus is not required, and the practical work is intended to be done

mainly by the scholar himself. In some cases the theory is developed from the experiments, while in others the theory is stated first and the illustrative experiments follow: so that in the main a balance is maintained between the heuristic and didactic methods.

This book has been prepared by Mr. John Satterly, M.A., D.Sc., but is based on the well-known *School Magnetism and Electricity* by Dr. R. H. Jude. Broadly speaking the new book follows the lines of the old, but much greater stress has been laid on practical work in the new version and the harder parts of the older book have been omitted so as to afford the opportunity for a thorough experimental treatment. The experiments have been separated out from the remainder of the text, and both experiments and diagrams have been largely increased in number.

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PART I.

MAGNETISM.

CHAPTER I.

GENERAL PHENOMENA.

1. Natural and Artificial Magnets. The name *Magnet* was applied at a very early date to pieces of a mineral found at Magnesia in Asia Minor. This substance, now known as *Magnetite* or *Magnetic iron ore* (Fe_3O_4) or *lodestone*, was found to possess the property of attracting iron and steel;* thus if a piece of it be dipped in iron filings the latter will adhere to it, more especially round the ends (Fig. 1).

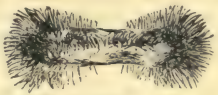

Fig. 1.

A lump or strip of magnetite constitutes a *natural magnet*. In practice such are rarely used, because very much better ones can be made by artificial means. Artificial magnets are bars of steel or iron which have been treated in certain ways of which we shall learn later; they are sometimes straight (Fig. 2), when they are called *bar magnets*; and sometimes are bent round so that their ends come near together, when they are called *horse-shoe magnets* (Fig. 3). In our experiments we shall generally suppose artificial magnets employed; natural ones possess, however, just the same properties, but as a rule much weaker.

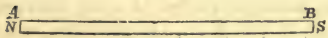

Fig. 2.

Fig. 3.

* This property of lodestone is mentioned by Pliny and Lucretius. The chief advance in the subject was made by Dr. Gilbert of Colchester, who was physician to Queen Elizabeth and who published in his book, *De Magnete*, the chief facts and principles of Magnetism.

2. Magnets and Magnetic Substances. Any substance capable of conversion into a magnet (whether it actually is one or not) is said to be a *magnetic substance*, and when we so convert it we are said to *magnetise* it. Thus, ordinary iron and steel are magnetic substances, but it is not until after certain treatment that they become magnets. The only other distinctly magnetic substances are the metals cobalt, nickel, manganese, and chromium, but their magnetic powers are somewhat feeble compared with iron and steel, and they are seldom used. Iron or steel in its ordinary or unmagnetised state is said to be *neutral*.

Exp. 1. *To distinguish between magnetic and non-magnetic substances.* Bring the ends of a horseshoe magnet against small pieces of various substances—glass, wood, iron, bone, nickel, copper, steel, paper, tin-tacks, a stair-rod (stair-rods are often made of iron covered with brass), etc. Classify these into (1) magnetic substances (those attracted), and non-magnetic substances (those unaffected).

3. All Forces are Mutual Actions. It will help us to avoid mistakes to notice that forces are of the nature of mutual actions between two bodies. This principle is a mechanical one and applies to all kinds of forces. Thus a horse exerts a certain forward pull on a cart, and the cart exerts an equal backward pull (resistance) upon the horse. Similarly a magnet attracts a neutral piece of iron, and the neutral piece of iron attracts the magnet with an equal force. In Exp. 1 matters were arranged that the substances under test were free to move while the magnet is held fast, but it is easy to arrange them the other way; thus if a magnet be balanced on a pivot and a neutral piece of iron held in the hand near it the magnet will swing round and point to the iron.

4. Test of Magnetisation. Two pieces of neutral iron or steel do not attract one another, but if one of them be magnetised it attracts the neutral one, and at the same time (see above) the neutral one attracts the magnetised one. Hence to test whether a body is magnetised or not we must see whether it attracts a piece of *neutral* iron or steel;* if so it is a magnet, otherwise it is not. A simple plan is to dip the body into iron filings and see if they adhere.

* A more elaborate method of testing by means of a magnetic needle will be given in Exp. 5.

It should be observed in Exp. 1 that a magnet will attract *magnetic substances only*; it has no action on brass, wood, paper, etc.

Exp. 2. *To show that a freely suspended magnet tends to set in a certain direction.* Make a small piece of paper into a stirrup and suspend it by a long strand of unspun silk. Insert a light bar magnet or a magnetised knitting-needle A B (Fig. 4) into the stirrup so that it rests evenly. After swinging for some time the magnet takes up a position of rest and sets approximately north and south. If disturbed from this position observe that it goes back to it. If the end that is pointing north be turned so as to point south observe that it will not stop there but will go back to the north again. Mark the end which points north with a file or a piece of stamp paper.

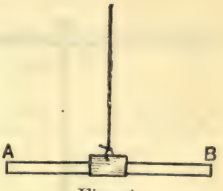

Fig. 4.

Exp. 3. Suspend a horseshoe magnet and find how it sets.

5. Magnetic Poles and Polarity. The last experiment shows us that the two ends of a magnet have distinct properties. The magnet may be said to be *polarised* and the ends are called *poles*; the end pointing north is called the *north pole*, and the one pointing south the *south pole*. For a reason which will appear in § 46 some confusion has arisen with regard to these terms, and some writers call the end that points to the north the *south pole*. Others prefer to drop these terms altogether and speak of the end that points to the north as the *north-seeking pole*. Throughout this book we shall follow the usual English custom and use the term *north pole* to denote the pole that *points to the north*.

It will be observed that we have defined the poles of a magnet with a certain degree of vagueness as its "ends," and have not located them as definite points. We shall refer to this again in §§ 13, 41.

In order to distinguish at a glance between the poles of a magnet, a common practice is to paint the north pole red and the south pole blue.

Another convenient plan is to have sharp points about $\frac{1}{8}$ of an inch long fixed near each end, on which bits of painted cork or pith can be temporarily stuck; the latter contrivance is especially convenient for magnetic needles

As it is awkward to suspend bar magnets in stirrups and get them to come quickly to rest, it is usual for this purpose to employ *magnetic needles*. These are small magnets (Fig. 5) fitted with a brass or agate cup at the centre which can rest on a fine pivot. The friction at the pivot greatly damps the vibratory motion and the needle quickly sets north and south.

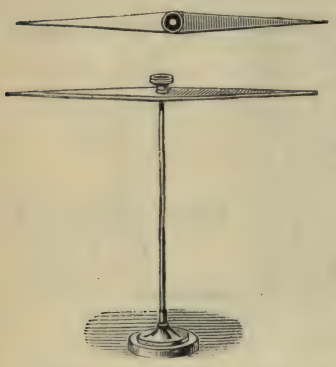

Fig. 5.

Magnetic needles are sometimes mounted so as to turn in a vertical plane; those turning in a horizontal plane, as in Fig. 5 (by far the most usual arrangement), are generally called *compass needles*.

The property of magnets shown in the possession of distinct poles is called *polarity*; we shall frequently employ this term and shall speak of the *north* pole as having *northern* polarity and the *south* pole *southern* polarity.

The properties of magnets were at one time accounted for by supposing them to possess some mysterious fluid called *Magnetism*, and there was conceived to be more of this at the poles than elsewhere. It is, however, better to regard "magnetism" as a property inherent in the molecules * of the magnetic substance, but manifested only under suitable conditions (§§ 14, 16). The influence of this property is transmitted to another magnetic substance across the air, or other intervening medium, in some way not fully understood, and of which for elementary purposes nothing need be said.

6. Magnetic Attraction and Repulsion. Action of the Earth.

Exp. 4. Take a bar magnet or a magnetic needle, A, suspend it by a fibre or mount it on a pivot so that it can turn horizontally, wait till it comes to rest, and then mark the poles, say, by putting a red pith ball on the north pole and a blue one on the south. Set this magnet aside with

* That is the smallest particles.

the markers on it; then take another magnet, B, and treat it in the same way. Now, keeping either one of the magnets, say B, suspended, bring the north pole of A gradually near the north pole of B: *repulsion* ensues. After B has come to rest, do the same with their south poles: again we get *repulsion*. Now (B having again come to rest) bring the north pole of A gradually near the south pole of B: *attraction* ensues; and the same happens if we use the south pole of A and the north pole of B.

We thus get the following law:—**Like poles repel and unlike attract.** This is the fundamental law of magnetic action.

With regard to the fact that a magnetic needle tends to set north and south, a preliminary note should here be made of what will receive detailed attention in Ch. IV.—viz. that *the earth itself behaves as a magnet*: its northern regions attract the north pole and its southern the south pole of the needle. It is really the force exerted by the earth pulling one pole northwards and the other southwards that is the cause of the needle pointing in these directions.

7. Test for Polarity. Suppose now we have a steel (or iron) bar, A B, and we wish to examine its magnetic condition, that is, to determine whether it is magnetised at all, and if so which end is a north pole and which a south. We proceed thus:—

Exp. 5. Set a compass needle on a pivot and bring one end A of the bar gradually towards the north pole of the needle; if *repulsion* ensues A is the north pole of the bar. Then bring the end B in like manner towards the south pole of the needle, the repulsion indicates that B is the south pole. If A had *attracted* the north pole of the needle, then so far as we can tell A may be a south pole, or it may be neutral; for § 3 teaches us that neutral iron attracts a magnet;* we must therefore go a step further and bring the *same end A of the bar* towards the *south* pole of the needle; if we now get *repulsion* A is a south pole, but if we *still get attraction* A is neutral. We can then examine B in like manner.

The point always to bear in mind is that repulsion is a sure test of polarity, but attraction is not. It is a common practice to dip the bar into iron filings (§ 4) prior to these experiments, so as to know beforehand whether it is magnetised or not; if the latter we need go no further, while if the former, then since neutrality is excluded the attraction of an end by the north (or south) pole of the needle proves that end to be a south (or north)

* As will be explained in § 10, the iron is not neutral *while near the magnet*, but it may have been originally neutral, and it is *original* state that we are concerned with.

pole; it is, however, important in any case to bring the bar gradually towards the needle from a distance, otherwise confusion might arise, from "induction" (see § 12).

8. Magnetic Induction.

Exp. 6. *To show that magnetic qualities may be transmitted through steel.*—Support a magnet N S (Fig. 6) vertically in a clamp. Bring a short piece

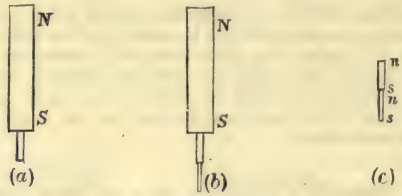

Fig. 6.

of steel near the bottom end of the magnet. It is attracted and remains supported (Fig. 6 *a*). Now bring another small piece of steel near the bottom of the first piece. It, too, is attracted (Fig. 6 *b*), and if the magnet

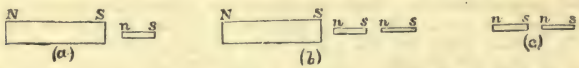

Fig. 7.

be strong enough will be supported by the first piece. Now gently detach the upper piece from the magnet, the lower piece will still cling to it (Fig. 6 *c*), and test the two pieces separately, as in Exp. 5; each will be found to be a magnet with poles as shown in the figure.

Exp. 7. Modify the last experiment, as indicated in Fig. 7, by not allowing the magnet and the pieces of steel to come into actual contact. The results will be exactly the same except that the pieces are not so strongly magnetised.

The production of a magnet, as in these experiments by the influence of another magnet (either with or without actual contact), is called magnetisation by **induction**, and the phenomenon in general is termed **Magnetic induction.**

9. Differences in the Magnetic Behaviour of Iron and Steel. Permanent and Electro-Magnets. It is a matter of general knowledge that steel and iron are not exactly the same: steel is in general harder than iron; thus while the latter can be easily filed or turned in a lathe it is much more difficult to do so to the former. Different varieties of steel differ greatly in hardness: "mild" steel is nearly as soft as iron, "hard" steel is very hard indeed, "medium" steel is intermediate. In general if a steel bar be made hot and then *suddenly* cooled it becomes harder, while if made hot and *gradually* cooled it becomes softer.

An ordinary knitting-needle is of medium steel; it can be filed with moderate difficulty, is tough, and requires considerable force to bend it; if it be made red-hot and quenched in water it becomes very hard and brittle, is scarcely touched by the file, and can be snapped into pieces by the fingers. If a needle thus hardened be again made red-hot and buried in hot ashes so as to cool very slowly it becomes so soft as to be very readily filed, and can be bent like a piece of copper wire.

Different specimens of *iron* also differ much in hardness. Cast iron is harder and more brittle than wrought. Generally in magnetism when "iron" is spoken of, good "soft" or "annealed" iron is understood.

Now, although both iron and steel are magnetic substances they differ magnetically in two important respects. We shall first *state* these, and then describe experiments illustrating them:—

1. Iron is more readily magnetised than steel.

2. Steel when it *is* magnetised retains its magnetic properties much better than iron.

Both these points of difference may be shown by the following experiment.

Exp. 8. Set up two strong bar magnets vertically, and have ready a dozen or so little rods of steel (as in Exp. 6) and the same number of iron of the same weight. Wire nails will do for the iron rods and pieces of knitting-needle of equal length for the steel; or pins for the iron rods and sewing needles for the steel rods. To one of the magnets attach the steel rods one under another (as in Exp. 6), and see how many will hang on; there will probably be only two, the magnetic strength of the second rod being too feeble to support a third. To the other magnet attach the iron rods: probably six or eight will hang on. This proves the first of the foregoing statements.

Next very carefully detach the upper steel rod from the magnet and carry it away, then, as already seen (*c*, Fig. 6), each *retains its magnetism.** Do the same with the upper iron rod; all the rest will fall off, and when separately tested, as in Exp. 5, will be found neutral. This proves the second statement.

The remarks of this section apply not only to magnetism *by induction* but by all methods, *e.g.* by the electric current (§ 207).

The details depend a great deal upon the hardness of the steel; if very hard only one piece can be supported, if medium two or three, if mild more. In any case the piece in contact with the magnet becomes more strongly magnetised the softer it is, but if kept for some months the harder specimens would retain their magnetism best.

[It is worthy of special note as a point of importance to the electrical engineer that there is a *particular variety* of steel known as "cast dynamo-steel," now largely employed, which can be magnetised with as great, or even greater readiness than the best wrought iron. On the other hand, the so-called "manganese-steel" is practically non-magnetic.]

It is usual to classify magnets as *permanent* or *temporary*. Permanent magnets are those which retain their magnetism when the original magnetising influence is removed; the simplest forms are the ordinary bar and horseshoe magnets. They must be made of *steel* of a suitable degree of hardness, or, as it is technically expressed, suitably *tempered*. Temporary magnets are those which are magnets only by virtue of some external influence then and there acting upon them, and which lose their magnetic properties as soon as that influence is removed; thus a soft iron bar touching a pole of a permanent magnet is a temporary magnet. The most important members of this class are *electro-magnets*; they consist of soft iron bars technically termed *cores*, wound round with copper wire through which an electric current can be passed; the current makes the core into a very powerful magnet, but when the current is switched off the core loses its magnetic properties.†

Electro-magnets will be more fully considered in connection with Voltaic electricity (Part III., Ch. XIV.).

* A body is frequently said to "retain its magnetism" or to "lose its magnetism." The phrases are relics of a now discarded theory (§ 5); they are, however, convenient, and do no harm provided we regard the word "magnetism" as a mere abbreviation for "magnetic properties."

† Unless under special circumstances (§ 17).

10. Polarity due to Induction. Referring to Figs. 6 and 7 it will be seen that the *south* pole (S) of the bar magnet has been used, and that the end of each of the little bars nearest to it has become a *north* pole. This is, of course, proved by actually testing with the magnetic needle (Exp. 5). If the *north* pole (N) had been employed the ends nearest it would have been *south* poles. We thus have the following law, known as the "law of induced polarity": *When a bar is magnetised by induction the end of it nearest the inducing pole acquires polarity of the opposite kind.* The student should note this very carefully; it may equally well be expressed by saying that the end *farthest from* the inducing pole acquires polarity of the *same kind.*

This law is equally true of iron and steel. But in the case of the former it is somewhat more troublesome to prove it, because the bars when detached from the magnet lose their polarity altogether. The following experiment, however, meets the case.

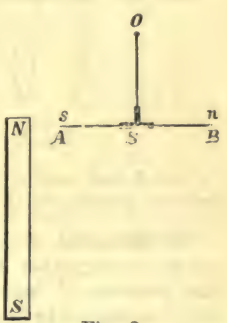

Fig. 8.

Exp. 9. Set a bar magnet vertically as shown in Fig. 8, and near it suspend a light iron rod or piece of iron wire, A B, from a point, O, by means of a thread and stirrup, S. Bring the north pole of another bar magnet gradually towards B (Exp. 5); it will be repelled, B is therefore a *north pole.* Then bring the south pole of the magnet towards A; it is repelled, and is therefore a *south pole.*

11. Why Neutral Bodies are Attracted. In Fig. 7 (*a*) let *n s* represent a piece of originally neutral iron or steel; the magnet N S acts inductively upon it as shown. By the law of § 6, S attracts *n* and repels *s*. But *n* is nearer than *s*, consequently the force of attraction exceeds that of repulsion, and *on the whole the little bar is attracted by* S. This explains the apparent attraction of neutral bodies; they are really not neutral, but are magnetised by induction. The fact is often expressed thus: **Induction precedes attraction**, or **induction is the cause of attraction.**

Strictly speaking, we should take account of the influence of the other pole (N), both as respects inductive action and force

exerted on the little bar; but on account of the relatively great distance of N, its effects are small, and are usually neglected.

It should be carefully noted that inasmuch as attraction (in the case of otherwise neutral bodies) is the consequence of inductive magnetisation, and iron is magnetised more readily than steel (§ 8), a given magnet attracts iron with considerably greater force than steel. A hard steel ball clinging to the pole of a magnet can be pulled off with very little force, while a comparatively large force is required to pull off a similar ball of soft iron. The difference is also well shown by the following experiment.

Exp. 10. Arrange equal-sized pieces of soft iron and a piece of hard steel on each side of the pole of a compass-needle, as shown in Fig. 9. If the iron and steel be placed at equal distances from the pole the attraction of the former will be greater, and the pole will be deflected towards it; by removing the iron to a greater distance, as shown by the dotted outline, the two forces may be made equal; they will then balance, and the needle will go back to its natural position.

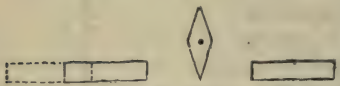

Fig. 9.

12. Reversal of Polarity by Induction. Not only does induction magnetise a bar originally neutral, but it may reverse the polarity of one already magnetised. Thus, in Fig. 7 (*a*) suppose the little bar to have been originally a magnet, with the end nearest S a south pole, then the inductive action of N S tends to make it a north pole, and if strong enough will reverse the previous polarity; in fact, this frequently happens in practice when weak magnets are allowed to touch strong ones.

We can now see why it is necessary in Exp. 5 to bring the end of the bar under examination *gradually near to the pole of the needle from a distance.* For suppose the pole of the needle to be a S one, and suppose that the end of A of the bar approaching it is also S, then the pole of the needle will, by induction, tend to make A *north*, and if the induction be strong enough the inductive action will reverse A's original state, and there will be *attraction* which might lead us to infer that A was originally N. But by bringing it up gradually we can generally catch the repulsive action before induction becomes strong enough to reverse the original polarity. Of course, repulsion is always perfectly reliable; the danger is that we may lose the chance of getting it through too great hurry. In the same way, provided we have already ascertained that the body under examination is not neutral, attraction is reliable *if it is genuine* and not due to reversed polarity.

13. Distribution of Polarity in Magnets. Lateral and Terminal Polarity. Simple Magnet. Point Pole.

Exp. 11. Dip an ordinary bar or horseshoe magnet into iron filings (or tin tacks). Take it out and gently shake off the excess filings. Note that most of the remaining filings will be found adhering to the ends, the number getting fewer and fewer as the middle is approached, while at the middle there is none at all.

Exp. 12. Test different parts of a bar magnet by presenting them in turn to a compass needle as in Exp. 5 (Fig. 10). Begin at the end and travel towards the middle of the magnet. Note carefully how the deflection of the needle varies. You will find that in travelling from the north pole to the middle all the parts repel the *north* pole with a force which gradually diminishes as we approach the middle, that in travelling from the south pole to the middle all the parts repel the *south* pole with a gradually diminishing force, while at the middle neither pole is repelled.

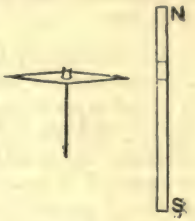

Fig. 10.

We thus learn that the polarity of the magnet is not confined to the poles, but is *distributed in a varying degree all over its surface.* The polarity along the sides of the magnet is termed *lateral polarity,* that at the ends being sometimes, for distinction, called *terminal polarity.*

Fig. 11 shows graphically the distribution of lateral polarity in an ordinary bar magnet. N S represents the magnet, and at a number of points taken at equal distances along its length perpendiculars are erected proportional to the strength of the polarity at that point. If the perpendiculars are drawn to the left of the magnet for northern polarity and to the right for southern, then the curve, *n o s,* drawn through their extremities gives the desired graphic representation. It clearly shows the gradual weakening of polarity from either pole to the middle; the latter region when the polarity is *nil* is sometimes called the *magnetic equator.*

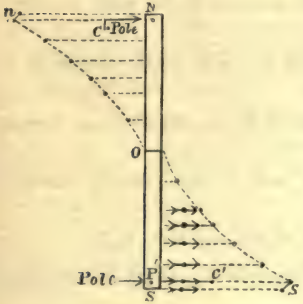

Fig. 11.

The polarity at any point of a magnet is frequently said to be due to the *free magnetism* at that point. The phrase is a relic of the old fluid theory (§ 5), but is sometimes convenient. Thus the above curve is said to indicate the *distribution of free magnetism* (see also Exp. 37).

Exp. 13. Repeat Exp. 12 with a very thin magnet such as a magnetised knitting-needle. You will find slight polarity along the sides, although at the ends it might be strong. It is permissible to deduce from this that an indefinitely thin magnet would have its polarity entirely concentrated at its ends.

An indefinitely thin magnet (whether straight or not) is called a *simple magnet*, and a mere point regarded as possessing polarity is called a *point-pole*; it corresponds to a point-charge in electrostatics (§ 142). A simple magnet is thus the only kind of magnet that really possesses point-poles; it cannot, of course, be realised in practice, though a *thin* needle, or ordinary compass needle, if properly magnetised, is a sufficiently close approximation to it (cf. § 5).

SUMMARY.—CHAPTER I.

1. Distinction between magnets and magnetic substances (§ 2).

2. Poles and polarity (§ 5). The pole that points *to the north* is called the *north pole*.

3. *Like poles repel and unlike attract* (§ 6). *The earth behaves as a magnet.*

4. Use of compass-needle to test polarity (§ 7); *repulsion is always reliable*, but attraction is not.

5. Magnetic induction (§ 8), and the law of the polarity thereby produced (§ 10).

6. Differences in the magnetic behaviour of iron and steel (§ 9).

7. *Induction precedes attraction* (§ 11). Reversal of polarity by induction (§ 12).

8. The polarity of a magnet is in general not confined to its poles, but distributed in a varying degree over its surface. Terminal and lateral polarity. The only cases where lateral polarity is entirely absent is that of a *simple magnet*, which *has true point-poles* (§ 13).

EXERCISES I.

1. How would you exhibit the polarity of a horseshoe magnet?

2. It is doubtful whether a steel rod is neutral or slightly magnetised: how could you find out by trying its action upon a compass-needle?

3. What is the magnetic condition of a bar of soft iron held horizontally above and parallel to a bar magnet of the same size resting horizontally on a table?

4. A bar magnet is laid on a table with its N end projecting over the edge. A soft iron ball clings to the under side of the projecting end. State and explain what happens when the S pole of a second bar magnet is brought above and near the N pole of the first?

5. Supposing the first magnet and ball arranged as in the preceding question, state and explain what happens: (1) When the N pole of another magnet is placed underneath the ball a little way from it, (2) When the S pole of another magnet is placed in the same position.

6. If a compass-needle is deflected when a steel bar is brought near it how can you find out whether the deflection is due to magnetism already possessed by the bar, or to the bar becoming magnetised by the compass-needle at the time of the experiment?

7. A compass-needle and a straight strip of soft iron of the same length are fastened together so as to be in contact with both ends. Will the force tending to make the combination point north or south be the same as that which would act on the compass-needle alone? Give reasons.

8. Two bars of soft iron are so placed to the east and west of the north pole of a compass-needle that the latter still points N and S. If the iron to the E of the needle be replaced by a bar of hard steel of exactly the same size and shape as itself, will the direction in which the needle points be altered? If so, in which direction will it move, and why?

9. A compass-needle is deflected by a bar magnet placed some distance away from it. How is the deflection modified (if at all) when a bar of soft iron is placed parallel to, but not touching, the magnet?

10. A vertical steel rod, of which a portion of length less than half that of the rod is stuck into the earth, is found to be rather strongly magnetised. If you were given a compass-needle and a foot-rule, how could you without disturbing the rod form an estimate of the length of the buried portion?

11. One pole of a magnet made of soft iron and only weakly magnetised is found to be repelled by the north pole of a strong magnet when the latter is some distance away but to be attracted when the magnets are brought close together. Explain this.

CHAPTER II.

NATURE OF MAGNETISM.

14. Magnetism a Molecular Property. Fundamental Experiment. It was pointed out in § 5 that magnetism is not something which can be put on or taken off a body, but is to be regarded as a property inherent in the molecules of iron and steel, and that magnetisation simply consists in so treating these substances that this property shall manifest itself. Why it does not do so under ordinary circumstances, and what is done to the molecules in the process of magnetisation to make it do so, will appear in due course.

Exp. 14. Take a thin magnetised steel bar (a hardened* knitting-needle or a piece of clock spring is very convenient), as A B (Fig. 12), and break it at any place C. Test each part, A C, C B (as in Exp. 5) and show that each *is itself a complete magnet.* A and B being the original north and south poles respectively will remain so, while C will be found to be a south pole and C′ a north. Further, if either of the bits be broken again the same thing will be found to be the case, and so on, no matter into how many small pieces the original magnet be divided.

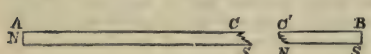

Fig. 12.

It is even possible to grind a magnet to powder, and still each fragment is a magnet, as is proved by the fact that the powder is attracted by *neutral* iron. It is, moreover, impossible by any known means to obtain a piece of substance containing one *kind of polarity only.* It is thus clear that magnetism is a molecular property:† the only question is, what is the difference between a magnetised and an unmagnetised bar? Does the process of magnetising confer the property on the molecules, or does it merely turn them into positions favourable to its manifestation? We shall see as we proceed that there is every reason for accepting the latter as the true explanation.

* See § 9.

† Of course the particles of powder, however fine, are not actual molecules, but there can be no reasonable doubt that if we could obtain the molecules separately they would be found to be magnets.

15. Magnetic Chains.

Exp. 15.—Take two bar magnets of the same size and strength, and place them together as in Fig. 13. Then bring either of the pairs of poles, N_1S_2 or N_2S_1, near the pole of a compass needle; note that they will produce much less deflection than a single pole of one of the magnets would have produced, and a similar effect will be observed if we test them by their inductive action on a little steel rod as in Exp. 6.

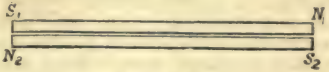

Fig. 13.

It thus appears that the two magnets tend to neutralise one another *so far as the external field* is concerned.* Of course there is no diminution of polarity of *the magnets themselves*, it is simply that one pole more or less neutralises the *effect* of the other so that the pair possess very little *free* polarity: in all cases it is *free polarity* that alone can produce an external field. A similar phenomenon is observed with the arrangement in Fig. 14, or with any number of magnets of the same strength disposed so as to form a closed geometrical figure, the north pole of each touching the south pole of the next; such an arrangement possesses comparatively little free polarity.

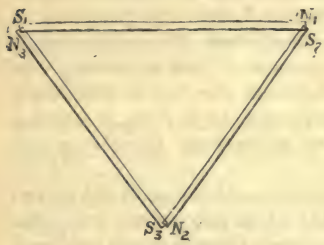

Fig. 14.

Any such arrangement is called a *closed magnetic chain.*

It is easily understood that with ordinary bar magnets we can never get *complete* neutralisation of external effect, partly because their thickness prevents their ends coming into contact at all points, and partly on account of lateral polarity (§ 13). But if the magnets were simple magnets (§ 13), a closed chain made up of them would produce no external effect whatever. In other words, *a simple closed magnetic chain has no free polarity.*

Suppose now that in a closed chain, *e.g.* Fig. 14, we separate two of the poles, say N_1 and S_2, then the free polarity is at once

* Any region in which magnetic action is exerted is called a magnetic field.

increased and the external field strengthened: the chain is now said to be *open*.

Exp. 16. Take a bar magnet and support it horizontally. Hang on from the N pole a number of iron or steel nails. Take a similar bar magnet, hold it horizontally with the S pole resting on the S pole of the fixed magnet (the north poles of the two magnets pointing in opposite directions), and then gradually move it along towards the N pole of the fixed magnet. As it nears this N pole the nails gradually drop off, showing that magnetic chains are being linked up through the two magnets and hence there is a decrease of free polarity.

16. Molecular Theory of Magnetism. This theory, which is the one now universally adopted and has been much elaborated by Professors Hughes and Ewing, supposes that *every molecule of a magnetic substance is a minute simple magnet.* How it comes about that such is the case is not known*, but the property is supposed to belong to *the molecule itself*, and the molecules of an unmagnetised steel or iron bar possess it just as strongly as those of a magnet. The difference between the two is that *when unmagnetised the molecules are so arranged as to form a number of closed magnetic chains, while when magnetised the chains are more or less open.*

The exact arrangement of the molecules in either case is unknown; in an unmagnetised bar it is probably of an altogether irregular character, while in an ordinary bar magnet there is much evidence

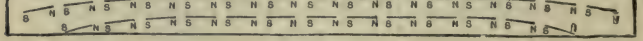

Fig. 15.

that the arrangement is as Fig. 15. In this figure each little line N S represents a molecule, and for convenience of drawing, gaps are left between the N of one molecule and the S of the next, though in reality they are supposed to touch. It will be seen that throughout the interior of the magnet every molecular pole touches an opposite pole of its neighbour, so that the two neutralise one

* See § 231 for Ampère's and the modern theory.

another and the interior possesses no free polarity; on the other hand, at the ends and along the sides all the molecular poles are free, which accounts for the terminal and lateral polarity of the magnet; the accumulation of free molecular north poles at the end A makes that end the N pole of the bar, while the free molecular south poles at B make that the S pole.*

Fig. 16 shows an ideal arrangement of the molecules in a magnetised bar; it corresponds to the absence of lateral polarity, and unless (§ 13) the bar is very thin does not fairly

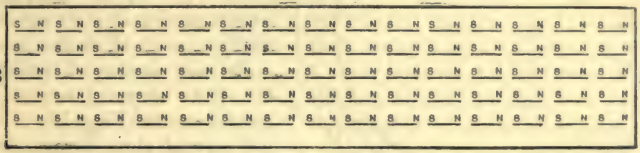

Fig. 16.

represent its true state; for convenience it is, however, usually adopted when, as frequency occurs, we are not concerned with the lateral polarity.

The molecular theory at once explains the effect of breaking a magnet (§ 14). For since the molecules are by their very nature unbreakable, the fracture must take place *between* and not *through* them; referring, then, to Fig. 15 or 16, it is clear that each fractured face must contain an accumulation of free molecular poles, north on one and south on the other. The sudden freeing of these two sets of poles which, prior to the fracture, neutralised one another, accounts for the development of strong free polarity at parts of the magnet where previously none existed.

17. Internal and External Magnetisation. Circumferential Magnetisation. If we contrast Fig. 15 or 16 with the irregular arrangement of the molecules in an unmagnetised bar, we see that although the interior of a magnet possesses no free polarity it is in a very different state from the unmagnetised bar; it possesses in fact what is known as *internal magnetisation*, the free polarity on the surface being termed *external magnetisation*.

* In the diagram only five chains of molecules are shown, but of course in reality there are many millions.

As we have seen (§§ 14, 16), as soon as the magnetised bar is broken or cut across, some of its internal magnetisation becomes external and manifests itself by acting on a compass needle, etc., but breaking an *un*magnetised bar does not produce any such effect.

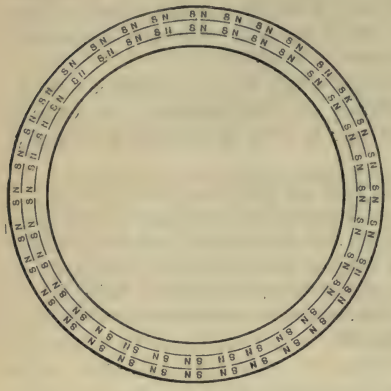

Fig. 17.

If we take a steel ring all cast in one piece it is possible by suitable treatment to magnetise it *circumferentially*, *i.e.* so that the molecules are arranged parallel to the circumference all round as in Fig. 17, without any of their poles abutting on the sides of the ring. In this case the magnetisation is *entirely* internal, and no matter how strong it may be the ring produces absolutely no external field, and affects a compass-needle merely as a piece of neutral steel would do. But if we file through the ring at any part and then slightly separate the cut faces they at once exhibit free polarity, one northern and the other southern, and produce the usual effects. If we reclose the ends these effects again in a great measure disappear, but never entirely, as the process of filing displaces some of the molecules and develops more or less lateral polarity near the cut.

18. Other modes of Magnetisation. In addition to the modes of magnetisation indicated in Figs. 15, 16, 17, there are innumerable others all of small importance. Thus it is possible to magnetise a bar so that its molecules set *sideways* as in Fig. 18. The poles of such a magnet would be its *faces* A B and C D, and when suspended so as to be free to turn horizontally, A B would face northwards and C D southwards. Such magnets are rarely met with and are difficult to make.

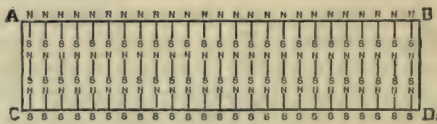

Fig. 18.

With regard to the bar and other magnets hitherto considered, they have been what may be termed *regularly* magnetised. The characteristic of such is that their poles are of equal strength and that their lateral polarity is uniformly distributed over the two halves, the magnetic equator being in the middle (§ 13). But it is quite possible to make *irregular* magnets for which these statements are not true.

19. Explanation of Magnetisation. Magnetising Force. Magnetic Saturation. According to the molecular theory the process of magnetising a piece of iron or steel whether effected by simple induction or by any of the methods to be subsequently considered, consists in making its molecules rotate from a position in which all their chains are closed to one in which at any rate some of them are open, and as a somewhat curious piece of direct evidence in favour of this may be mentioned the facts that a bar is found to increase in length very slightly during magnetisation, and also that when suddenly magnetised by a strong electric current (§ 207) it gives a sharp click: both these circumstances appear due to some kind of molecular rearrangement. Exp. 22 furnishes further support of this view.

Any influence tending to magnetise iron or steel is termed a **magnetising force**, and such a force exists in every magnetic field. Suppose now a neutral bar of iron or steel be subjected to a weak magnetising force, it acquires weak polarity; if then the magnetising force be increased it acquires stronger polarity, and so on. But the acquired polarity *does not increase indefinitely with the magnetising force*, there is a certain point which varies for different specimens, beyond which it is impossible to increase the polarity however great the magnetising force be made; this point is called **magnetic saturation**, and when the specimen has attained it it is said to be *saturated*. The explanation is that as the magnetisation increases *as more and more molecular chains are opened*, so soon as they are *all* opened of course the process must stop. *Magnetic saturation thus corresponds to the opening out of all the molecular chains.*

20. Explanation of Demagnetisation. When for any reason a magnetised bar sustains a diminution or loss of magnetisation we must, according to the same theory, regard it as due to a closing up of some or all of the molecular chains that were previously open.

All magnets tend to become weaker with time, so that this closing tendency would appear to be always present; its cause is doubtless the mutual attractions between unlike poles of neighbouring molecules, though the exact manner in which these attractions operate is often difficult to trace.

21. Molecular Rigidity. We have already seen (§ 9) that it is more difficult to magnetise steel than iron, but that when magnetised it remains so far longer than iron. This is explained by supposing that magnetic substances offer a certain opposition to the rotation of their molecules which varies in different specimens, being very small indeed in soft iron but considerable in steel. The opposition in question is sometimes called **coercive force**, but this expression is open to objection: in place of it the term **molecular rigidity** is now more commonly employed; it is a mechanical property closely allied to that upon which hardness depends, for the harder a specimen of iron or steel the more difficult it is either to magnetise or demagnetise—that is, the more opposition it offers to the free movement of its molecules.

22. Retentivity. The power which a substance possesses of "retaining its magnetism" is called its *retentivity*; it is in general a consequence of molecular rigidity. There is, however, a kind of artificial retentivity or retention which does not depend upon molecular rigidity at all, but upon *making the magnet part of a closed magnetic chain.*

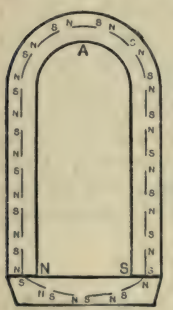

Fig. 19.

23. Keepers afford an example of this artificial retentivity. When a permanent magnet (steel) is left to itself there is a tendency for the molecules to leave their regular order and form little closed magnetic chains, thus lessening the effective magnetism of the magnet. This tendency is prevented by the use of keepers, which are pieces of soft iron used to join up poles of opposite natures. Fig. 19 shows a horseshoe magnet N A S fitted with a keeper I I'. The magnetic chains in the magnet would, if the magnet were left to itself, gradually break down, but when the keeper is placed in position it is magnetised by induction and the magnetic chains are

continued right through it. Closed magnetic chains have little tendency to rearrange themselves, hence the magnet retains its magnetism for a long time.

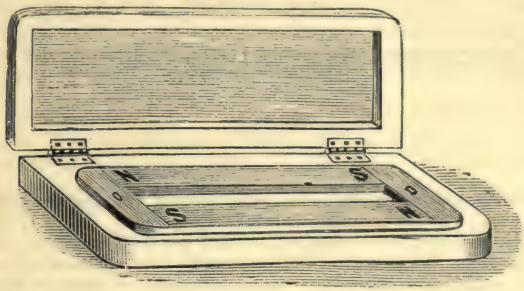

Fig. 20.

It is usual to put the horseshoe magnet away in a box (or on a hook) with the keeper across its poles. With bar magnets the practice is to keep them in pairs, generally with a strip of wood between and a keeper at each end, as in Fig. 20.

In § 15 we saw that a closed magnetic chain produces a very weak effect in the external field, hence we should expect that a pair of unlike magnetic poles with a keeper across them would exert but slight external action. That this is the case may be shown as follows :—

Exp. 17.—Set a compass-needle on a pivot so that it is free to swing. It sets N and S of course. Hold some little distance from it a horseshoe magnet without a keeper so that it deflects the needle considerably. Then place the keeper across its poles ; note that the needle goes back nearly to its original position. (See also § 45.)

24. Consequent Poles. Magnets are sometimes made, either accidentally or intentionally, which instead of having simply a north pole at one end and a south at the other contain other poles. Thus there may be a north pole at each end and a south somewhere between ; or there may be a north at one end, a south at the other, and two intermediate ones, as in Fig. 21, etc. All such intermediate poles are termed *consequent poles*. There can never be two poles of the same kind without one of the opposite kind somewhere between them.

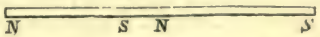

Fig. 21.

The explanation of consequent poles according to the molecular theory is made clear in Fig. 22. Here A B represents a magnet with a north pole at each end and a south pole at some intermediate point, C; it will be seen that at C the south poles of two adjacent sets of molecules face each other;

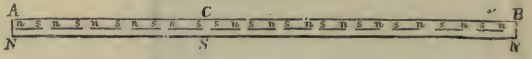

Fig. 22.

in other words, two molecular south poles follow one another instead of a north following a south as in the normal arrangement. This is the origin of the word "consequent" applied to such poles (the literal meaning is "following on"). We have, in fact, practically two magnets, A C and C B, with their south poles touching. Now, two poles of *opposite* kinds when placed together tend to neutralise each other so far as the external field is concerned, but the reverse is the case with two of *the same* kind, and hence it is that the point C exhibits strong southern polarity.

25. Explanation of Magnetic Induction. Magnetic induction is readily explained by the molecular theory. Thus, in Fig. 15 consider to begin with the bar A B in its neutral state with its molecules arranged irregularly in closed chains, and suppose the south pole of a magnet then held near A. By the law of attraction and repulsion (§ 6) this south pole tends to turn all the molecular north poles towards A and all the molecular south poles towards B, thus making them take more or less the positions in Fig. 15 or 16, so that A becomes a north and B a south pole in accordance with the law of induced polarity (§ 10).

In the case of soft iron the smallness of the molecular rigidity permits the molecules to turn round easily throughout the entire length of a fairly long bar, so that magnetisation by induction is easy. But in hard steel the molecules turn but slightly, and we cannot effectually magnetise a steel bar by simple induction; to do so we must either move the inducing pole so as to bring it successively into the immediate neighbourhood of all the molecules (§ 26), or we must do something temporarily to diminish the molecular rigidity (§ 27), or, which is by far the best method, we must surround the whole bar by a strong electric current which places every molecule in a powerful magnetic field (§ 207).

26. Magnetisation of Steel by Rubbing. The simplest way of magnetising a steel bar is to rub it in a suitable manner with a magnet or magnets. There are three so-called "methods" of doing this; they differ only in detail. The principle in each is the same—*viz.*, to bring each molecule of the steel in turn into the strongest part of the magnetic field in such a way as to make it turn in the desired direction. We shall consider the methods in order:—

Method of Single Touch. This is shown in Fig. 23. A B is the bar to be magnetised. Suppose we wish the end A to become a north pole, we take a bar magnet, N S, and keeping it

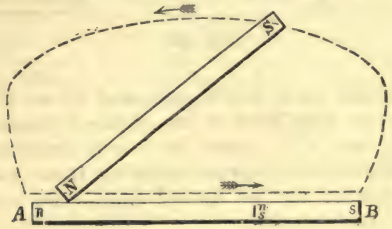

Fig. 23.

in a somewhat inclined position, draw its north pole from A to B, repeating the process a great many times, *but always stroking A B in the same direction.* Each time N arrives at B it is lifted and brought back at a distance from A B, so that on the whole it describes the dotted line in the direction of the arrow.

The method may be modified by employing a horseshoe instead of a bar magnet, and stroking the bar from A to B with its north pole, and from B to A with its south pole. It will be observed that in all cases *the end of the bar where the stroking pole leaves it is of the kind opposite to that pole.*

The explanation of the action is as follows:—In Fig. 23 consider a single molecule *n s*, of the bar A B. This, to begin with, forms part of a closed chain. As N *approaches* this molecule, it turns *n* somewhat towards B, but as it *leaves* it it turns *n* towards A. Thus every time N passes along, all the molecules of the bar are left with their north poles inclined towards A. The inclination produced at each stroke is small on account of molecular rigidity; but it accumulates, and after several strokes the bar is distinctly magnetised. It should, however, be noticed that *the approach of* N

towards any molecule partly reverses the effect of its previous recession, so that after N has passed over it, it is left less inclined than it otherwise would be; thus the method is prone to produce weak magnetisation.

Method of Divided or Separate Touch. This is shown in Fig. 24. Supposing we require the end A of the bar A B to become a north pole, it is supported on the poles of two fixed

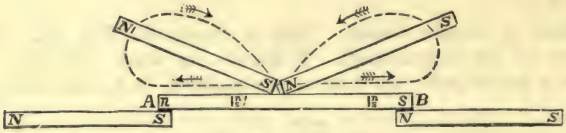

Fig. 24.

magnets, A resting on a south pole and B on a north. Two other magnets are then held in an inclined position with their poles as shown, and with these the bar is stroked a good many times from the middle outwards, bringing them back well above the bar, as indicated by the dotted lines: the final effect is to magnetise the bar as shown.

The explanation is similar to that of the method of single touch:—Every time the stroking pole N passes over a molecule *n s* it *on leaving it* inclines its north pole towards A, and the same is true of S as it passes over a molecule *n' s'*. The stationary magnets help to keep the molecules in the inclined positions acquired, and thus to prevent the stroking pole reversing its previous action on *approaching* the molecule; the method thus produces stronger magnetisation than the preceding one.

Method of Double Touch. This is shown in Fig. 25. The arrangement is similar to that of the method of divided touch,

Fig. 25.

except as respects the movement of the stroking poles. The latter are kept a little distance apart by means of a pellet of wood or cork, and are moved *both together to and fro along A B*;

thus starting at the middle, we move the pair of them to B, then back from B to A, then from A to B, and so on, finishing off at the middle.

Again the explanation is very similar : every time a molecule *n s* comes under the space between the stroking poles, it is in a magnetic field tending to incline its north pole towards A, but this field being much stronger than that at command in either of the previous methods, the resulting magnetisation is in general stronger. It will, however, be seen by reference to the diagram that the weak part of the field beyond the poles tends to turn the molecule the wrong way, not only as the pair of poles approaches, *but also as it leaves* ; for this reason there is a possibility of the method producing consequent poles (§ 24), though as a matter of fact this rarely happens.

Exp. 18. *Make a magnet.* Employ the method of single touch. Stroke a knitting-needle 30 or 40 times from one end to the other with one pole (say that marked N) of a permanent magnet. Slowly rotate the needle about its longitudinal axis during the operations so that the stroking pole may move along different parts of its surface. Test the magnetism by dipping the needle into iron filings or tin-tacks. Observe by testing as in Exp. 5 that the polarity of the end at which stroking began is the same as that of the stroking pole.

Exp. 19. Make a magnet with consequent poles. Stroke a needle with one pole of a bar magnet, beginning at each end and finishing at the middle. Test with filings as before.

In all these methods the action is mainly confined to the superficial layers of the bar ; indeed, they are only well adapted to thin ones, such as knitting-needles. For thick bars the best of them is the modified method of single touch, using the poles of a very strong electro-magnet. But in no case can the inner layers of a thick steel bar be effectually magnetised. The strongest permanent magnets are of the kind shown in Fig. 26 : they consist of a number of thin steel bands which have been magnetised separately, and then clamped or riveted together with their similar poles side by side. Such magnets are termed *compound* or *laminated* magnets.

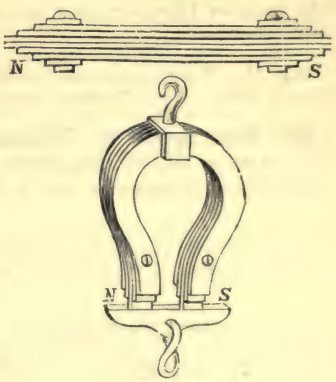

Fig. 26.

Exp. 20. *Make a Compound Magnet.* Separately magnetise seven or eight knitting-needles. Test each for consequent poles, and neglect any needle that has a consequent pole. Bind the magnetised needles together so that the poles are all at one end.

27. Effect of Heat and of Agitation. Every blacksmith knows that heat softens iron and steel. It diminishes their molecular rigidity, and we should therefore expect it would facilitate magnetisation and demagnetisation. This is actually the case. If we make a steel bar red-hot and hold it between the poles of a strong horseshoe magnet, allowing it to cool in that position, it is found afterwards to be well magnetised. Unfortunately, however, the gradual cooling softens it, so that it will not retain its magnetism well, and the method is therefore not a good one for the manufacture of magnets.

If a magnetised steel bar be made red-hot and then cooled away from magnetic influence it becomes demagnetised; here the molecular chains are open to start with, the heat diminishes the molecular rigidity, and they close up in accordance with their natural tendency (§ 20).

The effect of agitation is very similar. If we place one end of a knitting-needle against a pole of a bar magnet and then hammer it so as to temporarily loosen its molecules while under the magnetising influence, it is afterwards found to be more effectually magnetised than if it had been left quiescent. In like manner hammering a magnet or allowing it to fall tends to demagnetise it.

Exp. 21. Take a magnetised needle. Make it red-hot in the fire or a Bunsen flame. Test its magnetisation when cool. It has vanished.

28. Experiment in Illustration of the Molecular Theory. The explanation of the difference between magnetised and unmagnetised substances, set forth in § 16 as due to the turning round of the molecules, is strikingly illustrated by the following experiment.

Fig. 27.

Exp. 22. Take a thin narrow glass tube, *a*, Fig. 27, closed at both ends, and containing unmagnetised steel filings. Test it by a delicate magnetic needle as in Exp. 5, to make sure that it is destitute of polarity. If the filings be looked at closely they will be seen to be arranged irregularly. Now keep the tube firmly fixed, and stroke it from one end to the other several times with the north pole of a bar magnet, precisely as in the method of single touch: on watching carefully, many of the filings will be seen to place themselves

lengthwise (*b*, Fig. 27). Now test the tube by the magnetic needle: it will be found to possess distinct polarity, the end at which the stroking N pole left it being a south pole. Next shake the tube vigorously so as to make the filings resume their irregular arrangement and again test; all trace of polarity will be found to have disappeared.

The filings, therefore, behave both during magnetisation and demagnetisation very much as we have conceived the molecules of a bar to do.

In an improved variation of this experiment a glass tube containing fine iron particles suspended in water is employed. The experiment is done in front of an optical lantern with the tube end-on to the light. Before magnetisation no light can get through the tube owing to the irregular arrangement of the particles. A current is now sent through a coil surrounding the tube, so that the magnetic field produced is in the direction the light is travelling. Immediately the light flashes up, showing that the particles have now set themselves with their lengths in the same direction.

SUMMARY.—CHAPTER II.

1. Effect of breaking a magnet (§ 14).

2. Magnetic chains. *A simple closed magnetic chain has no free polarity* (§ 15).

3. The molecular theory of magnetism. Cause of difference between unmagnetised and magnetised iron or steel, according to the molecular theory. Explanation of terminal and lateral polarity (§ 16).

4. The *interior* of a magnet possesses magnetisation but not free polarity. Circumferential magnetisation (§ 17).

5. Explanation of magnetisation and of magnetic saturation (§ 19); also of demagnetisation (§ 20).

6. The magnetic difference between iron and steel due to the greater molecular rigidity of the latter (§ 21).

7. Two kinds of magnetic retention, one due to molecular rigidity the other to the formation of closed chains (§ 22). Keepers promote the latter kind of retention (§ 23).

8. Consequent poles (§ 24).

9. Methods of magnetisation by rubbing (§ 26), especially the modified method of single touch in which both poles of a horseshoe magnet are used.

EXERCISES II.

1. A steel bar magnet and a steel knitting needle, one end of which has been notched with a file, are given you. Describe exactly what you would do in order to magnetise the knitting-needle in such a way that, when hung by a fine thread fastened to the middle, the notched end of the needle shall turn southwards.

2. Three equal bar magnets without keepers are given you. How would you arrange them so that, when not in use, they might preserve their magnetism? Give a sketch.

3. A glass tube with its ends marked A and B, and nearly full of steel filings, is stroked several times from A to B with the north-seeking end of a strong magnet. The tube is then brought with its end B near to the south-seeking pole of a compass-needle. What is the effect upon the needle?

The tube is now shaken so as to mix the filings, and put near the needle as before. What is the effect upon the needle? Why is the effect on the needle different in the two cases?

4. A horseshoe magnet is placed near a compass-needle so as to pull the needle a little way round. On laying a piece of soft iron across the poles of the horseshoe magnet, the compass-needle moves back towards its natural position. Explain this.

5. It is suspected that a magnetised bar of steel has consequent poles. How would you ascertain whether this is so or not?

6. A bar magnet which has two consequent poles at its middle point is broken into two pieces. What will be the magnetic condition of each piece according as the break is made exactly at the middle point or at some distance from it?

7. A sewing-needle is magnetised by stroking it from the eye to the point with the north pole of a steel magnet. How will the needle be magnetised? If it is then broken into three equal pieces, describe and explain the magnetic state of the fragments in relation to their positions in the unbroken needle.

8. Two rods of the same size, one of soft iron and the other of hard steel, are each rubbed from end to end with one pole of a strong bar magnet. How will the rods affect a compass-needle to which they are successively brought near?

9. Distinguish between terminal and lateral polarity, and show how the existence of both is explained by the molecular theory.

10. A straight steel watch spring is magnetised and is then bent so that its ends are in contact. What effect would it have on a compass-needle to which it was brought near? And how would this effect be modified by the spring being cut into two parts while the original ends were still held together?

11. Two bar magnets are placed with the north pole of one against the south of the other: prove that although these poles weaken each other's effect on the external field they tend to preserve each other's actual polarity.

12. Two bar magnets are placed with the north pole of one against the north pole of the other: prove that though these poles strengthen each other's effect on the external field they tend to destroy each other's actual polarity.

13. Describe how you would by the method of single touch (i) magnetise a knitting-needle so as to have a south pole at each end and a north in the middle; (ii) magnetise a ring circumferentially.

14. If a common magnet be placed in nitric acid for a short time it is found to have lost a great deal of its magnetism: explain this.

15. A glass cylinder with flat ends contains water in which is suspended finely divided magnetic oxide of iron. The tube has a coil of copper wire wound round it, and is fitted in a lantern so that the light falls on one of the flat ends. In front is placed a screen, and the circle of light on it is very dull. An electric current is now passed round the copper wire, when the light becomes much brighter. Explain this.

16. Would it do to employ keepers of hard steel? Give reasons for your answer.

CHAPTER III.

MAGNETIC FIELD, FORCE, AND FLUX.

29. Different kinds of Field. The student will from the preceding chapter be already familiar with the term **magnetic field**; it simply means any region subject to magnetic influence. The simplest field is that due to a single point-pole such as belongs to a simple magnet (§ 13); strictly, however, it can exist only in imagination, for however thin a magnet may be it still falls short of the ideal simple magnet, and, besides, we cannot obtain a magnet with one pole only. But we can obtain a very close approximation to such a field by using a long and fairly thin magnet and confining our attention to the region near either pole, which is practically uninfluenced by the other.

The next simplest field is that due to two point-poles either of the same or of opposite kinds. The field round an ordinary bar magnet is an approximation to the latter, while an approximation to the former may be obtained by placing two long bar magnets in a line, with like poles facing each other, the region between and near these poles being practically due to them alone. By placing two magnets in irregular positions, or by using several magnets, magnetic fields of varying degrees of complexity may be obtained.

The region outside a magnet or magnets is called the *external field*, that in the material of the magnets themselves the *internal field*. Whenever the field simply is spoken of, the *external* field is usually intended.

30. Force in a Magnetic Field. Consider any magnetic field, and imagine an isolated north pole * placed at a given point P in that field: it will experience a force in a certain direction and of a certain magnitude. The magnitude of the force depends, among other things, upon the strength of the pole; but its

* That is "point-pole"; in future the context will in general indicate where the word "pole" is to receive this interpretation.

direction, with which alone we are at present concerned, depends only upon the nature of the field and the position of the point P.

Now we must be very careful what we mean when we speak of the "direction" of a force. A force may, for example, pull straight up or straight down; in both cases, in so far as it acts along *the same line—viz.*, a vertical one, its direction may be said to be the same, but the *way it pulls* is precisely opposite in the two cases. In Mechanics the word "sense" is often employed to include both the line along which a force acts and the way it pulls, but in magnetism it is more usual to make the word *direction* include both these: thus we should speak of a force as having an upward direction or a downward direction.

Now if at our point P we place a *north* pole, it will be urged in a certain direction, but if at the same point we were to place a *south* pole it would be urged in a precisely opposite direction; when, however, any consideration is involved, not merely of the line along which the force acts, but of the way it pulls, it is invariably understood, in speaking of *the direction* of the magnetic force at a point, that we mean the direction in which a *north* pole placed at that point would be urged. Sometimes for the sake of emphasis this is called the *positive* direction; the term, however, is hardly needed.

The force with which we are dealing is often called the *resultant* or *total* force at the point, in order to distinguish it from certain partial forces or "components."*

Since in practice it is impossible to obtain an isolated pole, what we do is to employ a *very short, straight, and thin* freely suspended† magnetic needle; both its poles are then practically at the same point, and it sets itself along the line of the magnetic force at that point *with its north pole pointing in the (positive) direction of the force.* Such a needle, when used for examining the direction of the force at different points of a field, is spoken of as an *exploring* needle or *explorer.*

* At this point a student not familiar with the composition and resolution of forces should refer to the Appendix (§§ 245, 246) or for a fuller study to a text-book of mechanics. The knowledge will be required later.

† A magnetic needle is said to be *freely suspended* when it is free to turn in all directions—*e.g.*, when supported at its centre of gravity by a piece of fine silk or cotton; a compass-needle (Fig. 5) is not freely suspended, for its mode of mounting restricts its movements to the horizontal plane.

31. Exploration of a Field. The general method of using the explorer will be understood from Fig. 28, where it is shown in various parts of the field of a bar magnet. Only one explorer is

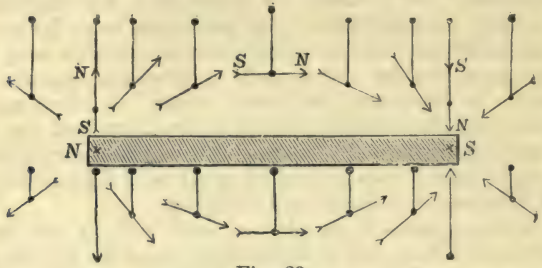

Fig. 28.

used; it is moved about from place to place, its direction then altering. The arrow-tip represents the north pole of the explorer, and in all positions it is pointing in the direction of the resultant force. It will be observed that at the magnetic equator the line of the force is parallel to the bar, while near either pole it passes practically through one of the points marked ×. These two points are sometimes erroneously called the "poles" of the magnet (cf. § 41).

32. Lines of Force. The explorer mentioned above can be used at all points in the neighbourhood of a magnet. There are other more simple means by which we can explore the field in any given horizontal plane, and this is in practice all that we want. Also the field about any simple magnet is symmetrical, so that if the field is explored in any one plane which contains the magnet we know the distribution of magnetic force in all other planes which contain the magnet. If, then, we keep to one plane the easiest and most striking way of exploring a magnetic field is by means of fine iron filings, as described in the following experiment.

Exp. 23. To explore the magnetic field in the neighbourhood of a magnet. Place a sheet of glass or stiff white cardboard over the magnet (or magnets), and then loosely and evenly scatter filings over it from a pepper-box or flour-dredge provided with fine holes. Each filing becomes by induction a little magnet, and is ready to act as an explorer provided it were free to turn. This, however, it cannot do on account of the friction of the sheet.

Therefore gently tap the latter so as to give the filings a little more freedom; each one then sets itself in the line of the resultant magnetic force at the particular point it occupies, and remains in the position thus acquired. Now it is natural that filings which are near to one another should in most cases point in nearly the same direction. The filings therefore arrange themselves in definite curves (cf. Figs. 29, 30): these curves are called *lines of force.**

Now considering any point, P, on one of the curves it is obvious that the filing at P lies along the tangent † to that curve at P; whence, since the filing also lies along the direction of the magnetic force at P, it follows that *the tangent to any one of the curves at any point on it is in the direction of the magnetic force at that point:* this is a most important property and the one whence the curves derive their name. It should be noticed, too, that in the strict sense of the term the "lines of force" are mere geometrical lines in the field possessing the property in question, and exist *in the absence of the iron filings;* the latter serve merely to render them visible to the eye. The term *magnetic curves* is sometimes used to distinguish the lines thus rendered visible from the mere mathematical lines.

Lines of force have numerous uses in the higher part of the science of magnetism, and also in electrical engineering; their immediate and most valuable elementary characteristic is that by virtue of the foregoing property they afford us a bird's-eye view of the direction of the magnetic force at any and all points of the field.

It should be noticed that the lines of force in any field always *run clear of one another, i.e.* no two can intersect, for if they did it would indicate that the resultant force at the point of intersection was in two different directions, which of course is absurd.

In § 30 we have explained what is meant by the (positive) direction of the force at a point in a field. In like manner

* To fix the filing chains spray gum mastic over the cardboard and allow to dry, or use paper that has been soaked in paraffin wax and brush over with a Bunsen flame or warm in hot air. Allow to cool or dry before removing the magnets. Another way is to use smooth photographic printing out paper, expose to sunlight till the paper is darkened; then shake off the filings and fix in hypo.

† Lines of magnetic force are very similar to those of electric force treated in Part II., Ch. X.; the student will find it instructive to compare the two.

by the (positive) direction of a *line of force* is meant the direction in which an isolated north pole would travel along it if free to move, or that in which the north pole of an explorer or a filing points.

In examining a field by an explorer we can always tell the positive direction because the north pole of the explorer is marked, but this is not the case with the filings; usually, however, we know beforehand which is the general positive direction (*e.g.* in the field of a bar magnet we know that it runs *from* the north *to* the south pole), and merely want the filings to show the specific forms which the lines take. If we use as an explorer a small compass-needle we get the direction of the force as well as forms of the lines.

It is usual in diagrams to employ arrows to indicate the positive direction of the lines of force (cf. § 34).

33. Special Cases of Lines of Force. The form of the lines of force varies according to the nature of the field. The simplest field is that due to a single pole: they are then straight lines

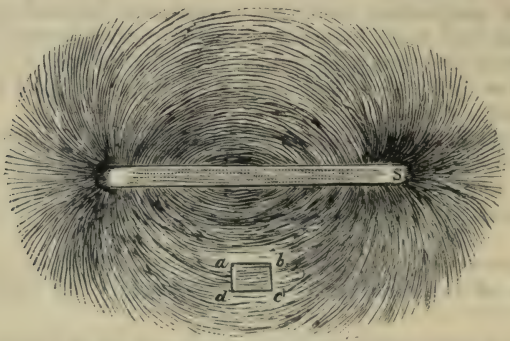

Fig. 29.

passing through the pole. If the pole be a north one their positive direction is *from* it, if a south one *towards* it. They may be approximately shown by setting a long bar magnet vertically, placing a piece of cardboard over its upper end and

sprinkling filings on it: on gently tapping the cardboard the filings will arrange themselves in straight lines passing through the pole.

A very important field is that due to an ordinary bar magnet, and Fig. 29 shows the lines of force in this case.* They form a series of oval curves many of which strike not the ends but the sides of the magnet; this is on account of lateral polarity (§ 13). With a very thin magnet most of them strike the ends, and in the ideal simple magnet they would *all* do so (see Fig. 43).

Fig. 30 depicts the field of a horse-shoe magnet. The region above the line joining the poles is similar to that between the poles in Fig. 29, while between the prongs the lines of force are nearly straight: here and also on the outer sides of the prongs we see the effect of their lateral polarity.

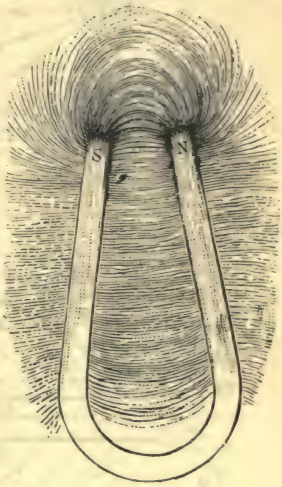

Fig. 30.

It will be noticed in these figures that no filings collect *actually over the magnets*, or at any rate they do not there arrange themselves in definite curves. This is sometimes said to be because there are in this part of the field no lines of force, but that is of course absurd, the whole region round the magnets is a magnetic field, and lines of force strike them on the top just as much as on the sides. But as these lines are in vertical planes the magnetic action would tend to make the filings stand more or less on end, and this their weight prevents them from doing, consequently they fall into positions unfavourable to the inductive action of the field, and simply get shaken off. If, however, the magnets be very strong it is quite possible to get a good many of them to stand on end in the way described.

34. How to sketch Lines of Force. Students are frequently required to make neat sketches of the lines of force in simple magnetic fields. It is best to make them diagrammatic rather than pictorial, and we append diagrams for the most important

* The piece of iron *a b c d* shown in the figure is for another purpose: see § 35.

Fig. 31.—Single North Pole.

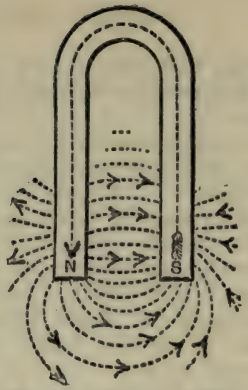

Fig. 32.—Horseshoe Magnet.

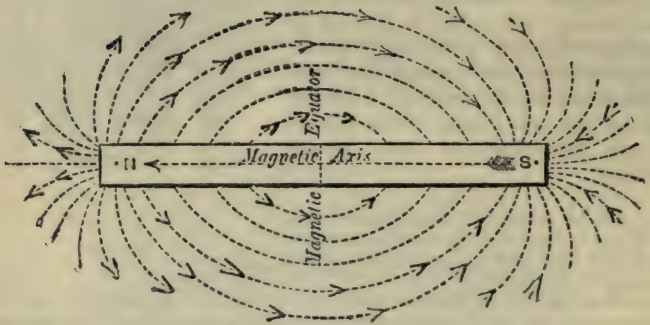

Fig. 33.—Bar Magnet.

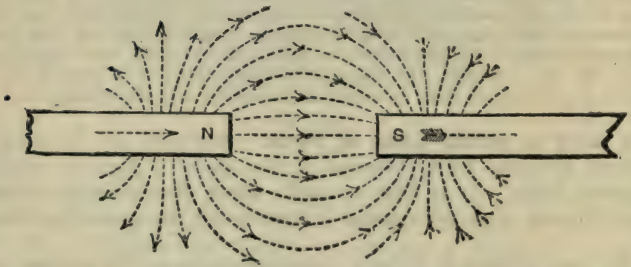

Fig. 34.—Between two *Unlike* Poles.

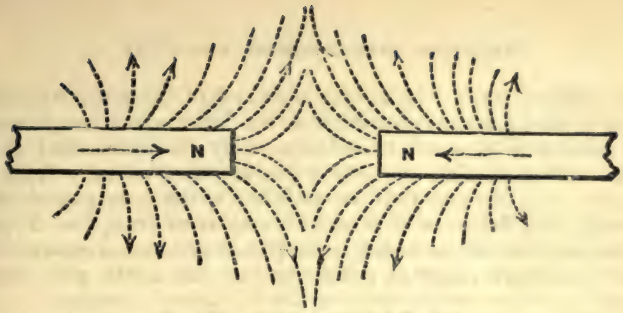

Fig. 35.—Between two *Like* Poles.

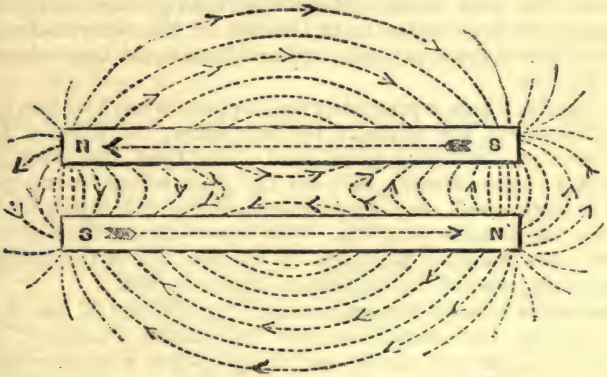

Fig. 36.—Pair of Parallel Opposed Bar Magnets.

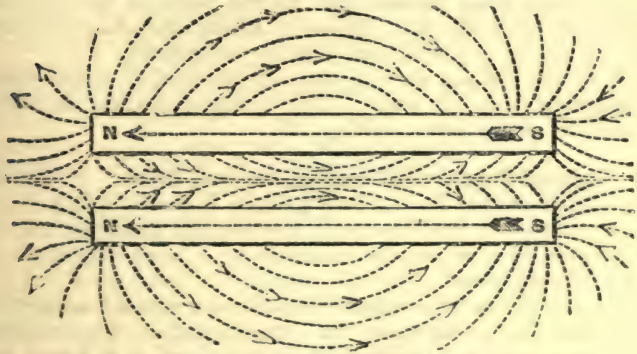

Fig. 37.—Pair of Parallel Concurrent Bar Magnets.

fields (Figs. 31-37). It is very essential in all cases to notice which is *the* (*positive*) *direction* of the lines, and when this has been settled it is usual to indicate it by arrows marked along them. Thus in Fig. 33, which is the diagrammatic form of Fig. 29, it is obvious that an isolated north pole placed anywhere in the field would tend to travel away from the N pole of the magnet, and accordingly we put in arrows, as shown; the arrow is always made to point the way the north pole would travel.

In Fig. 34 the two unlike poles are the near ends of two magnets; the lines in the centre of the field go from one pole to the other, the lines on the right and left of the centre belong to magnets each taken separately, and are distributed approximately as in Fig. 33.

In Fig. 35, which represents what happens when two like poles of two magnets are adjacent to each other, the lines of force from each of the facing poles bend away from the other, and there is a region directly between the poles where the magnetic force is very small, and where consequently the filings do not take up two definite positions. The curves on the right and left of the poles belong to the parts of the field where the influence of the other poles of the magnets is distinctly felt.

The curves *between* the magnets in Figs. 36 and 37 are somewhat peculiar, but comparison with the standard cases of Figs. 33, 34, and 35 should enable us to recall them when necessary; thus in Fig. 37 the lines between N and N resemble those in Fig. 35, while those between the central parts of the magnets resemble those in Fig. 33. Reference to the same standard cases often enables us to predict the lines for other fields.

Iron filings will not serve to map out the field at points where the magnetic force is weak. Tiny compass-needles, delicately pivoted, are of use here (see Exp. 32).

35. Induced Polarity in Terms of Lines of Force. In § 10 we have given a simple law for the polarity induced in a piece of iron or steel when subjected to a magnetising influence, and we now proceed to consider the same law in a more extended and often preferable form.

In Fig. 7 (*a*) the big magnet N S will cause induced magnetism in the small bar as shown by the letters *n* and *s*. Now consider those lines of force of the inducing magnet N S which run

through the small bar. Since the lines of force on the right of the magnet N S run (following their positive direction) from right to left (see Fig. 33), they enter *n s* on the right and leave it on the left. Also the little bar *n s* acquires southern polarity on the right, *i.e. where the lines of force enter it.*

Now it is just the same whatever field we are dealing with, and wherever we place the iron which is to be magnetised by induction; *e.g.* in Fig. 29, if *a b c d* represent a piece of iron placed in the position shown, its side *a d* will acquire southern and *b c* northern polarity. We have then the following perfectly general law—*viz.* that when a piece of previously neutral iron, steel, or other magnetic substance is placed in a magnetic field *it acquires southern polarity where the lines of force enter it and northern where they leave it.*

36. Strength of Pole. Law of Force. Unit Pole. Conceive two point-poles, placed at a certain distance apart. Call them P and P′ (Fig. 38), and suppose they are both north poles. Then P repels P′ with a certain force, and P′ repels P with an equal force. For a given pair of poles this force can be proved by experiment (§ 63) to be inversely proportional to the square of the distance P P′. This is termed the *law of inverse squares.*

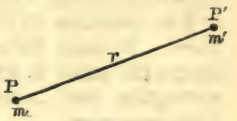

Fig. 38.

Again, for a given pair of poles at a given distance, the force will be definite in amount. Now, still keeping them at this distance, let us suppose one of them, P′, to remain always the same, while the other, P, is altered; then the force will alter, its magnitude depending simply upon P. It is agreed that this force shall be taken as the measure of P's strength: in other words, *the strength of a pole is defined as measured by the force it exerts on another given pole at a given distance from it, so that the said force is proportional to the strength.*

If, then, m be the strength of P, and F the force it exerts on P′ at the given distance, we have

$$F \propto m.$$

In like manner, if we keep P always the same, and alter P′, and if m' denote the strength of the latter, then (at the same distance) we shall have

$$F \propto m'.$$

Hence, by the law of compound proportion, if we alter both P and P′, but still keep them at the same distance apart, we get

$$F \propto m\,m' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Again, if we alter the distance (r), keeping the poles the same, we get by the law of inverse squares

$$F \propto \frac{1}{r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence finally, combining (1) and (2) by the law of compound proportion, we obtain

$$F \propto \frac{m\,m'}{r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

We have supposed that both poles are north poles. If both are south poles the result will be precisely the same; if one is north and the other south, the only difference will be that F, instead of being a force of repulsion, will be a force of attraction.

The relation (3) constitutes the complete law of magnetic force. It may be enunciated thus:—**The force between two magnetic poles is directly proportional to the product of their strengths, and inversely proportional to the square of their distance apart.**

So far the law is merely one of proportion; it remains to reduce it to an equation. Now (3) may be written

$$F = c\frac{m\,m'}{r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where c is a constant depending *only on the units of measurement and on the medium*, and not on the strength of their poles or their distance. Let us then agree to express r in centimetres and F in dynes,* and let us take as our **unit pole** a pole *of such strength that, when placed in air one centimetre from another pole of the same strength, it exerts upon it a force of one dyne.* If, then, we put $r = 1$ (cm.) and m and m' each $= 1$ (unit pole), we must have $F = 1$ (dyne). Hence from (4) we obtain

$$1 = c\frac{1 \cdot 1}{1^2},$$

i.e. $$c = 1,$$

* See Appendix, § 243.

whence (4) assumes the simpler form

$$\mathbf{F} = \frac{\mathrm{m}\,\mathrm{m}'}{\mathrm{r}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The unit pole is sometimes called the "unit of magnetism."

N.B.—In applying mathematical formulae to magnetism, the opposite kinds of polarity must be affected with opposite signs, northern polarity being usually taken as + and southern as − . If in (5) we take m and m' to be unlike poles this makes F negative. Thus the formula will be in strict accordance with the algebraic rule for products of positive and negative quantities if we regard F as a force of *attraction* when *negative* and a force of repulsion when positive.

EXAMPLES :—1. Two poles of strengths 4 and 6 are placed nine inches apart. Also two other poles of strengths 5 and 8 are placed four inches apart. Compare the forces exerted in the two cases.

Let F_1 denote the force in the first case, and F_2 in the second, then by (3) :—

$$\frac{F_1}{F_2} = \frac{\dfrac{4 \times 6}{9^2}}{\dfrac{5 \times 8}{4^2}} = \frac{\dfrac{24}{81}}{\dfrac{40}{16}} = \frac{24 \times 16}{40 \times 81}$$

which reduces to $\frac{16}{135}$; that is,

$$F_1 : F_2 :: 16 : 135.$$

2. A pole of strength 200 (*i.e.* 200 unit poles) when placed 4 cms. from another pole, P, is found to exert on it a force equal to the weight of 510 milligrammes. Find the strength of P.

Let x (unit poles) be the required strength. By (5) the force is $\frac{200x}{4^2}$, *i.e.* $\frac{25x}{2}$ *dynes*. But a dyne is equal to the weight of 1·02 milligrammes (Appendix, § 243), and therefore the force is equal to the weight of $\frac{25x \times 1{\cdot}02}{2}$ mgr. Hence

$$\frac{25x \times 1{\cdot}02}{2} = 510,$$

which gives $x = 40$

37. Intensity of Magnetic Field. Consider any point, P, in a magnetic field. Place an isolated point-pole at P. The force on the pole will depend on its strength and on the position of P. Denote its strength by m', and the force on it by F, then for a given position of P, $F \propto m'$, *i.e.* the *ratio*

$$\frac{F}{m'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

is independent of m'; it depends only on the field and the position of P. This ratio is called *the strength or intensity of the field at P.* Since it is independent of m' we may put

$$m' = +1,$$

i.e. (§ 36) unit north pole; then

$$\frac{F}{m'},$$

which is the intensity, becomes equal to F, the force. This gives the usual definition, viz. *the intensity of a field at any point is the force which the field would exert on a unit north pole placed at that point.*

It being agreed to estimate our forces in dynes, the intensity clearly represents so many *dynes per unit pole*, or, as they are sometimes called, *gausses*. The intensity at any point of a field is usually denoted by I, and is of course a *vector* quantity, *i.e.* has both magnitude and direction. Like all forces, it can be resolved on mechanical principles*, and if H be its component in a direction making an angle θ with its own direction,

$$H = I \cos \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Sometimes, for emphasis, I is termed the *total* or *resultant* intensity, and H the resolved or component intensity in the direction θ. A very important instance is afforded by the earth's magnetism, in which case θ is the dip, I the total, and H the horizontal intensity of the earth's field (§ 48).

If I be the intensity of the field at a point P, *i.e.* the force on a pole of 1 unit placed at P, then the force, F, on a pole of m' units placed thereat is given by

$$F = I\,m' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

* See Appendix, § 246.

and similarly, if H be the component intensity in the direction α, the resolved part of F in that direction is

$$Hm' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

If in (3) we make m' negative, *i.e.* a *south* pole, F becomes negative, indicating that *its direction is reversed*; it is for this reason that in the last definition of intensity we imagine a unit *north* pole placed at the point; this definitely specifies *the direction* as well as the magnitude of the intensity.

EXAMPLES :—1. The intensity at a certain point of a magnetic field is 20 dynes per unit pole ; find the force that would be experienced by a pole of strength 50 placed at that point.

On a unit pole the force is 20 dynes, hence on a pole of 50 units the force is 20×50, *i.e.* 1,000 dynes.

2. In the preceding question find the resolved part of the force at an angle 60° with the total intensity.

By mechanics the result is obviously 1,000 cos 60°,* *i.e.* 500 dynes or we may first resolve I, and then apply (4); the component of I is 20 cos 60°, *i.e.* 10, whence by (4) the required result is 10×50, *i.e.* 500 dynes.

38. Graphic determination of the Direction of the Total Intensity at any point of a Field due to Point-poles. This may be effected by means of the law of inverse squares (§ 36) and the parallelogram of forces.† It will suffice to consider the field due to a simple magnet (Fig. 39). Let N, S be its poles, and P the point in the field at which the intensity is required. Join P N, P S, and imagine a unit north pole at P; the latter will be repelled along N P and attracted along N S, and since N and S are of the same strength the forces of attraction and repulsion are inversely proportional to N P and S P^2 respectively. Produce N P, and mark off lengths P Q, P R, inversely proportional to N P^2, S P^2; then P Q, P R represent the two forces in magnitude and direction. Complete the parallelogram P Q T R, then by the parallelogram of forces *P T is in the direction of the total intensity at P.*

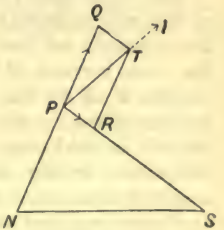

Fig. 39.

From this it is clear that *P T is a tangent to the line of force at P*; the student should roughly convince himself of this by

* Cos 60° = ·5. † See Appendix, § 245.

comparing Fig. 39 with Fig. 43, which shows the lines of force of a simple magnet.

39. Tubes of Force. Magnetic Flux. In mapping out lines of force by iron filings or drawing them on paper we are obliged to confine ourselves to the part of the field in the plane of the paper, but of course the lines exist in the entire field. Consider then any field, *e.g.* a room in which there are magnets. It is full of lines of force, and we may fix our attention upon as many or as few of them as we please. Let us in imagination pick them out in such a way as to enclose a number of little tubes; these are called *tubes of force* and resemble pipes whose breadth is in general different at different parts: see A B C D, Fig. 40. The tubes may be conceived as drawn to any scale we please, but it is usual to adopt a certain scale based upon considerations belonging to the higher parts of the subject, and when drawn to this scale they are called *unit tubes*.

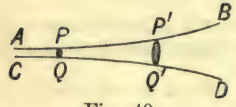

Fig. 40.

Now these unit tubes possess the property that they are narrow in the strong parts of the field and broaden out in the weak parts, the strength of the field being in fact inversely proportional to the cross section of the tube; thus, in Fig. 40 if the area of the section P′ Q′ be four times that of the section P Q, the field will be four times as strong at P Q as at P′ Q′. Let us now imagine the whole field mapped out in unit tubes, so that the particular one shown in Fig. 40 is surrounded by others, and let us imagine a certain area, say a square centimetre, placed so as to overlap P Q, and another equal area placed so as to overlap P′ Q′. Then clearly if P′ Q′ have four times the area of P Q, four times as many tubes will fall on the square centimetre at P Q as on the one at P′ Q′.

It thus appears that the strength of the field at any point is directly proportional to the number of unit tubes that fall in a square centimetre, placed at that point perpendicular to the sides of the tube. Adopting the definition of intensity or strength given in § 37, convention has decided that the intensity shall be not merely proportional but *equal* to that number, *e.g.* if there were $17\frac{1}{2}$ tubes per square centimetre at a particular part of the field, the intensity there would be $17\frac{1}{2}$ dynes per unit pole, *i.e.* $17\frac{1}{2}$ gausses.

Now the number of unit tubes that fall in any area placed perpendicular to their sides is called the **magnetic flux**, or simply the FLUX across that area; we thus have the theorem that *the intensity of a magnetic field at any point is equal to the flux per square centimetre at that point.*

This theorem, and the conception of magnetic flux generally, are of very great importance in Electrical Engineering, as they are the basis of the whole theory of the action and design of dynamos, transformers, etc.

The student should cultivate the habit of picturing to himself any magnetic field as riddled with unit tubes whose characteristic feature as above explained is that they *narrow up in the strong parts of the field and broaden out in the weak ones*, thus affording a heavy flux in the former and a light one in the latter.

Electrical engineers frequently speak of the number of "lines" of force that fall on a given area. Strictly, this phrase can have no meaning, for *every point* of a magnetic field has a line of force passing through it, and the number of lines that strike any area must therefore be infinite; whenever the term "lines" is used in this connection it must be understood as meaning *unit tubes*, the "number of lines" per given area signifying the *flux* across that area.

If we now turn to Figs. 29, 30, it will be seen that the filings congregate most closely near the poles of the magnets, *i.e.* in the strongest parts of the respective fields; this is what always happens—the filings, for reasons into which we need not enter, do not map out the whole infinitude of lines of force but select certain ones, and indicate roughly the unit tubes, so that an inspection of them gives a general idea of the relative intensity of the several parts of the field.

40. Uniform Field. Directivity of Uniform Field. DEFINITION. *A magnetic field (in air) at every point of which the intensity is the same, both in magnitude and direction, is said to be uniform.* In such a field the lines of force are obviously all straight and parallel, and the unit tubes are all of the same sectional area; the spaces between equidistant lines thus fairly depict the unit tubes.

Now consider a uniform field and let the dotted line *a* (Fig. 41) be the direction of its lines of force. Let a straight simple magnet (§ 13) *n s* be placed in the field in a direction *not* coincident with its lines of force. Then the needle experiences equal forces, P and Q, on its two poles, each parallel to *a* but in opposite directions. These two forces constitute what is, in Mechanics, called a *couple*,* and the effect of a couple is to *turn the needle round* until it sets parallel to *a*, but in no way to make it *move bodily*.

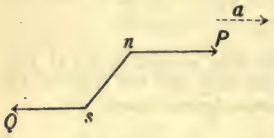

Fig. 41.

If, however, the field be *non*-uniform, the forces in the poles are not equal and parallel. They can be resolved into a couple and a single force, and the result of the combined action of the couple and the force is to make the needle not only take up a definite direction, but also move as a whole.

In the higher parts of the subject it is shown that the same effects are produced on *any magnet whatever*.

These results are usually expressed by saying that *a uniform field is purely directive*, while *a non-uniform field is both directive and translatory*: they will be of service in the next chapter.

As will be explained in Part III., Ch. XIV., when an electric current is passed through a spiral of wire, the region in and round about the spiral becomes a magnetic field. This affords the readiest means of procuring a uniform field of any desired strength; for this purpose it is only necessary to take a *long* straight glass tube, wind on it a spiral of insulated copper wire with the turns close together and pass the current: the field inside the tube is then (except near the ends) perfectly uniform, its lines of force running lengthwise through the tube.

If the tube be *curved* the field is uniform *in magnitude only*, *i.e.* the magnitude of the intensity is the same at all points, but its direction varies, the lines of force following the bend of the curve.

We shall see in the next chapter that the *earth's* field in any given locality is uniform; it is, however, too weak for many purposes.

* See Appendix, § 248.

41. Precise Meaning of Poles. Magnetic Axis. Magnetic Moment. If in § 40 we employ any magnet other than a simple one, the forces of the field are distributed irregularly all over its surface. But it may be shown on mechanical principles that *if the field is uniform* their action on the magnet is precisely the same as if all its northern polarity were concentrated at a certain point within it, and all its southern polarity at a certain other point within it. These two points are defined as the *true poles* of the magnet, and are what must always be understood whenever the word "poles" is applied in the sense of definite points to magnets in general.

It should be expressly noticed that it is *only* in respect of the action between the magnet and a uniform field that the poles can be regarded as representing the actual magnet; *e.g.* the intensity at a point in the external field does *not* follow the law of the inverse square of the distance from the pole, and the construction of § 38 does not hold accurately: a magnet, in fact, unless it be a simple one, has in a general sense no point-poles whatever.

The line joining the poles of a magnet as above defined is called the *magnetic axis*; in the case of a properly magnetised thin needle, or a compass-needle of the shape shown in Fig. 5, it practically coincides with the line joining its tips.

In a properly magnetised bar magnet the poles are somewhere about the position of the points marked × × in Fig. 28, but as a rule a little further from the ends.

In any magnet whatever the total amounts of northern and southern polarity, *i.e.* units of surface magnetism, are equal, and either of these imagined concentrated at a true pole constitutes the "pole-strength" of the magnet.

The product of the pole-strength of any magnet (estimated in unit poles) by the distance between the poles (estimated in centimetres) is called the **magnetic moment** of the magnet; if the magnet is a simple one the magnetic moment becomes the product of its pole-strength and *its length*. There is no special name for the unit of magnetic moment.

42. Internal Field. Lines of Induction. Magnetic Circuit. Leakage. The *actual substance* of a magnet whether a permanent one or otherwise (§ 9) is a field of magnetic force just as truly as the region outside, and constitutes the *internal field*; the positive direction of the line of force at any point of this field

is that in which an isolated north pole placed at that point would travel, provided its motion were not impeded by the substance of the magnet.

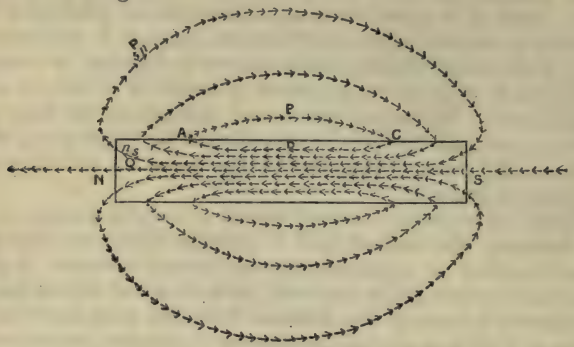

Fig. 42.

What we have called internal lines of force are frequently termed *lines of induction.*

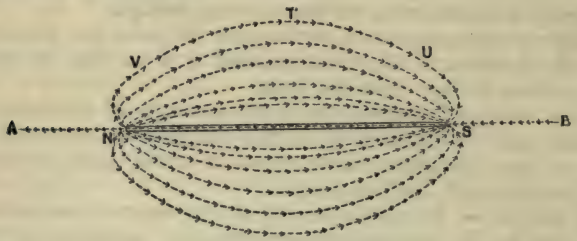

Fig. 43.

Figs. 42 and 43 show the arrangement of the lines of induction and of force for an ordinary bar magnet and a simple magnet respectively, the arrows indicating their positive direction,* and the diagrams bring out two very important points which are true

* It is a point for discussion whether the molecules of a magnet (cf. Fig. 15) set themselves exactly along its lines of induction, though their general trend is certainly the same: the matter will not, however, concern us.

not only for the magnets here considered but in every case; these are:—

1. That when any line of induction, *e.g.* C D A, Fig. 42, emerges from the magnet it is *continued without any break* into a line of force, A P C, in the external air, and similarly when any line of force enters the magnet it is continued into a line of induction. Any complete line of force, *i.e.* internal and external together, thus constitute *a closed curve.** One or any collection of such closed curves is called a *magnetic circuit.* The trespassing of lines of force into any place when they are not wanted is spoken of by electrical engineers as *leakage*; we may accordingly say that there is a leakage of internal lines at the sides of a bar magnet (Fig. 42).

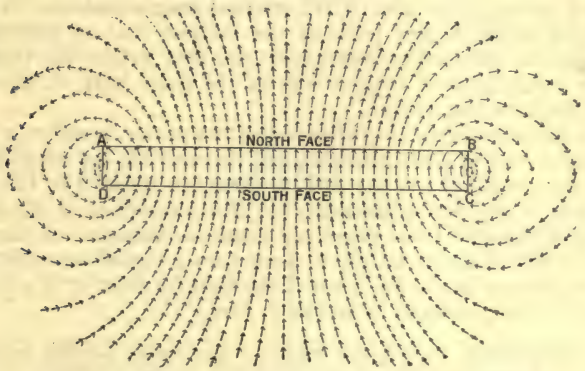

Fig. 44.

2. That free polarity is always exhibited on those parts of the surface of the magnet *where the internal lines emerge into the air, and nowhere else*; thus in Fig. 42 their leakage at the sides is accompanied by lateral polarity, while in Fig. 43, which represents the simple magnet, all the lines emerge at the ends.

Fig. 44 shows the external and internal lines of a bar magnetised sideways (cf. Fig. 18).

* A closed curve is a curve which has no ends, *e.g.* a circle.

In the case of circumferential magnetisation (Fig. 17) there are *no* external lines and the internal ones run round the ring following the trend of the molecules.

43. Internal Flux. Case of a previously Neutral Bar placed in a Magnetic Field. The lines of force in the actual substance of a magnet will aggregate themselves into unit tubes, which, when they emerge into the external field, run direct into the unit tubes of force of the air-field, completing the magnetic circuit. We thus have not only an external but also an internal flux.

By far the most important case is that of a previously neutral bar of iron (steel or other magnetic substance) introduced into a magnetic field. It then becomes magnetised by induction, in accordance with the law of § 35 and acquires of course, not only surface polarity, but also (§ 17) internal magnetisation. Now it is shown in the higher parts of the subject that in general, *by virtue of the internal magnetisation*, the flux within the iron becomes vastly greater than it was in the same part of the field when the iron was absent.

The numerical relation between the two varies, however, a good deal according to circumstances, and depends among other things upon the length of the bar and its direction relative to the lines of force of the field; according to the researches of Professor Ewing a flux of 5 (unit tubes per square centimetre) in air produces a flux of about 10,000 in long bars of good wrought iron set lengthways in the field, but if the bars be short or if set sideways the flux produced is decidedly less.

44. Permeability. The power which a previously neutral substance possesses of acquiring a flux when subjected to magnetic influence is called its permeability.

The permeability of all magnetic substances as compared with that of air is very great under ordinary conditions,* it is greatest for annealed wrought iron and cast dynamo-steel (§ 9), while for other materials it is considerably less. The permeability of all *non*-magnetic substances such as copper, brass, wood, etc., is practically equal to that of air, *i.e.* to unity.

45. Practical View of Permeability; Principle of Least Reluctance. In the early days of the science of magnetism attention was devoted exclusively to poles and surface polarity, but at the present time far greater

* At the temperature of red heat the permeability of iron is practically the same as that of air; in other words iron at a temperature of red heat may be said to be non-magnetic.

importance is attached to fluxes and permeability, chiefly because they are much better adapted to the needs of the electrical engineer.

For practical purposes the best view to take of permeability is this :— We regard every substance as offering a certain amount of opposition to the passage of lines of force, this opposition being termed *magnetic resistance* or *reluctance*, or (to put the same idea another way) as possessing a certain power of transmitting or conducting the lines. Permeability then simply means conducting power or the reverse of reluctance. Suppose now we have a field occupied partly by air and partly by iron : the lines of force will, so to speak, try to get through it *with as little trouble as possible*, or as it is technically expressed will *pursue the path of least reluctance*; they will therefore tend to crowd themselves into the iron, thus making the unit tubes narrow and the flux dense, while in the air they will tend to spread themselves out, making the unit tubes broad and the flux light. This principle is known as the *Principle of Least Reluctance.*

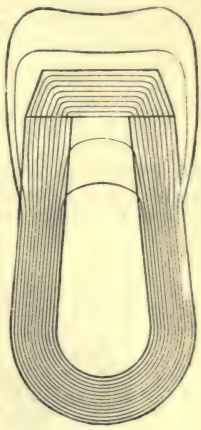

Fig. 45.

Consider now Fig. 45, which, when contrasted with Fig. 30, shows the effect of placing a keeper across the poles of a horseshoe magnet. In both cases the lines of force which emerge from the north pole have to get round to the south pole. In Fig. 30 the only path open to them is the air, and as this possesses small permeability they are obliged to spread themselves out into it so that there is a bountiful flux and a fairly strong field for some distance beyond the poles. But in Fig. 45 the keeper affords a very much easier path, accordingly the lines travel almost entirely through it, very few entering the air-field beyond, so that the latter is very much weakened.

Another important aspect of the Principle of Least Reluctance is that, if circumstances compel the lines of force to go through the air *at all*, they will make that part of their journey as short as possible. Thus consider Fig. 46 *a*, which shows the effect of introducing a piece of iron between the opposite poles of two bar magnets. The path of least reluctance between pole and pole is that which on its way passes through the piece of iron. Hence the lines of force distribute themselves so that the majority crowd into the iron and the spaces between the iron and the two poles. The field away from this region is much weaker than if the iron had not been present.

Exp. 24. Arrange two magnets, with unlike poles facing, and a piece of iron as in Fig. 46 (*a*), and obtain the filing diagram as in Exp. 23. Show that the statements made above are correct.

Exp. 25. Obtain and draw the filing chains for a horseshoe magnet and its keeper. Try the following cases : (1) magnet without keeper, (2) magnet with keeper half an inch away from poles, (3) magnet with keeper on.

Exp. 26. Obtain filing chains for a bar magnet and soft iron rod of equal length placed parallel an inch apart. Observe that there are very few or no chains beyond the soft iron rod; the magnet's influence fails to work through the soft iron (see Fig. 46 (*b*)). This illustrates the value of a thick piece of iron as a *magnetic screen.*

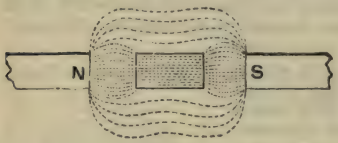

Fig. 46 (*a*).

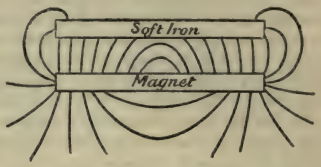

Fig. 46 (*b*).

Exp. 27. Place a thick ring of soft iron in the magnetic field between the unlike poles of two strong bar magnets and obtain the filing diagram. Fig. 47 illustrates the result. The magnetic lines of force which would

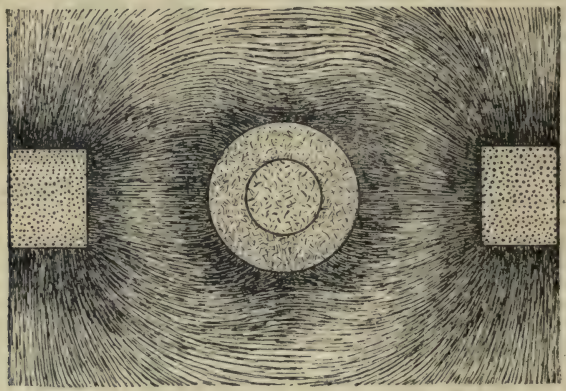

Fig. 47.

have traversed the region in the centre of the field are now crowded into the metal of the ring. The field within the ring is therefore of practically zero strength, and a compass-needle placed inside the ring would be very little affected by the magnet outside. A thick iron cylinder thus makes a good magnetic screen.

Exp. 28. Repeat Exp. 1, and test whether the attractive force of the magnet can be exerted through thin pieces of paper, glass (microscope cover-slip), iron (cover of a tin), silver (a sixpenny piece), or through thin layers of water and other liquids. Note that only iron has the screening effect.

The student should guard against the not uncommon error of supposing that a magnetic screen can act by *breaking off* the lines of force; this cannot possibly happen, they are never broken, but only deflected into another part of the field.

With any one of the magnetic arrangements described above tap vigorously the cardboard on which the filings rest. Observe the movements of the filings. Those scattered in the neighbourhood of the magnet or magnets do not travel along the curves, *i.e.* along the lines of magnetic force, but at right angles to them and tend to form flatter and shorter chains. Those on the surfaces of the magnet stand aslant and during the tapping march towards the edges of the magnet or magnets (these filings are really the ends of chains that tend to be produced above the magnet). If the tapping is prolonged the filings concentrate along the shortest lines between the poles. This is an instance of a general principle that magnetic bodies always tend to move towards the strongest parts of the magnetic field.

SUMMARY.—CHAPTER III.

1. Meaning of a Magnetic Field (§ 29).

2. *The* (*positive*) *direction* of the force at any point of a field is the direction in which an isolated north pole placed at that point would be urged, or in which the north pole of an explorer situated there would point (§ 30).

3. *Lines of force* are geometrical lines in a magnetic field, which possess the property that the tangent to any one of them at any point thereon is along the line of the resultant force at that point. *The* (*positive*) *direction* of a line of force is the direction in which an isolated north pole would travel along it if free to move. The lines of force of a field always run clear of one another. The visible forms which the lines take when mapped out by iron filings are called "magnetic curves" (§ 32)

4. Special cases of lines of force, particularly Figs. 29, 30, and the corresponding diagrammatic forms, Figs. 33, 32 and also 34, 35. In all cases note the positive direction as shown by the arrows.

5. When a piece of previously neutral iron or other magnetic substance is placed in a magnetic field, it acquires *southern* polarity where the lines of force *enter* it and northern where they leave it (§ 35).

6. The law of magnetic force, $F = \frac{mm'}{r^2}$, and definition of unit pole or unit of magnetism (§ 36).

7. Meaning of Intensity of Field (§ 37).

8. Every magnetic field is to be pictured as riddled by *unit tubes of force*; these are narrow in the strong parts of the field and broad in the weak parts. The number of them that strike any area perpendicular to their sides is called the MAGNETIC FLUX across that area, and *the flux per square centimetre at any point of the field is equal to the intensity at that point* (§ 39).

9. By a *uniform field* is meant one at every point of which the intensity is the same, both in magnitude and direction: in such a field the lines of force are all straight and parallel to one another. *The action of a uniform field on a magnet is purely directive*; but the action of a *non*-uniform field is both directive *and translatory* (§ 40).

10. When *any* magnet is placed in a uniform field we may regard the whole magnet as represented by the sp.cific points at which its polarities are concentrated, but except in the case of a simple magnet we cannot do so under any other circumstances; these points are the "poles" of the magnet in the precise sense. The line joining these poles is called the *magnetic axis*, and the product of the strength of either of them into the distance between them the *magnetic moment* (§ 41).

11. Lines of force in the actual substance of a magnet are often called *lines of induction*. The lines of induction and of force together constitute a set of closed curves forming the *magnetic circuit*. Free polarity is only exhibited where the lines of induction strike the surface of the magnet (§ 42).

12. When a previously neutral bar of iron or other magnetic material is placed in a magnetic field it acquires a much greater flux than existed in the same part of the field before it was put there, and it does so by virtue of its internal magnetisation (§ 43).

13. Definition of PERMEABILITY (§ 44). The permeability of all magnetic substances as compared with that of air is extremely great.

14. *Principle of least Reluctance* and its use to explain various magnetic phenomena (§ 45).

15. Magnetic screens depend for their action upon their high permeability. They do not *break off* the lines of force but draw them into their own substance and so away from the part of the field it is desired to screen. They must be thick, and even then are never perfect on account of "leakage" (§ 45).

EXERCISES III.

1. Sketch the lines of force for a bar magnet with consequent poles one-third of its length from each end.

2. Sketch the lines of force for a pair of similar bar magnets crossing each other at right angles at their centre.

3. Six bar magnets of equal strength are arranged with their poles round the circumference of a circle at equal distances apart ; a north and a south pole following one another alternately and all the magnets pointing towards the centre of the circle. Draw a diagram of the lines of force within the circle.

4. Two bar magnets are placed in a straight line with their north poles facing one another, and a soft iron rod is placed between and in a line with them. How will it be magnetised?

5. Two poles of strengths 3 and 8 are placed six inches apart. Compare their force with that of two of strengths 4 and 14 placed seven inches apart.

6. Two poles of strengths 4 and 6 are placed three inches apart. Two other poles of strengths 12 and x are put 4 inches apart. The force in the latter case is nine times that in the former. Find x.

7. Two poles of strengths 2 and 27, when at a distance of three inches apart, exert upon one another a certain force. Find at what distance two poles of strengths 3 and 8 must be placed in order to exert upon each other an equal force.

8. Find (i) in dynes, (ii) in milligrammes-wt., the force of attraction between two poles of strength + 18 and − 108 placed 6 cm. apart.

9. Two poles, one of which is twice as strong as the other, when placed 12 cm. apart, exert on each other a force of 50 dynes; find the strength of each.

10. Find the force experienced by a pole of 50 units when placed at a point in the field where the intensity is 100 units.

11. At a certain point, P, of a field the force on a pole of strength 150 is equal to the weight of 300 milligrammes. Find the intensity at P.

12. A pole of 30 units strength is placed at a point of a field where the total intensity is 120 gausses. Find the component of the force on it at an angle of 45°.

13. In London the earth's horizontal intensity is ·18 dyne per unit pole, and the dip is 66° 54′ (1910). Find (i) the magnitude of the horizontal force experienced by a pole of 500 units located there; (ii) the total intensity; (iii) the magnitude of the total force experienced by a pole of 1,000 units. [N.B.—Cos 66° 54′ = ·3923.]

14. Construct the direction of the resultant intensity at a point on the field of two point-poles of the same kind and of equal strength a few inches apart. P is a point in the neighbourhood of a bar magnet, A B, having a north pole at A. P A = 2 P B, and the angle A P B is a right angle. Show upon a diagram the direction in which a N pole at P would be urged. How would a small compass-needle with its centre at P behave? Indicate the direction of the line of force through P. [Note: The "bar magnet" must be treated as simple, otherwise (cf. § 41) the construction of Fig. 39 is not valid.]

15. What is the intensity at a point of a field where the cross section of the unit tube has an area of $\frac{1}{100}$ square centimetre?

16. What is the sectional area of the unit tube at a point of a field where the intensity is 40 dynes per unit pole?

17. What are the respective intensities at points of a field where the sectional areas of the unit tubes are (i) $\frac{1}{50}$ sq. cm., (ii) $\frac{2}{5}$ sq. cm., (iii) $\frac{3}{20}$ sq. cm.?

18. At a certain point of a field the intensity is 60 dynes per unit pole. Find the number of "lines" of force per square centimetre at that point.

19. A spiral of wire is wound outside a glass tube bent into the form of a ring. A current of electricity is passed through the wire. Draw a diagram of the lines of force within the tube.

20. A simple magnet is 4 cm. long and its magnetic moment is 80; what is its pole-strength?

21. A keeper is placed across the poles of a horseshoe magnet but with about half an inch air-space between; draw a diagram showing the path of the lines of force from pole to pole, and explain it in accordance with the principle of least reluctance.

22. A bar magnet lying on a table about a foot from a compass-needle produces a certain deflection of the needle. How is the deflection altered (if at all) when an iron bar the same size and shape as the magnet is placed on the top of the magnet?

23. A thick soft iron pipe is placed vertically between unlike magnetic poles; a card is placed over the poles, sprinkled with iron filings and tapped. *Very few filings collect over the hole.* Explain this.

24. An iron ball is held over a pole of a horseshoe magnet. Will the attraction exerted on the ball be altered if the poles of the magnet are connected by a soft iron keeper, and if so, in what way, and why?

25. Three precisely similar magnets are placed vertically, with their lower ends on a horizontal table. Iron filings are scattered over a plate of glass which rests on their upper ends, two of which are north poles and the third a south pole. Give a diagram showing the forms of the lines of force mapped out by the filings.

26. Iron filings are scattered on a piece of cardboard, which is placed over a horseshoe magnet and tapped. What difference would be observed in the arrangement of the filings when the ends of the magnet were joined in turn by bars of (1) soft iron, (2) steel, and (3) copper?

27. A compass-needle is placed one foot east from a short bar magnet which points towards it. How will the needle be affected (i) when a thick box of soft iron is placed over the needle, (ii) when the box is placed over the magnet?

28. Two equal magnets of the same strength are placed on a horizontal table parallel to each other and perpendicular to the line joining their centres, with similar poles in opposite directions. What changes would take place in the magnetic field produced by them close to the surface of the table if they were gradually moved parallel to themselves until they were in contact along the whole of their lengths? Give diagrams showing how your statement might be tested by means of iron filings.

29. What is meant by a line of magnetic force?

Draw diagrams showing the lines of force due to two equal bar magnets each 1 foot long placed in line 1 foot apart with (1) opposite, (2) like poles facing each other. Show how the lines of force are affected in each case by placing a bar of iron 10 inches long in line with the magnets and midway between them. Describe the magnetic state of the iron bar in each case.

30. What do you understand by a uniform magnetic field? By what special property as respects its action on a magnetic needle is such a field distinguished from any other?

CHAPTER IV.

TERRESTRIAL MAGNETISM AND MAGNETOMETRY.

46. The Earth regarded as a Magnet. We saw at the commencement of the subject that a compass-needle sets itself (approximately) north and south, and we agreed to call the pole which points northwards the *north pole*. There must clearly be some influence in the northern regions of the earth tending to attract that pole, and likewise an influence in the southern regions tending to attract the south pole. The earth, in fact, behaves like a huge magnet whose south pole is in its northern regions and whose north pole is in its southern, and it is accordingly agreed to look upon the earth as such a magnet: we do not, however, in any way commit ourselves to the *cause* of its magnetism.

The fact that the northern regions of the earth behave like the south pole of a magnet—that is, attract the north pole of another magnet—is the source of the confusion of nomenclature referred to in § 5: those writers who speak of the pole of a magnet which points to the north as the "south pole" intend thereby to call attention to the fact that it possesses the same kind of polarity as exists in the southern regions of the earth. But nothing is really gained by this phraseology, and we shall adhere to that hitherto employed. The pole which points *northwards* we term the *north pole*, and the kind of polarity which exists in the north pole of a magnet we term *northern polarity*. *The northern regions of the earth therefore possess southern polarity, and its southern regions northern polarity.*

47. The Earth's Magnetic Field. Line of Dip. Intensity of Earth's Field. The whole region on and around the earth's surface is a magnetic field and may be explored like any other magnetic field by the method of § 31. It is of course permeated by lines of force, and an explorer suspended at its centre of gravity always points along them with its north pole in their positive direction. The line along which the explorer sets at any point is called the *line of dip*, and is the direction of the earth's resultant force at that point. The intensity of the earth's field

at a point is in accordance with the general definition of intensity (§ 37), the force which unit pole there situate would experience; sometimes this is called the *total* or resultant intensity, to distinguish it from the horizontal and vertical intensities (§ 48).

The line of dip can be ascertained with great accuracy by means of the Dip Circle (§ 53), which is merely a very delicate explorer specially adapted to the purpose, while the intensity can be measured by methods explained in the mathematical parts of the subject; it is then found that both remain practically the same for a considerable distance, say several miles, round any given point; in other words (§ 40), *in any given locality the earth's field is uniform*; we may thus picture to ourselves each locality as mapped out by lines of force parallel to the line of dip.

The direction of the line of dip varies gradually from one locality to another; speaking broadly it may be said that its positive direction—in other words, the positive direction of the earth's line of force—slopes more or less downwards in the northern hemisphere and upwards in the southern.

48. Magnetic Meridian. Declination and Inclination. Horizontal and Vertical Intensities. Direction of Compass Needle. If at a given place we set up a thin freely suspended (§ 30) magnetic needle it will, as we have seen, set itself along the line of the earth's resultant force at that place, that is the line of dip. Imagine now a vertical plane drawn through this line; this plane is called the *magnetic meridian of the place*. It in general does not coincide with the geographical meridian, and therefore does not pass through the geographical north or south pole. The angle between the magnetic and geographical meridians at any place is called the *magnetic declination* or simply the *declination* at that place.

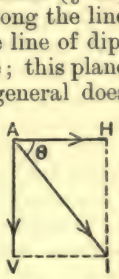

Fig. 48.

Now in Fig. 48 let A denote some point, and A I the (positive direction of the) line of dip sloping downwards towards the north, so that the north is on the right in the figure. The plane of the paper clearly coincides with the magnetic meridian. Through A draw A H horizontally towards the north and A V vertically downwards; then the angle H A I is called the *dip* or *inclination* at the place A.

Take any length, A I, to represent the resultant intensity of the earth's field at A, and complete the parallelogram V A H I. Then by the parallelogram of forces* the force A I is equivalent to two others represented in magnitude and direction by A V and A H respectively. Of these the former acts vertically downwards and the latter horizontally northwards; they are called the *vertical* and *horizontal components* respectively of the earth's force, or simply the earth's *vertical and horizontal intensities.*

It is clear from the figure that though A H is northwards it is not directly towards the geographical north, for A H is in the magnetic meridian, which (see above) does not pass through the geographical north pole. The direction of A H is called *magnetic* north. Moreover, it is evident by geometry that the angle between A H and the geographical north line is the same as that between the two meridians—that is, it is the declination; in fact, the two north lines are simply the traces of their respective meridians on the horizontal plane through A.

Now what is the practical significance of the magnetic north line? If we take a magnetic needle mounted in such a way that it is free to obey the horizontal component of the earth's force *only,* it will clearly set along A H with its north pole to the right. This is precisely what a compass-needle is designed to effect. In making a compass-needle, the agate or brass cup on which it turns is placed not exactly at its centre of gravity, but (for places in the northern hemisphere) a little nearer its north pole, so that the weight of the needle may just counteract the vertical component of the earth's force, leaving it subject to the horizontal component and that alone.

The magnetic north line at any place is therefore the direction in which a compass-needle at that place sets itself; in other words, *a compass-needle sets itself horizontally in the magnetic meridian.* It therefore does not point *true* (*i.e.* geographical) north, but *magnetic* north. Moreover, it is clear that the angle between the needle and the true north line is coincident with the declination; in fact, the declination is sometimes *defined* in this way.

The two meridians at any place cut the earth's surface in two great circles called *meridian lines*†; evidently the two north lines

* See Appendix, §§ 245, 246.

† These lines are themselves frequently called "meridians," but it is best to employ the latter terms to denote the *planes.*

are tangents to the respective meridian lines, and the angle between the latter is coincident with the declination.

It should be noticed that strictly speaking it is the *magnetic axis* (§ 41) of the needle that sets itself either in the line of dip or horizontally in the magnetic meridian according as it is mounted, and for very accurate observations special steps have to be taken for determining that axis. (See §§ 52, 53.)

If I be the total intensity at a place, θ the dip, and H, V, the horizontal and vertical intensities respectively, it is obvious (see Fig. 48) that

$$H = I \cos \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$V = I \sin \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$\frac{V}{H} = \tan \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

49. Directivity of Earth's Horizontal Force. Behaviour of Needle under Joint Influence of Earth and Magnet. Since the resultant force of the earth does not appreciably alter either in magnitude or direction over a considerable distance from a given point, the same must be true of its horizontal component; it thus appears that not only its resultant field, but also its *horizontal field* is uniform in any given locality, and therefore (§ 40) the horizontal field is, like the total field, purely directive.

The directivity of the earth's horizontal force may be shown by the following simple experiment.

Exp. 29. Take a broad basin or dish of water, float a flat cork or piece of wood at about the middle of it, and lay thereon a bar magnet (Fig. 49). The cork will *turn round* so that the magnet sets itself in the magnetic meridian, but it will not *move* as a whole in any direction. The motion is due to the earth's horizontal force, and it shows that the earth's field is directive only.

Fig. 49.

Now hold a large bar magnet with one of its poles at the edge of the basin, the cork travels across the water up to it, *i.e.* the force of the magnet is *not* simply directive.

The same experiment may be performed with a magnetised sewing-needle, which will rest on the surface of the water without any other support.

When a compass-needle is balanced on a pivot and a magnet is held near it, the pivot of course prevents translatory movement, and only the directive action of the magnet is operative. For simplicity consider the arrangement in Fig. 50: A B is the magnetic north-south line, A being northwards, and $n\ s$ a compass-needle balanced on a pivot, O; then under the earth's influence alone it sets in the direction $n\ s$, coincident with A B. Now let a bar magnet S N be placed as shown, its magnetic axis being perpendicular to A B (*i.e.* running magnetic east and west) and passing through O. The effect of this magnet alone would be to pull the needle round, making n point towards S. What really happens is that the needle assumes an intermediate position, $n'\ s'$, in which the influences of the earth and magnet just balance one another.

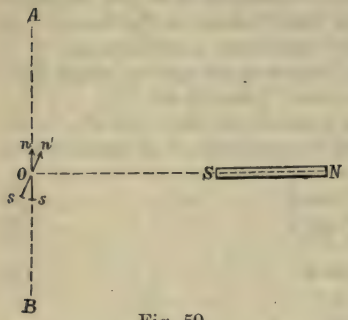

Fig. 50.

Clearly, the nearer the magnet is to the needle the greater will be its influence compared with that of the earth and the greater will be the angle $n\,O\,n'$: indeed, when the magnet comes very near, the influence of the earth is relatively negligible in comparison with the influence of the magnet, and the needle points practically due magnetic east.

The student should here be cautioned against a very common error arising from ignorance of mechanical principles, viz. that of supposing that the magnet must exert a certain force before the needle can *begin* to move away from the line A B; apart from friction on the pivot, which may be neglected, this is certainly not the case—the very least force will deflect the needle, though the deflection may not be noticeable without some delicate means of detection, such as is afforded by a mirror and a beam of light. (See § 210.)

50. Numerical Values of the Declination and Inclination at London. Let A (Fig. 51) denote any place in either hemisphere, A G the geographical, and A M the magnetic north line, so that the angle M A G is the declination at the place. The declination

is termed west or east according as A M lies on the west or east side of A G; the figure shows it west.

The value of the declination is different at different places, and even for the same place changes slowly with the lapse of time. At Kew in 1910 it was 16° 4′ W., and is decreasing at the rate of 4′ per year. The earliest record is for the year 1580, when it was 11° 17′ *east*; it then grew less, until in 1657 it was *nil*. After this it became west, and increased until it attained a maximum of 24° 38′ in 1815. It is now going back, and will presumably again become *nil* in 1975, after which it will become east, and so on, thus going through its complete cycle of changes in about 600 years.

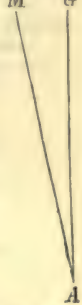

Fig. 51.

The value of the dip is different at different places. It is usual to name it "north" or "south" according as the north or south pole of the freely suspended needle points downwards, so that, broadly, it is N. in the northern and S. in the southern hemisphere.* Broadly, too, it increases as we travel north or south from the equator. It also changes slowly with time. At Kew in 1910 its value was 66° 58′, and it is decreasing at the rate of about 2′ per year. Its maximum value was 74° 42′ in the year 1720, but we have no record of its minimum value, so that its cyclic period is unknown.

51. Distribution of Terrestrial Magnetism. If a line be drawn through all the places on the surface of the earth where there is the same declination it is called an *isogonal* line. In the particular places where the declination is zero so that the magnetic needle points true north the line is called the *agonic* line. A line drawn through places of equal dip is called an *isoclinic* line, and if drawn through all places where the total force is equal it is called an *isodynamic* line.

The line of no dip is called the *magnetic equator* and it is in the neighbourhood of the terrestrial equator. To the north of it the north end of the needle dips downwards and this dip increases until the north magnetic pole is reached. To the south of the equator the south pole dips. The two magnetic poles are characterised by the two facts that the isogonal lines converge to them and the dipping needle stands vertical at either of them. The north magnetic pole is 70° 45′ N. and 96° 45′ W., and the south magnetic pole is 72° 25′ S. and 154° E.

The total force is not a maximum over the magnetic poles, but there are in each hemisphere two magnetic foci, at each of which the total force reaches a maximum value. These maxima are not equal.

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* Some writers speak of the dip as *positive* when the north pole points down and *negative* when up.

THE
BRITISH ISLANDS
ISOGONIC AND ISOCLINIC
LINES FOR THE
YEAR 1910
19°
20°
72°
72°
21°
18°
71°
71°
70°
70°
17°
69°
69°
68°
68°
67°
67°
16°
16°
Wick
Dingwall
Elgin
Aberdeen
Fort William
Dundee
Oban
Glasgow
EDINBURGH
Berwick
Ayr
Eskdalemuir
Londonderry
Port Patrick
Carlisle
Newcastle
Hartlepool
Belfast
Sligo
Barrow
Lancaster
York
Drogheda
Stonyhurst
Hull
Athlone
Liverpool
Manchester
Sheffield
Galway
DUBLIN
Bangor
Chester
Linoln
Wicklow
Shrewsbury
Hunstanton
Limerick
Leicester
Lynn
Norwich
Yarmouth
Aberdovey
Waterford
Birmingham
Aberystwyth
Cambridge
Valencia
Cork
Bantry
Buckingham
Brecknock
Oxford
Harwich
Swansea
Gloucester
LONDON
Margate
Tenby
Bristol
Reading
Cardiff
Dover
Taunton
Southampton
Bude
Lyme Regis
Brighton
Exeter
Ashburton
Plymouth
Portsmouth
Falmouth

In the United Kingdom the declination is westerly and increases from east to west, changing from about 16° in East Anglia to 21° in Connaught. In like manner the dip increases from south to north, being about 67° on the South coast of England and 72° in the Shetlands. If the true isogonal and isoclinic lines are drawn they are found to be very irregular over the land and these irregularities are probably due to the action of magnetic rocks, some of which are on the surface, like the Malvern Hills, and others may be of great depth.

The general trend of the isogonal and isoclinic lines over the United Kingdom is shown in the accompanying map. It will be seen, for example, that Bantry, Cork, Aberdovey, Shrewsbury, and Lincoln lie on the isoclinic of 68°, and that Lyme Regis, Bristol, Gloucester, Birmingham, Sheffield, and York lie on the isogonal of 17° W.

Exp. 30. *Find the magnetic meridian by a compass-needle.* Remove all iron or steel from the neighbourhood of the compass-needle. Set the needle oscillating. When at rest adjust two brass pins vertically and in line with, but a little distance from, the ends of the needle. (Tap the pivot base *gently* with a loosely held pencil to make sure that the needle comes to rest in the right position.) The line through the two pins is the magnetic meridian.

The next experiment is more accurate.

Exp. 31. *Find the magnetic meridian by a bar magnet.* Fix (by soft wax) two pins, A, B, perpendicular to the edges of a bar magnet and near its ends (Fig. 52). Place the magnet in a metal or cardboard stirrup suspended by a long silk fibre above a piece of paper fixed to the table: the points of A, B should just clear the surface. When the magnet is at rest fix pins C and D into the paper in line with A and B. Now turn the magnet right over so that A, B point upwards instead of downwards, and, as before, fix pins E, F in line with A, B. Join C D, E F and let O be their point of intersection. The bisector of the angle C O E lies in the magnetic meridian, and its trace on the magnet is the *magnetic axis* of the magnet.

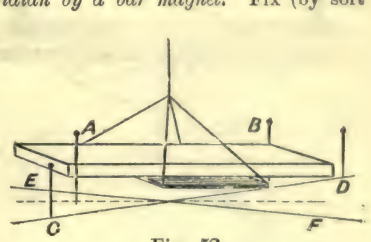

Fig. 52.

52. Determination of the Declination. Having found the magnetic meridian at any place by Exp. 31, it is an easy matter, if we know the direction of true north, to find the declination. The angle between the true north line and the magnetic meridian is the declination. One way of finding the true north is by

observing when the shadow of a vertical stick is shortest. It is shortest when the sun is highest in the heavens, *i.e.* when the sun is in the geographical meridian, hence the shadow then points due geographical north. This method is, however, not accurate. The accurate method is astronomical and is based on the observation of the transit of stars across the meridian with the transit circle.

53. Determination of the Inclination. The Dip Circle. To determine the inclination at a place, we must first find the line of dip and measure the angle it makes with the horizontal line in the magnetic meridian. It is rather inconvenient to employ a freely suspended needle (§ 30).

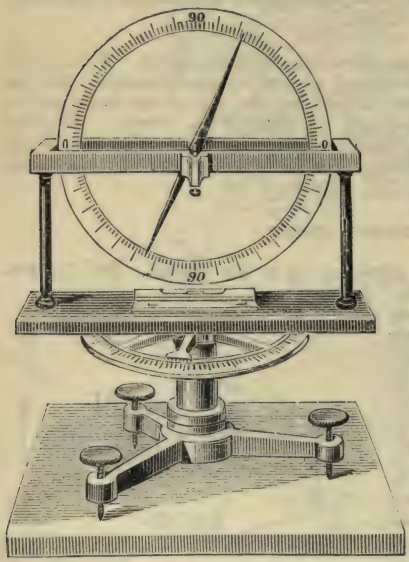

Fig. 53.

The best instrument for the purpose is the *dip circle* (Fig. 53). It consists of a vertical graduated circle with a horizontal axis at its centre, and perpendicular to its plane. On this axis turns the magnetic needle used for determining the dip, and termed the *dipping needle*; the axis should pass accurately through its centre of gravity, and the mounting be such as to diminish friction on the bearings as much as possible. The vertical circle and needle are mounted as shown on a frame, so that they can be turned bodily about a vertical axis on a horizontal graduated circle which is fixed. The instrument is further provided with levelling screws and a spirit-level, and care must be taken in using it that the horizontal circle is levelled properly.

Suppose now the instrument used at a place in the northern

hemisphere. Let the plane of the paper represent the magnetic meridian, the north being on the left hand; then the figure shows the position assumed by the needle *when the plane of the vertical circle is in the magnetic meridian*, and the point to be noticed is that it then sets in precisely the same way as if it were *freely suspended*. For the mode of mounting only precludes its tilting *perpendicular* to the plane of the vertical circle, and as the earth's resultant force is entirely *in* that plane, the mounting in no way impedes the needle's fully obeying it. Thus it sets along the line of dip, and the dip itself is read off on the vertical circle.

But what is the use of the horizontal circle? Well, if we knew beforehand the exact direction of the magnetic meridian, we could dispense with it. But of that direction we are in general ignorant, and the horizontal circle enables us to find it. To understand how, let us imagine the vertical circle set in a plane *perpendicular* to the magnetic meridian. The horizontal component of the earth's force being *in* the plane of the meridian simply tends to tilt the needle perpendicular to the plane of the circle, and this it cannot do by reason of the way it is mounted; this part of the force is therefore quite inoperative. But the *vertical* component tends to pull the needle into a vertical position, and to this the mounting offers no impediment. *Accordingly the needle then sets vertically.*

But it will not set vertically if the vertical circle be in *any other* plane: what we do then is to begin by turning the upper framework of the instrument until the needle sets at the reading 90°; we then know that the plane of the vertical circle is perpendicular to the magnetic meridian, and all we have then to do is to turn it through 90° of the horizontal circle when we know that it is *in* that meridian, and we can proceed to read the dip as explained above.

54. Elementary Representation of the Earth's Magnetism. If we take a long, thin bar magnet (Fig. 54), and hold an explorer directly in front of S, it will set itself along the magnetic axis B S N A, with its N pole towards S. If now we carry it along the line S U T V N it will at all places point as shown by the arrows; between S and T the north pole will incline downwards, at T the needle will be horizontal; and between T and N the north pole will incline upwards, until at N the north pole

will again point along the magnetic axis. This is very similar to the behaviour of a dipping needle when carried from north

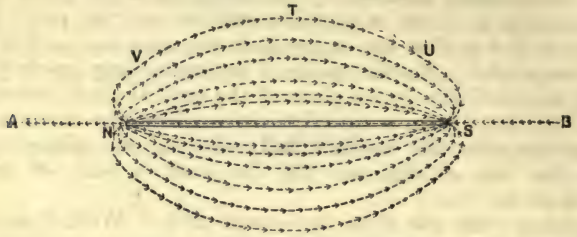

Fig. 54.

magnetic pole of the earth across the magnetic equator to the south magnetic pole.

A better mode of representing the earth's magnetic action is shown in Fig. 55. It represents a large wooden globe along one of whose diameters is fixed, as shown, a strong bar magnet *considerably shorter than that diameter*. The magnetic axis is now produced to cut the globe in the points B and X, and E E′ is drawn perpendicular to B X through the common centre, C, of the globe and magnet. Then this arrangement *roughly* represents the magnetism of the earth, B corresponding to the north magnetic pole* and X to the south magnetic pole,* while E E′ roughly represents the earth's magnetic equator. P, Q, Q′, P′ represent places in the N and S magnetic hemispheres. In all

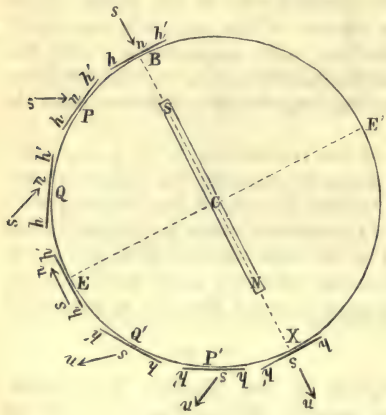

Fig. 55.

* The north magnetic pole is situated in latitude 70° 5′ N., longitude 96° 45′ W. in Boothia Felix, the most northerly part of Canada. The south magnetic pole is situated in latitude 72° 25′ S., longitude 154° E.

positions hh' represents the horizontal north-south line, sn a dipping needle, and the inclination of sn to hh' represents the dip. If we actually fix up such a globe, and carry an explorer round from B to X, it will set itself in the several positions shown by sn.*

It should be carefully noted that, in order that the dip at the several points of the model should be approximately equal to the actual dip at corresponding places on the earth, the length of the magnet SN must be suitably adjusted to the diameter of the globe; if not, though the general movements of the explorer will be similar, its dip will be very far wrong except quite near B (and X). And, even if the magnet be adjusted as well as possible, the model is still but a rough representation of the earth, many important details of the earth's magnetism being quite unindicated. But for rough elementary purposes the ideas conveyed by the model are often convenient.

Looking then at the globe in Fig. 55 as depicting the earth itself, we may for such rough purposes refer its magnetism to *an imaginary internal magnet S N*. The points S and N (which are many miles beneath the earth's surface) may be termed its "representative" magnetic north and south poles respectively; they must not be confused with the points B and X, which we still call the "magnetic poles." And care must be taken not to refer the earth's magnetism to polarity *concentrated at B and X*.

The student should distinguish clearly between the direction of the magnetic meridian and that of the great circle joining the magnetic poles. In the great majority of cases these directions are not identical: *e.g.* the magnetic meridian of London when produced passes about midway between the geographical and magnetic north pole, thus falling short of the latter by about 600 miles.

The student should remember that only a rough approximation to the state of the surface magnetic field of the earth is obtained by the hypothesis of the internal magnet mentioned above. Another approximation is obtained by assuming that

* In practice it is best to keep the explorer in one place, and turn the globe so that B, P . . . X comes in succession directly below it; the explorer then gradually turns round. This method has the advantage that the line hh' always *is* horizontal, so that it is easier for the eye to follow the movements of the needle.

the earth is a uniformly magnetised sphere. This gives rather better results, but as a matter of fact the earth's magnetism is exceedingly complex and cannot be adequately represented in any simple way. The interior of the earth (excepting at points comparatively near the surface) is too hot for any magnetic substance to be in a magnetic state there.

The magnetism of the earth is possibly due to several causes, such as magnetism in the cooler surface rocks, electrical currents in the earth and air, especially in the upper regions of the atmosphere, action of the sun, moon, etc.

55. The Mariner's Compass. The sailor traces his path across the sea by means of a compass-needle.* The needle always points along the magnetic meridian, which is not a constant direction, but, as declination tables have been prepared, it is easy for the navigator to find his true N., S., E., and W. The compass usually employed consists of eight light flat needles, 2 to 3 inches long, fastened parallel to one another on the underside of a card about 10 inches in diameter. The top side of the card is divided into degrees, and also into thirty-two divisions obtained by continual bisection of the right angles formed by the two lines joining the **cardinal points**, viz. the N. and S., and the E. and W. points. The division lines are called the **points of the compass**, and bear appropriate names.

Fig. 56.

By means of an inverted sapphire cup fixed to its underside, the card is then mounted on a hard iridium-cased steel pivot fixed in a cylindrical copper bowl (Fig. 56 shows a portable form of compass) provided with a glass top. To avoid the shaking of the compass bowl by the rolling and pitching of the ship at sea it is supported on knife edges in gimbals, *i.e.* two concentric rings whose axes of suspension intersect at right angles exactly at the iridium point. The outer gimbal rides in stirrups attached to

* The Chinese seem to have been familiar with this use of the compass needle from very early days, but it was reserved for Christopher Columbus to discover the variation of declination with variation of position on the earth's surface.

the pedestal by a system of stout springs. The pedestal of a ship's compass is mounted on a high part of the ship free from loose pieces of iron. Owing to the magnetism of the ship itself, sundry corrections have to be applied.

56. The tracing of Magnetic Lines of Force. Suppose that it is required to trace the magnetic field due to a magnet or magnetic combination placed on a sheet of paper fixed to a board, and that the survey is restricted to the horizontal plane of the paper.

Exp. 32. *To map or trace the lines of force in a magnetic field.* As an explorer use a small (watch chain) compass ($\frac{3}{4}$ inch long), in a case with a glass top and bottom (Fig. 57).

The method is:—

Place the compass-needle in the field, and mark the positions, A, B, of its ends (Fig. 57). Move the compass until the end that was against A is over B, mark the position, C, of the other end; again move the compass until the end that was over A is against C, mark the position, D, of the other end; continue the process until the line has been sufficiently traced. Join A B, B C, etc., as *each step is completed.* The curve through A B C . . . marks a magnetic line of force. Obtain others by starting the compass from different positions. Indicate by an arrow-head the + *sense* (that in which a north-pole *tends* to travel) *of the line of force.*

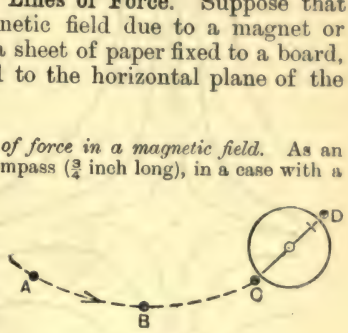

Fig. 57.

Try different fields, say—

(*a*) *Earth's field alone.* Fix a sheet of drawing-paper to the table with an edge lying roughly east and west. Start the compass from positions along this edge about an inch apart. If the lines of force are straight and parallel the field is uniform.

(*b*) *Earth's field together with that of a bar magnet in the magnetic meridian with its N-end pointing south.* Fix a large sheet of drawing-paper to a drawing-board. Place the magnet with its centre about a foot from the S.W. corner of the paper.

Fig. 58 shows the kind of map obtained. × is a *null* or *neutral* position. At this point the horizontal field due to the magnet is equal and opposite to that of the earth; hence the resultant horizontal magnetic field is zero, and the compass-needle will remain in any position in which it is placed. The only magnetic force at a null point is practically the vertical component of the earth's magnetic field. The complete map would be symmetrical about the axis of the magnet, and also about a line through its centre perpendicular to its axis. Hence there is a second null point on the lower side.

(*c*) *Earth's field together with that of a bar magnet in the magnetic meridian with its N-end pointing north.* You will find that two null points are now equidistant from the magnet on the line through its centre perpendicular to its axis.

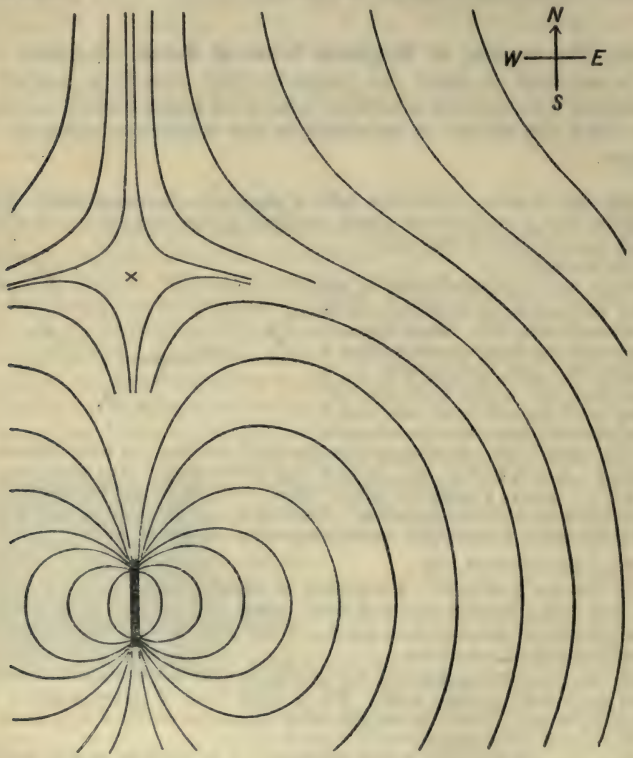

Fig. 58.

(*d*) *Earth's field with that of a bar magnet at right angles to the magnetic meridian.* Fig. 59 gives the sort of map obtained. There are two null points diagonally opposite.

The maps obtained in (*b*), (*c*), (*d*) above show a fairly well defined region in which the forces due to the magnet are predominant;

at greater distances the effects of the magnet are subordinate, and the lines appear to be those of the earth's field more or less distorted.

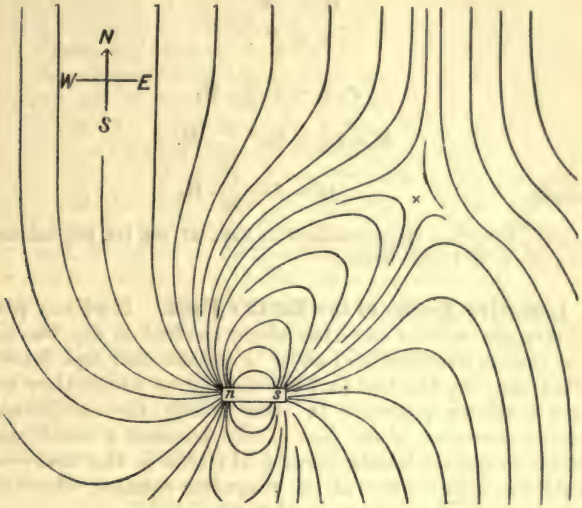

Fig. 59.

Filing chains afford a somewhat rougher and more restricted picture of the lines of force, but are more quickly obtained. By the compass method, however, the lines can be traced in these portions of the field where the forces are very small, areas where the filings would not be visibly affected. Practically when filing chains are produced the effect of the earth is negligible, and the curves show the lines of force due to the magnets alone.

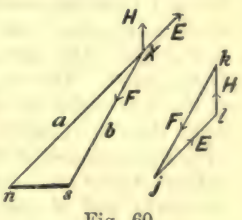

Fig. 60.

Relation between Moment of Magnet and the Horizontal Intensity of the Earth's Field. Let *ns* (Fig. 60) be a magnet of moment M, pole strength *p*, and length L. Let X be a null point at distances *a*, *b*, respectively from the N and S poles of the magnet. Imagine a unit N-pole at X: it will be in equilibrium and acted on by three forces, H due

to the earth, a repulsion E due to the north pole of the magnet, and an attraction F due to the south pole of the magnet. Draw the triangle of forces *jlk*. Then

$$\frac{E}{H} = \frac{jl}{lk};$$

but

$$E = \frac{p}{a^2}.$$

$$\therefore p = a^2 \frac{jl}{lk} . H,$$

and

$$M = Lp = La^2 \frac{jl}{lk} . H.$$

Similarly

$$M = Lb^2 \frac{kj}{lk} . H.$$

Exercise.—From the maps obtained in Exp. 32, (*b*), (*c*), (*d*), calculate M assuming H = ·18 C.G.S. unit.

57. Inductive Action of the Earth's Field. If at any place we hold a straight iron or steel bar along the line of dip, the lines of force of the earth's field will enter it at one end and leave it at the other, and, by the law in § 35, the end at which they leave it acquires northern polarity; in other words, *the end pointing in the positive direction of the line of dip becomes a north pole*; in the north magnetic hemisphere (§ 51) this is the lower end, in the south the upper, while at the magnetic equator, where the dip is *nil*, it is the end pointing northward.

Suppose now that instead of holding the bar along the line of dip, we set it magnetic north and south.* To understand what happens, let us resolve the earth's force into its horizontal and vertical components. The former magnetises the bar, so that the end pointing northwards becomes a north pole. The latter tends to produce side magnetisation, after the fashion indicated in Fig. 18. The latter effect is, however, very feeble, and may be disregarded. The magnetisation is therefore practically that due to the horizontal intensity only, and must clearly be weaker than when the bar is held in the line of dip.

In like manner, if the bar is set vertically, the magnetisation is that due to the vertical component of the earth's force only; the lower end of the bar becomes a north pole at places in the northern magnetic hemisphere, and *vice versâ* in the southern.

* That is, in the direction of the horizontal component of the earth's magnetism.

On the magnetic equator the vertical force is *nil*, and a bar held vertically is unaffected.

If the bar be held perpendicular to the line of dip* the component of the earth's force in the direction of its length is *nil*, and it is quite unaffected (except for a negligible side action). In fine, in whatever direction it be held it experiences simply the effect of the component of the earth's force *in that direction*.

Any and all of these effects can readily be shown with a long, soft iron bar. For example, if such a bar be set vertically (say in London), and a compass-needle be held near its lower end, the north pole of the needle is repelled, while when placed near its upper end the south pole is repelled. If the bar be now gradually tilted away from the line of dip, these effects grow weaker until, when nearly perpendicular to that line, the ends of the bar attract either pole of the needle indifferently, the action now being simply due to the inductive effect of the needle itself (§ 11). In order that these experiments should be successful, the bar should be of *very* soft iron, otherwise the earth's field is too weak to produce a good effect. The compass-needle should also be weak, otherwise its inductive action on the bar may overcome that of the earth (§ 12).

A hard iron or steel bar may be magnetised by the earth's inductive action, provided it be held in a suitable position and hammered so as momentarily to weaken its molecular rigidity (cf. § 27). Thus, if a common poker be held N. and S., and struck sharply on the handle a dozen times or so, it will, on removing and testing, be found magnetised. If it be now turned the other way round and again struck, its magnetism may be reversed or neutralised, though it is difficult to know how much to strike to secure *exact* neutralisation.

It not unfrequently happens that iron rods are found magnetised when we want them to be perfectly neutral. A good way to neutralise them is as follows:—Set up a compass-needle so as to determine very nearly the direction of the magnetic meridian, make a chalk-line on the table in this direction and take the needle away. Then hold the bar firmly in a direction as nearly as possible perpendicular to the chalk-line, and hammer it at one end. This loosens the molecules, and there being in this position of the bar no independent field which can appreciably influence

* Whether in the magnetic meridian, or perpendicular to it, or otherwise, is immaterial.

them, they simply (§ 20) turn round in obedience to each other's mutual attractions and form closed magnetic chains.

The student should examine Fig. 61, which shows the effect of introducing a bar of iron lengthways (*i.e.* along the lines of force) into the earth's or any other uniform field. Prior to its introduction the lines of force of the field are all straight and parallel, and their positive direction is downwards, as indicated by the arrows. On inserting the bar it acquires polarity in accordance with the law of § 35, while on account of its high permeability the lines of force become distorted as shown, the induction within the bar becoming very dense. Fig. 62 shows the effect of setting a bar *across* a uniform field; the letters N and S indicate the feeble side polarity.

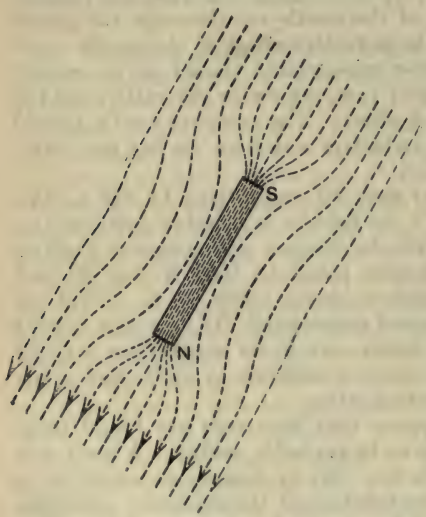

Fig. 61.

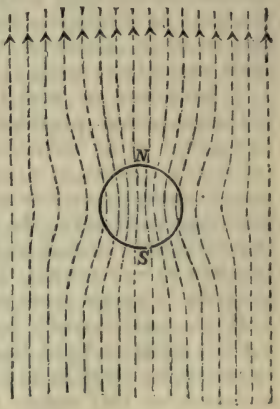

Fig. 62.

Exp. 33. *To illustrate induction due to the magnetism of the earth.*

(*a*) Hold a poker or soft iron rod vertically (say handle downwards), bring (1) its upper end near one pole of a suspended magnetised needle, (2) its lower end near: (3) and (4) repeat when the poker is reversed in position (handle upwards). Show that the upper end always repels the S. pole and the lower repels the N. pole of the magnet.

(*b*) Hold the poker or soft-iron rod horizontally east and west (magnetic), and bring its ends in turn up to the poles of a suspended magnetic needle. Observe that there is always attraction, that is, the poker is not polarised at its ends.

(*c*) Hold the poker vertically, hit it once or twice with a mallet. Test as in (*b*). Observe that the end that was lower is now a N. pole, the other a S. pole.

(*d*) Throw the poker down, or hit it violently. Test as in (*b*). Observe that it no longer has polarity.

Explanation.—The east and west position (*b*) is one in which the earth's induction produces scarcely any polarity. The induction is greatest along a line parallel to the dipping needle. Observe that in the northern hemisphere the lower and more northern end is magnetised N. In (*c*) the temporary magnet is made permanent, but is destroyed by violence in (*d*.)

58. Astatic Needles. It is sometimes necessary in physical experiments to have a magnetic needle upon which the earth can exert no influence; such a needle is called *astatic*. It is not possible to make a *single* needle which shall be astatic, but by combining two needles as shown in Fig. 63 the desired property is secured. The two needles must be exactly similar in length, strength, etc., and are firmly fixed on a common axis of brass or aluminium with their poles in opposite directions.

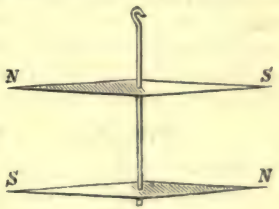

Fig. 63.

Suppose now such a combination suspended by a fine thread attached to a hook, and set so that the vertical plane through their magnetic axis makes any angle with the magnetic meridian. The horizontal force of the earth exerts a certain turning effect on the upper needle, tending to make the combination set with its left-hand portion northwards, and also a precisely equal turning effect on the lower needle, tending to make it set with its left-hand portion southwards. These two turning effects neutralise one another, and on the whole the earth exerts no influence whatever, so that the needle remains *in any position we please to put it.*

If the needles be not exactly similar, or be not fixed accurately parallel, the two turning effects will not exactly neutralise each other, and the combination will not be *perfectly* astatic; a needle *nearly* astatic is, however, sufficient for most purposes for which such needles are required (§ 209).

59. Vibrations of Suspended Magnets. Let a magnetic needle be suspended horizontally from an unspun silk fibre in a magnetic field, displaced from its position of equilibrium, and then let go. It will swing to and fro a number of times before coming to rest, and if the suspension be torsionless, as it very nearly is when the needle is suspended by a fine thread, the vibrations will continue a long time. In counting them we must make up our minds whether to take *single* swings, *i.e.* journeys *one way only*, or *double* swings, *i.e.* journeys *to and fro*; it is most convenient to take *double swings* and this we shall always suppose done. The number of swings per given time, usually per second, is called the *frequency*.

Now, if the field is uniform, or, which amounts to the same thing, if the needle is so small that the field in which it swings is practically uniform, it can be shown mathematically that *for the same needle** in fields of different intensity the square of the frequency is proportional to the intensity, or rather to what may be called the "operative intensity," that is the part of the intensity which really affects the needle; for a compass-needle oscillating in the earth's field this will be H, the horizontal component, while for a dipping needle it will be the total intensity I. With this understanding then we have

$$\text{(Frequency)}^2 \propto \text{Intensity of Magnetic Field} \quad . \quad . \quad (1)$$

It will be noticed that this is a mere *proportion*, the corresponding *equation* is considered in the higher parts of the subject and need not be considered further here.

For experiments on magnetic oscillations a convenient instrument is the *Vibration Magnetometer*.

60. Vibration Magnetometer. Fig. 64 illustrates a convenient arrangement. A wooden stool, A, about 10 inches high with narrow legs, supports a short glass or metal tube, B, into which is fixed a cork, C. A short metal rod, D, passes centrally through the cork. D is provided at the upper extremity with a disc or handle by which it may be turned round, and at its lower extremity with a hook. An unspun silk fibre carrying a metal or cardboard stirrup (the figure illustrates a metal stirrup) is

* The needle used should be magnetised to saturation (§ 19), so that its strength is unaffected by the fields in which it is placed.

hung by a loop at its extremity from the hook. The whole is enclosed in a glass shade, E, to protect it from draughts. Two narrow strips of gummed paper are fixed vertically and diametrically opposite on the sides of the shade.

A rough arrangement is to hang the magnet in a wide-mouthed bottle. The upper end of the suspending fibre is tied to a hook on the under side of the wooden cover.

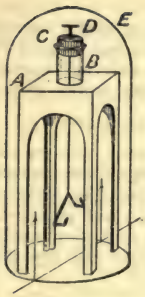

Fig. 64.

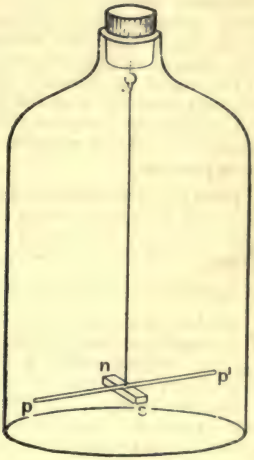

Fig. 65.

Another arrangement is to use a bell jar with an opening at the top. Fig. 65 shows such a jar. A short magnet, *ns* (say, an inch long) is suspended. In order that its vibrations may be more easily followed a glass pointer, *pp'*, is fixed by a touch of sealing wax to the magnet at right angles to its length.

Still another arrangement consists of a wooden box, say 8″ long, 3″ wide, 3″ deep. In each of the two long sides there is a glass strip that can be slid in and out. A glass tube rises from the top of the box. The suspending filament is attached to a cap at the upper end of this, passes down the tube, and supports the stirrup and magnet in the box. The cap can be twisted.

ADJUSTMENTS, ETC., OF THE VIBRATION MAGNETOMETER.

1. Mark roughly the magnetic line on the table (Exps. 30-32, *a*). Set the magnetometer of Fig. 64 so that the reference line passes, roughly, midway between its legs.

2. There must be *no twist to begin with* of the suspending filament. To avoid this place a small brass bar in the stirrup (the brass stirrup itself may be sufficient) and leave until vibration ceases—the fibre may then be assumed to be untwisted.

3. Observe the angle between the bar and magnetic line. Turn the cork through an angle equal to this so that the brass bar may rest over the magnetic line. Hold the stirrup so that no further twisting takes place, put the magnet in it and remove the brass bar. Now allow the magnet to hang freely and let it come to rest—this may be assisted by touching it lightly and carefully with the fingers. Place the glass shade over it, and adjust so that the marks on it (edges of paper strips) are on the magnetic reference line. Care must be taken not to disturb the magnet when the cover is put on.

4. *Start the suspended magnet swinging* by bringing slowly towards it a bar magnet presented end-on (when done remove this bar magnet from the neighbourhood). If there is an appreciable side to side movement of the suspension and magnet, stop and start again. *A twist only is required.*

5. Make the observations when the *arc of vibration is small.*

6. *Method of timing.* Sit in line with the magnetic line on the bench and the marks on the glass shade. Note by a watch, or start a stop watch when the magnet makes a transit, *i.e.* swings over the reference line. Again observe the time of transit when the magnet has performed, say, 50 complete swings. Divide the interval by 50 and the result is the periodic time of vibration. Repeat two or three times till you get a constant result for the period.

The simplest use of the Vibration Magnetometer is to compare horizontal intensities of fields at two places.

EXAMPLE.—A needle suspended by a silk fibre makes 27 vibrations per minute in London and 36 vibrations per minute in Sydney. If the value of H in London = ·18 dyne per unit pole, find the value of H in Sydney. By the formula we get

$$\left(\frac{\text{Frequency in London}}{\text{Frequency in Sydney}}\right)^2 = \frac{\text{Horizontal field in London}}{\text{Horizontal field in Sydney}};$$

$$\therefore \left(\frac{27}{36}\right)^2 = \frac{\cdot 18}{\text{Horizontal field in Sydney}}.$$

$$\therefore \text{Horizontal field in Sydney} = \cdot 18 \times \left(\frac{4}{3}\right)^2$$

$$= \cdot 32 \text{ dyne per unit pole.}$$

61. The Vibration Magnetometer in Compound Fields. Let there be a certain field of horizontal intensity I_1, and let n_1 be the frequency of vibration of a certain needle in it. Also let there be another field of horizontal intensity I_2, and let n_2 be the frequency of vibration of the same needle in it. Now let the two fields be superposed with their lines of force in the same direction so as to produce a compound field: the intensity of this will be $I_1 + I_2$. Let x be the frequency of vibration of the same needle in the compound field; it is required to find x in terms of n_1 and n_2.

By the formula of the preceding article we have

$$\frac{x^2}{n_1^2} = \frac{I_1 + I_2}{I_1} = 1 + \frac{I_2}{I_1} = 1 + \frac{n_2^2}{n_1^2},$$

whence

$$x^2 = n_1^2 + n_2^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

If the fields be superposed so that their lines of force point in *opposite* directions, and if I_1 be the stronger of them, the intensity of the compound field will be $I_1 - I_2$, and we have

$$x^2 = n_1^2 - n_2^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

If the lines of force of the two *cross one another* neither (1) nor (2) holds.

(1) and (2) may both be embodied in the statement:—

$$\left.\begin{matrix}\text{Square of frequency} \\ \text{in compound field}\end{matrix}\right\} = \left\{\begin{matrix}\text{sum (or difference) of} \\ \text{squares of frequencies} \\ \text{in separate fields} \quad . \quad . \quad .\end{matrix}\right. \quad (3),$$

it being, of course, understood that we take the *sum* if the lines of force of the separate fields point the same way; *i.e.* if the fields *assist* one another, and the *difference* if they are *opposed*.

When a needle vibrates in the neighbourhood of a magnet it is really in a compound field, viz. that of the magnet *and the earth*, and one of the most useful applications of (3) is to enable us to allow for the effect of the earth.

EXAMPLE.—A compass-needle* under the influence of the earth alone makes 24 vibrations per minute. When a bar magnet is placed (magnetic) south of the needle with the north pole pointing to the needle, the number of vibrations becomes 51 per minute. Find the number of vibrations per minute *due to the magnet alone*.

* The "compass-needle" referred to in this and similar questions should in practice be the magnetic needle *us* of the vibration magnetometer; an ordinary compass-needle mounted on a pivot is unsatisfactory on account of friction.

Let x be the required number. On drawing a figure it is seen that the magnet and earth *assist* one another, hence in (3) we must take the *sum*, this gives

$$51^2 = 24^2 + x^2,$$

whence $x = 45$. That is, if the earth were done away with and the magnet and needle occupied the same positions, the latter would make 45 vibrations per minute.

62. Comparison of Magnetic Moments and Pole Strengths. The field at a distance from a magnet depends upon the pole strength of the magnet, its length, and the distance of the point considered from the magnet.

The full relation may be worked out mathematically, but it will suffice to say here that if the distance of the point considered from the magnet is great in comparison with the length of the magnet the field is proportional to the magnetic moment of the magnet and inversely proportional to the cube of the distance of the point from the centre of the magnet. If, on the other hand, the magnet is very long in comparison with the distance of the point considered from a pole of the magnet, the field is practically that due to the near pole only, and is equal to the pole strength of the magnet divided by the square of the distance of the point from the near pole.

By using a small needle in a vibration magnetometer we can therefore compare the magnetic moments of short magnets and the pole strengths of long magnets, and, as shown in § 63, also check the inverse square law on which all our formulae are based. There is no satisfactory direct method of comparing the pole strengths of short magnets.

Exp. 34. *To compare the magnetic moments of two short magnets.* Mark out the magnetic meridian or magnetic line (Exps. 30-32) on the bench. Place the magnetometer near the south end of this line and find the frequency of vibration of the needle under the action of the earth's field alone. Then place one of the bar magnets, say A, some distance to the magnetic north of the magnetic, and with its S pole pointing directly to the centre of the needle. Find the frequency of vibration of the needle now. Replace A by the other magnet B, with its centre at exactly the same distance from the needle and pointing the same way. Again find the frequency of vibration of the needle. From the three frequencies find the ratio of the magnetic moments of A and B. An example will make the working clear.

EXAMPLE.—A small compass makes 12 swings per minute under the influence of the earth alone. When a bar magnet, A, is placed with its

centre magnetic north of that of the needle, and distant 20 cm. from it, and with its south pole pointing direct to the centre of the needle, the number of swings is 30 per minute. When another bar magnet, B, is substituted for A with its centre in a similar position, the number of swings is 40 per minute. Compare the magnetic moments of A and B.

Let M_A and M_B be the moments of the two magnets. By § 62, since *both magnets are placed at the same distance*, the fields which they produce at the compass-needle are proportional to their moments, and may therefore be represented by kM_A and kM_B.

The three fields concerned in the different observations are (1) the earth's, (2) the earth's plus that due to A at the given distance, (3) the earth's plus that due to B at the same distance. Then these fields are H, $H + kM_A$, $H + kM_B$, where H is the horizontal intensity of the earth's field

The strengths of these fields are proportional to the numbers 12^2, 30^2, 40^2, *i.e.* proportional to 36, 225, 400, therefore

$$\frac{H}{36} = \frac{H + kM}{225} = \frac{H + kM}{400}$$

or

$$\frac{H}{36} = \frac{kM}{225 - 36} = \frac{kM}{400 - 36}$$

or

$$\frac{M_A}{M_B} = \frac{225 - 36}{400 - 36} = \frac{189}{364} = \frac{1}{2} \text{ very nearly.}$$

Exp. 35. *To compare the pole strengths of two long magnets.* Draw the magnetic reference line on the bench and place the vibration magnetometer near the south or north end of this line, say to the north this time. First find the frequency of vibration of the needle under the action of the earth's field alone. Then take one of the long magnets, N S, and fix it as shown in Fig. 66 with its N pole in the magnetic meridian, a given short distance south of the needle, and the S pole if possible vertically over the needle. By this means the field due to the S pole is solely vertical and has no influence on the frequency of vibration of *n s*.

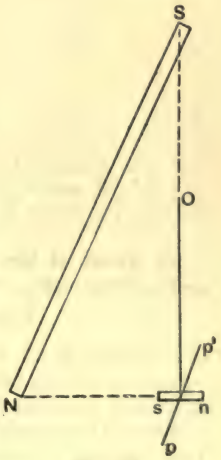

Fig. 66.

If N S is not long enough or the magnetometer is so made that N S cannot be put into this position, lay N S flat and in the meridian with S to the south. Find the frequency of vibration of *n s* now and again when N S is replaced by the other long magnet N′ S′, N′ being placed in the same position as N and the magnet similarly disposed. From the three readings calculate the ratio of the pole strength, as in the following example.

EXAMPLE.—A small compass-needle makes 10 vibrations per minute under the earth alone. A long bar magnet, A, is placed as described above with its north pole magnetic south of the needle and at a certain short

distance from it, when the number of vibrations becomes 35 per minute. A is then removed and another long magnet, B, similarly placed and at the same distance, and now the number of vibrations is 25 per minute. Compare the pole strengths of A and B.

Let H = strength of the earth's field, m_1 and m_2 the pole strength of the magnets, and r the distance of the N pole from centre of the needle. Then in the three cases the fields in which the needle is vibrating are H, $H + \frac{m_1}{r^2}$, $H + \frac{m_2}{r^2}$. This is on the assumption that the field due to the S pole has no horizontal component in the neighbourhood of the needle. The frequencies of vibrations are proportional to the numbers 10, 35, 25, therefore the fields are proportional to $(10)^2$, $(35)^2$, $(25)^2$, *i.e.* to 4, 49, 25.

Therefore
$$\frac{H}{4} = \frac{H + \frac{m_1}{r^2}}{49} = \frac{H + \frac{m_2}{r^2}}{25}$$

or
$$\frac{H}{4} = \frac{\frac{m_1}{r^2}}{49 - 4} = \frac{\frac{m_2}{r^2}}{25 - 4}$$

or
$$\frac{m_1}{m_2} = \frac{49 - 4}{25 - 4} = \frac{45}{21} = \frac{15}{7}.$$

If r is measured both m_1 and m_2 may be expressed in terms of H for $m_1 = \frac{4.5}{4} r^2 H$ and $m_2 = \frac{2.1}{4} r^2 H$. If r is 10 cms. and H may be taken as ·18 dyne per unit pole, then $m_1 = 202$ and $m_2 = 94$ C.G.S. units.

63. Proof of the Law of Inverse Squares. In the mathematical reasoning referred to in § 59 whereby the law

$$(\text{Frequency})^2 \propto \text{Intensity of Field} \quad . \quad . \quad . \quad . \quad (1)$$

is established, no assumption is made of the law of the inverse square or of any law of action between magnetic poles. We are therefore justified in employing (1) to discover what that law is.

This is Coulomb's method.* It is necessary to employ a long thin magnet, which is practically a simple magnet having point poles at its tips; the method then is as follows:—

Exp. 36. Set the magnetometer on the table and find its frequency in the earth's field: call this e. Next set a long thin magnet vertically, with its north pole at a measured distance, d_1, magnetic south of the needle, and again find the frequency: call this n_1. Then shift the magnet, so that

* Another of his methods was that of the Torsion Balance (cf. § 144), but this instrument has lost all but historical interest, on account of its lack of precision.

its north pole is at a new measured distance, d_2, magnetic south of the needle, and ascertain the frequency again: call it n_2. Then if I_1, I_2 are the field intensities at distances d_1, d_2 from the pole, we have

$$\frac{H}{e^2} = \frac{H + I_1}{{n_1}^2} = \frac{H + I_2}{{n_2}^2},$$

or

$$\frac{I_1}{I_2} = \frac{{n_1}^2 - e^2}{{n_2}^2 - e^2}.$$

Now you will find, within the limits of experimental error,* that over a great range of values of d_1 and d_2 the value of

$$\frac{{n_1}^2 - e^2}{{n_2}^2 - e^2} \text{ is equal to } \frac{{d_2}^2}{{d_1}^2}.$$

Hence you have proved that

$$\frac{I_1}{I_2} \text{ is equal to } \frac{{d_2}^2}{{d_1}^2}$$

or

$$I_1 : I_2 = \frac{1}{{d_1}^2} : \frac{1}{{d_2}^2}$$

which is the law of the inverse square.

Exp. 37. Use a vibrating short magnetic needle to determine the distribution of free magnetism in a bar magnet. Graph your results. (See § 13 and Fig. 11.)

64. Perpendicular Magnetic Fields. If two uniform magnetic fields are superposed on each other the resultant field is also uniform. Its direction and intensity are determined by the parallelogram of forces.

Let O P represent in length and direction the intensity, F, of one field, and similarly O Q the intensity, H, of the second field. Then on completing the parallelogram we get O R, which represents in magnitude and direction the resultant field.

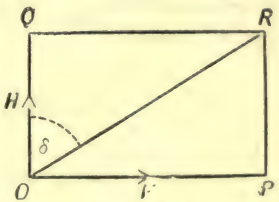

Fig. 67.

Hence when the fields are at right angles †

$$\frac{F}{H} = \frac{Q\,R}{O\,Q} = \tan \delta$$

* That is, any discrepancies which occur should be small enough to be regarded as due to unavoidable experimental errors.

† This is the only important case.

where δ = the angle between H and the resultant field. Also the intensity of the resultant field = O R = $\sqrt{F^2 + H^2}$.

If a freely suspended magnet is placed in a magnetic field it sets itself along with its magnetic axis in the direction of the resultant field. If both component fields are horizontal the resultant field is horizontal, so that instead of using a freely suspended magnet an ordinary compass-needle will do to find the direction of the resultant field. Let us suppose then that F is the strength of a horizontal magnetic field at right angles to the horizontal component H of the earth's field, then the magnetic axis of a compass-needle sets itself at an angle δ with the direction of H where

$$F = H \tan \delta.$$

Hence if H is known and δ is measured the value of F can be calculated.

Note that the position assumed by the compass-needle does not depend upon its own moment pole-strength or length. It is, however, frequently necessary to use short magnets because the magnetic fields obtained in practice are only approximately uniform within narrow limits. (See § 169 on the Tangent Galvanometer.) The magnets too should be strongly magnetised so that the effects of friction at the pivot or the stiffness of the suspending fibre are insignificant in comparison with the forces exerted magnetically.

SUMMARY.—CHAPTER IV.

1. The earth is to be regarded as a magnet whose *northern* regions possess *southern* polarity, and *vice versâ* (§ 46).

2. The direction of the earth's resultant force at any place is called the *line of dip*, and is indicated by a freely suspended needle or by the needle in the dip circle when properly set. The earth's field in a given locality is uniform, and the positive direction of its lines of force slopes downwards in the northern *magnetic* hemisphere and upwards in the southern (§§ 47, 51).

3. The vertical plane through the line of dip is called the *magnetic meridian* of the place. This does not in general coincide with the geographical meridian. A straight line drawn horizontally in the plane of the magnetic meridian constitutes the *magnetic* north-south line at the place, and it is along this that the compass-needle sets. The angle between

the magnetic north-south line and the geographical north-south line is called the *declination* of the place, and that between the magnetic north-south line and the line of dip the *inclination* or *dip* of the place. The declination is east at some places and west at others, and the dip is "north" or "south" according as the place is in the northern or southern hemisphere (§§ 48, 50, 51).

4. The intensity, I, at any point of the earth's field may be resolved into two components, one horizontal, H, and one vertical, V; it is the horizontal component alone which affects a compass-needle. The equations connecting these components with the total intensity and dip θ are

$$H = I \cos \theta ;$$
$$V = I \sin \theta \text{ (§ 48).}$$

5. Experimental proof of the directivity of the earth's *horizontal* field; behaviour of the needle under joint influence of earth and magnet (§ 49).

6. Determination of declination and dip (§§ 52, 53).

7. Elementary representation of dip and declination, and its unsatisfactoriness (§ 54).

8. The Mariner's compass (§ 55).

9. Effects of placing an iron bar (i) in the line of dip, (ii) magnetic north and south, (iii) vertically, (iv) perpendicular to the line of dip. Magnetisation and demagnetisation of a steel bar by the help of the earth (§ 57).

10. Astatic needles (§ 58).

11. Law of magnetic vibrations, viz.

$$(\text{Frequency})^2 \propto \text{Intensity (§ 59).}$$

12. Law of frequency in compound field (§ 61).

13. Comparison of Magnetic Moments and of Pole Strengths (§ 62).

14. Proof of Law of Inverse Squares (§ 63).

15. Resultant of two magnetic fields. Deduction of the formula

$$F = H \tan \delta \text{ (§ 64).}$$

EXERCISES IV.

1. Given that at Plymouth the dip = 67° and H = ·182 dyne per unit pole, find I and V. (Given cos 67° = ·391, sin 67° = ·921.)

2. How will a magnetic needle behave if the magnet be set along the meridian passing through the needle, (1) with its N pole pointing to the S pole of the needle, (2) with its N pole pointing to the N pole of the needle?

3. A short bar magnet is laid on a table with its north pole pointing due north (magnetic). A small pivoted compass-needle is placed on the table at some distance due east (magnetic) of the centre of the magnet, and is then moved up slowly to it in a straight line. Describe its behaviour as it approaches the magnet.

4. Describe and explain the movement of a small compass-needle placed in the middle of a horizontal circle round which the north pole of a long vertical magnet is carried, which produces at the centre of the circle a magnetic field less strong than that of the earth.

5. Show that there are certain localities where the north pole of a compass-needle points (geographical) south, east, and west respectively.

6. You are given a magnetised piece of flat steel plate and told that the magnetic axis lies in the plane of the plate. Describe how you would determine its direction.

7. An unmagnetised needle is suspended by a fine thread through a small hole at its centre and is balanced so as to hang freely in a horizontal position. What change in the position taken by the needle will occur if it is magnetised and then suspended as before?

8. Explain why the inclination of a dip needle at any place is less when the needle swings in the magnetic meridian than when it swings at right angles to the same.

9. A long bar of soft iron lies on the table in the magnetic meridian, and the north end of it is slowly raised until the bar is vertical. What changes in magnetism due to the inductive action of the earth does it undergo?

10. How would you place a rod of soft iron for it (i) not to be magnetised, (ii) to be magnetised as much as possible, by the earth's inductive action? Give your reasons.

11. When a rod of soft iron a yard long is held in a vertical position, each end when held downwards attracts one pole of a compass-needle and repels the other; but when the rod is held horizontally, and in a certain direction, both ends attract both poles of the needle. Explain these facts.

12. A tall iron mast is just forward of the compass of a wooden ship. Explain how this will affect the direction of the compass when the ship is sailing (i) to the east, (ii) to the north, in the northern hemisphere.

13. A compass-needle is suspended over a table. When a long bar magnet is placed in the magnetic meridian with its centre directly below that of the needle and its north pole southwards, the needle makes 35 swings per minute; but when the magnet is turned the other way round the needle also turns round and makes 5 swings per minute. Find the number of swings per minute due (i) to the magnet alone, (ii) to the earth alone.

14. A compass-needle has a frequency n_1 in one field and n_2 in another. Show that if the fields be superposed so that their lines of force are at right

angles to one another, and if x be the frequency in the resulting compound field,

$$x^4 = n_1^4 + n_2^4.$$

15. A compass-needle makes 100 oscillations in 8 minutes in the earth's field. When a magnet, A, is made to assist the earth, the needle makes 100 oscillations in 5 minutes. When another magnet, B, is substituted for A, also assisting the earth and at the same centric distance from the needle, the oscillations increase to 100 in 4 minutes. Compare the moments of A and B.

16. A small compass-needle has a frequency 15 under the earth alone. When the north pole of a long bar magnet, A, is placed 30 cm. (magnetic) north of it, it *does not* turn round but the frequency is reduced to 5. When A is removed, and the N pole of another long bar magnet, B, placed 25 cm. north of it, it *does* turn round and the frequency becomes 15 again. Compare the pole strengths of A and B.

17. How might the pole strengths of two *short thin* magnets be compared?

18. If you wish to support a uniform bar magnet horizontally on a pivot, how is it that the pivot must be placed nearer to one end than to the other? To which end must it be nearest in this country?

19. An astatic combination consists of two magnets at right angles instead of parallel to each other. If it be suspended as usual, what position will it assume with regard to the magnetic meridian? Illustrate your answer with a diagram showing the forces which act upon the magnets.

20. A large soft iron rod lies on a table in the magnetic meridian, and a dipping needle is placed at some distance and at about the same level, (1) due south, (2) due north of it. How will the magnitude of the angle of the dip be affected in each case? (Neglect any inductive action between the needle and the bar.)

21. Given a magnet and the means of suspending it. How will you determine (1) the magnetic meridian, (2) in which direction *north* lies? It is assumed that you do not know which end of your magnet is a north and which a south pole.

22. Give some account of the Mariner's Compass and of Magnetic Charts. In a recent Antarctic Expedition under Captain Scott, at a certain part of the voyage it was desired to go *east*, but orders were issued to steer *west* by the compass: explain this.

23. A magnetised needle is concealed within a wooden sphere but is known to lie along a diameter with its centre at the centre of the sphere. How would you find the position of the needle if the direction of the magnetic meridian were given? How would you use the sphere to find the direction of the magnetic meridian without knowing the position of the needle?

24. How would you magnetise a thin steel rod 12 inches long so that it might have north poles of equal strengths at the ends and a south pole 4 inches from one end? What would be the strength of the south pole?

How would such a magnet behave if placed horizontally upon a cork floating in water?

25. An iron ship is built with its bow to the north. How would you expect its permanent magnetism to affect a compass-needle on deck amidships, (*a*) when sailing eastward, (*b*) when sailing southward?

26. What is meant by a uniform magnetic field?

A steel rod hangs vertically from the pan of a balance and its weight is observed. It is then magnetised strongly and weighed again with its N pole pointing downwards. Will any change be observed? What will be the effect upon the apparent weight of the rod before and after magnetisation of holding under it a thin disc of soft iron, (1) with its plane faces vertical, (2) with its plane faces horizontal?

Give reasons for each part of your answer.

27. A magnet 10 cm. long is placed in the magnetic meridian, the north end of the magnet being to the south. The force due to this magnet just counterbalances the earth's horizontal magnetic force (·18 C.G.S. units) at a place 35 cm. from the centre of the magnet along its axis produced. Find the strength of each pole of the magnet.

PART II.

ELECTROSTATICS.

CHAPTER V.

ELECTRIFICATION.

65. Some Fundamental Experiments and Facts.

Exp. 38. Rub a piece of sealing-wax with a dry flannel or cloth and hold it near (Fig. 68) some small scraps of paper or some light bodies such as feathers, bits of cotton-wool, pieces of pith, bran, etc. These bodies jump up to the sealing-wax. Note their further behaviour.

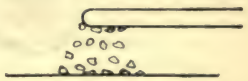

Fig. 68.

The sealing-wax when rubbed exerts a force tending to pull bodies towards it; a force which does this is said to be a force of attraction and the sealing-wax is said to attract the paper, etc. Other substances will produce this same effect.

Exp. 39. Dry well a piece of brown paper before the fire,* rub it with dry flannel, and repeat Exp. 38.

Exp. 40. Take a glass rod about a foot long and half an inch in diameter, dry it well before the fire, and repeat Exp. 38, using dry silk as a rubber.

Exp. 41. Rub a dry ebonite rod with fur and repeat Exp. 38. The ebonite behaves very well indeed; with it fairly large pieces of paper, cork, etc., can be lifted.

The first thing that was ever known to possess the property of attracting light bodies when rubbed was *amber* (the Greek word for amber is elektron, from which we get the word *electric*); this

* All bodies used in electrical experiments must be warm and quite dry. To make them so place them on a hearth in front of a good fire.

fact was discovered about the year 600 B.C. by Thales of Miletus, but it was not until the time of Queen Elizabeth that it was found out that many other things would behave in the same way: this was done by her physician Dr. Gilbert (see also § 1).

Do not confuse the *force* of attraction with the *movement* of the attracted bodies: thus, if a small piece of cork be held in the fingers, and an ebonite rod rubbed with fur be placed two or three inches from it, the cork, of course, cannot move because it

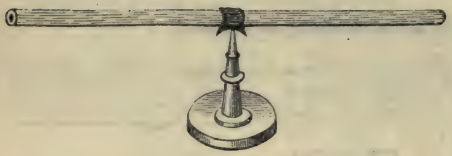

Fig. 69.

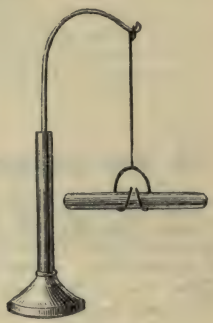

Fig. 70.

is held fast; but the rod is attracting it all the same. If the rod be held over a heavy body, such as an iron or wooden bar, the latter is not lifted simply because the attractive force is not strong enough to overcome its weight; but it is only necessary to balance the bar on a pivot as in Fig. 69, or a stirrup as in Fig. 70, and hold the rubbed ebonite rod a few inches from it towards its side, when it moves readily towards it.

Exp. 42. Balance a rod (a metre scale or a lath) on a convex surface such as an egg in an egg cup, or on a curved watch-glass or on a round-bottom flask fixed upside down, or suspend it horizontally by a thread. Hold a rubbed piece of ebonite near one end and to the side. Attraction occurs and motion ensues. Try whether a metal rod or a dry wooden rod behaves the better.

66. Electrification. Whenever a body shows this power of attracting things it is said to be *electrified*, or to possess an *electrification*. It is also said to be *charged*, or to possess a *charge*. The two latter terms will be assigned a more definite

meaning in §§ 88-90. Bodies in their ordinary state do not, of course, exhibit this property; such are said to be *neutral*. Hence the general facts of § 65 may be summed up in the statement that **electrified bodies attract neutral ones.**

67. All Forces are Mutual Actions. As with magnetic forces, electric forces are mutual. An electrified body attracts a neutral one, and *the neutral one attracts the electrified one with equal force.* In the preceding experiments matters are so arranged that the neutral one is free to move while the electrified one is held fast, but it is easy to arrange them the other way; thus, if an electrified ebonite rod be balanced on a pivot, and a neutral wooden one held in the hand near it, the ebonite will move up to the wood.

Exp. 43. Take the piece of brown paper of Exp. 39, again brush it, and hold it near the papered wall of the room. It is strongly attracted and may adhere to the wall for nearly a minute.

68. Electricity. Now, when it was discovered that so many things could be electrified, men asked *why* it was; and they began to make out several theories. But they really had not enough facts to build a theory upon; and, even now that very much more is known, scientific men are not certain as to the cause. But we are sure that there must be *something* which is *somehow or other* affected in such a way as to produce the attraction, and to that something we give the name *electricity*.

We must be very careful not to confuse between electricity and electrification. "Electrification" is merely a name given to *the condition of the electrified body*, while "electricity" is the mysterious "something" which renders it possible to produce that condition. Nobody knows for certain what electricity is, or what happens to the electricity when a body is electrified, though on this latter point we shall see, as we go on, we can make a very reasonable supposition (§§ 88-90).

One thing, however, we must guard against from the beginning: we must never suppose that when we electrify a body we *make any electricity*; it is much more likely that we only alter the condition of what already exists, so that it can do what it could not do before,—in something the same way as, when we wind up a watch, we do not *make* the spring, we merely alter the arrangement of its coils, and then it can do what it could not do before—viz. make the watch "go."

69. Induction or Influence. In § 66 we have said that "electrified bodies attract neutral ones." But here a question arises. A piece of cork is lying on the table: it is "neutral." We then bring over it an electrified rod, and it is attracted. *Is the cork neutral while the rod is near it but not actually touching it?* This question can be answered only by experiment, and we shall learn in the next chapter that the cork is certainly *not* neutral, but that it becomes electrified by the *external influence of the electrified rod.*

Such electrification is called *induced* electrification, or electrification by *induction* or *influence.* The same is true in every case: the attracted body, which we say is neutral, is really not so, but is electrified by induction or influence. When, therefore, we say that an electrified body attracts a neutral one, we mean, not that the latter is actually neutral *while it is being attracted*, but that *it would be neutral but for the presence of the attracting body.*

We shall deal more particularly with induced electrifications in the next chapter. An electrification which is not the mere result of external influence, but, so to speak, belongs properly to the body considered, is sometimes spoken of as a *free electrification*, or a *free charge*: such is the charge imparted to a body by rubbing. In general, when we speak of "electrification" or "charge," we mean it to be free, unless the contrary is specified.

Fig. 71.

70. Pith-ball Electroscope. This is a convenient little instrument for ascertaining practically whether a body is electrified or not. It consists (Fig. 71) of a small ball of elder-pith (which is very light) hung on a stand by a fine silk thread. To test a body we simply hold it near the pith ball and observe whether the latter moves towards it; if it does the body is electrified, if not it is neutral. Care must of course be taken that to begin with the pith ball is neutral, otherwise, for the reason in § 67, the experiment will be misleading. Should the ball have become electrified by accident, or as the result of some previous experiment, it may be rendered neutral by gently squeezing in the fingers (§ 79). The process of rendering an electrified body neutral is termed *de-electrifying* or *discharging.*

A double pith-ball electroscope is described in Exp. 51.

71. Can All Substances be Electrified? This is one of the questions Dr. Gilbert set himself to answer, and the pith-ball electroscope enables us to do so for ourselves. We find by experiment that any of the substances mentioned in LIST 1 (see below), if held in the hand and rubbed with almost any soft material, will afterwards attract the pith ball, and is therefore electrified.

But if we treat any of those in LIST 2 *in the same way* it will not affect the pith ball, and is therefore not electrified.

LIST 1.

Amber.	Gutta-percha.
Sealing-wax.	India-rubber.
Resin.	Glass.
Sulphur.	Ebonite.

LIST 2.

ANY METAL, *e.g.* iron, copper, brass, etc.	Paper under ordinary circumstances.
Ordinary wood.	Vegetable tissue.
Bricks, stones, etc.	Animal tissue.

If we take any of the things in LIST 2, and *hold them in the hand and rub them*, they do not become electrified; but this is a very different thing from saying that they cannot be electrified *by any means whatever*.

72. Insulators and Conductors.

Exp. 44. Make the instrument shown in Fig. 72. A is an ebonite rod with a socket D D, and C is an iron rod with an end put in the socket and held fast (if necessary by the screw B). Now hold A in the hand (taking care not to touch B or C), and gently beat C with a dry rubber and then hold it near the ball of the pith-ball electroscope; it will attract it strongly. By this means, therefore, *the iron rod has been electrified*, and any of the substances in the second list of § 71 can be electrified in the same way.

The essential point to be remembered when it is required to electrify things in LIST 2 is that they must not be held directly in the hand, but must be *mounted on a handle made of one of the substances in the first list*, the best being ebonite. Dr. Gilbert did

not know this: he thought that none of the things in the second list could be electrified, so he called them *non-electrics*, while those in the first list he called *electrics*; but these terms convey quite wrong ideas and are now obsolete. The things in the first list are now called *insulators*, and those in the second *conductors*. We shall learn in § 75 *why* these terms are used; just now we will regard them as mere names.

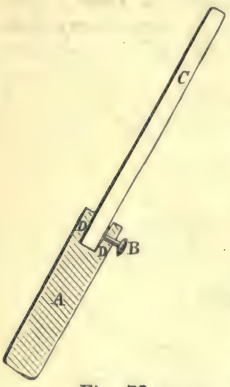

Fig. 72.

73. Discharging a Conductor. If in Exp. 44, after rubbing the rod and satisfying ourselves by the pith ball that it is electrified, we touch it with the finger and again try it with the pith ball, we shall find that it will no longer affect it; we thus learn that touching a freely electrified conductor reduces it to a neutral state, *i.e.* (§ 70) *deelectrifies* or *discharges* it. The same effect is produced by causing the electrified conductor to come into contact with the walls of the room or the floor. If a metal or wooden rod is stuck in the floor and an electrified conductor allowed to touch the top of it, the conductor will be at once discharged, but if the rod is of glass or ebonite or gutta-percha it will remain electrified.

74. General Conclusion. Now, remembering that metals, wood, stone, human flesh, etc., are what we have termed conductors, while glass, ebonite, etc., are what we have termed insulators, we learn from the experiments in §§ 72, 73, that *a conductor can be freely electrified when and only when an insulator is interposed between it and the earth, and also that after a conductor has been thus electrified it becomes discharged by making a conducting path from it to the earth.*

When an insulator is interposed between a conductor and the earth the conductor is said to be *insulated*, while, when there is a conducting path all the way, it is said to be *earthed*. Thus *we can freely electrify an insulated conductor but not an earthed one, while earthing a freely electrified conductor at once discharges it.*

75. Explanation of Difference. An electrified conductor remains electrified while insulated, but becomes at once de-electrified when earthed. And the most natural explanation *so far* is that the electrification is due to *something which can flow along conductors but not along insulators*; to this "something" we give the name *Electricity* (cf. §§ 88-90). When the electrified conductor is earthed this electricity flows along the conducting path into the earth and is lost to *us*, though it by no means follows that it is *really* lost, while when insulated the electricity remains on the conductor. And now we see why the terms "conductor" and "insulator" are used: a conductor is a substance which allows electricity to flow along it, that is, *conducts* it; while an insulator or non-conductor is one which does not do so, but blocks the way against its passage and so keeps it in its original place.

It is clear now why we cannot electrify a conductor while it is held in the hand: the human body is a conductor and cannot block the way, so that the electricity cannot remain on the conductor. Of course, with a non-conductor the electricity cannot flow along it, so that it serves as its own insulator. Incidentally we learn too that the main body of the earth is a good conductor, otherwise, of course, the electricity could not so readily pass into it.

Conductors may be roughly compared to water pipes, and insulators to taps *turned off*. The tap blocks the flow of the water, and the insulator blocks the flow of electricity. Turning on the tap corresponds to earthing the conductor.

76. Part played by Air and Moisture. Dry air is an excellent insulator, so that when a conductor is on an ebonite stand it is insulated from the earth all round; if the air were a conductor, the ebonite stand would, of course, be useless, as the charged conductor would give up its charge to the air.

Unfortunately *water* is a conductor, and therefore anything loses its insulating power if wet. None of the substances in List 1, § 71, can be electrified by holding in the hand and rubbing if they be damp themselves, or if the rubber be damp. For numerous electrical experiments, we require electrified conductors, and it is best to have a supply of metal balls, rods, etc., mounted

on ebonite supports. Not unfrequently glass supports are employed; but they are not nearly so good, because the moisture of the air sticks very persistently to the surface of the glass, which it does not do to ebonite; the evil may be to some extent remedied by coating the glass with a layer of shellac varnish. Silk is a good insulator, hence sometimes metal balls are hung up by silk threads: unspun silk answers best,—it absorbs very little moisture.

It should be observed that *moist air* itself is an insulator, for it is merely air containing the *vapour* of water, and this possesses very little if any conducting power; it is only water in the liquid form that is a good conductor

When electrified bodies lose their charge by exposure to damp air it is because such air deposits a thin layer of *liquid* moisture on the insulating supports and so renders *them* conductors.

77. Classification of Conductors and Insulators. The conducting and insulating powers of different substances vary greatly, and it is not possible to draw a sharp line between conductors and non-conductors; some substances, such as wood and paper, can hardly be called the one or the other. It is usual to give some such classification as the following, it being understood that the "good insulators" must be dry, as otherwise they cease to be such.

Bad Insulators.	Metals. Gas coke. Charcoal. Graphite. Acids. Metallic salts. Water. The Body.	Good Conductors.
Partial Insulators.	Linen. Cotton. Wood. Stone. Marble. Paper. Ivory.	Partial Conductors.

Good Insulators.	Oils. Porcelain. Wool. Silk. Sulphur. Gutta-percha. Shellac. Sealing-wax. Ebonite. Paraffin-wax. Glass. Air.	Bad Conductors.

78. Distribution of Electrification. If what we have said in § 75 as to *flow* is true, we should expect that if we took a metal rod (mounted on an insulating handle) and rubbed one end, the electrification would distribute itself all over the rod, whereas with an ebonite or glass rod we should expect it would remain at the end rubbed. Experiment tells us that this is the case, for if in Exp. 44 we rub the top of the iron rod all parts of it will attract the pith ball, while if we rub the top of an ebonite rod it is only the top that will attract. Thus *an electrification distributes itself all over a conductor*, but not over an insulator unless, of course, the whole insulator be rubbed.

A word of caution is needful here: we say an electrification distributes itself all *over* a conductor, not all *through* it. We shall learn later that the electrification on a conductor is always *all on the surface*,—it does not penetrate the interior at all; whereas, curiously enough, in non-conductors it *does* penetrate the interior to some extent (§ 122).

79. Transference of Electrification; Electrification by Contact. The view set forth in § 78 leads to another important point.

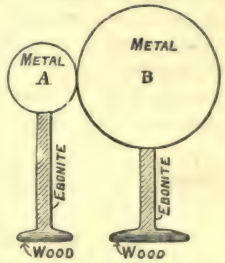

Fig. 73.

Exp. 45. Take two conductors, say two brass balls*, each mounted on an ebonite support as in Fig. 73, and place

* Brass door-handles or bed-post knobs are convenient. Balls covered with tinfoil can be obtained at toy shops. If bases are not provided the conductors may be supported in clamps or suspended by white silk thread.

them touching each other as shown in the diagram, or some distance apart and connected by a copper wire. Then they are in conducting or "electrical" communication, and, indeed, from an electrical point of view are *all one conductor*. Now electrify A, then separate them and test each separately with the pith-ball electroscope; each will be found to attract the ball, hence the electrification has distributed itself over *the entire surface*, both A and B being electrified.

Exp. 46. Keeping the balls quite separate again electrify A and afterwards put it into conducting communication with B (care of course being taken not to earth A or B). Then separate A and B and test them with the electroscope. Both will be found to be charged. This way of electrifying B is called *electrifying by contact*: the electrification of B is manifestly due to part of the electricity which existed on A flowing on to B, the rest remaining on A.

There is no difference between electrification obtained by contact and that obtained by direct friction, except as respects the way of giving it; but it should be noticed that, since electricity will not flow over non-conductors, we must, if we wish effectually to electrify them, employ direct friction: we cannot impart any considerable charge to or take such from them by contact. If we touch an insulated metal ball with an electrified ebonite rod, the ball will receive a certain electrification, but only from those points of the rod which the ball has actually touched, so that the electrification acquired will be small. In like manner, if an electrified ebonite rod be touched by the finger it will be de-electrified at the place touched, but nowhere else: in order to de-electrify it completely it must be squeezed in the hands along its entire length. Elder-pith is not a very good conductor, and hence it is (§ 70) that in discharging the pith ball we must squeeze it all over.

It is more convenient to have our pith ball a conductor. To do this moisten the ball with the least amount of gum and roll the ball on some gold leaf.

By the contact method substances such as water can be electrified when direct friction is out of the question.

Exp. 47. Place a small glass of water on an insulating stool,* and touch the surface of the water with a strongly electrified conductor. Prove by the pith-ball electroscope that the water is electrified.

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* An insulating stool or stand is a very small table mounted on glass or ebonite legs (ebonite being preferable), or simply a piece of wood resting on two glass tubes or some blocks of paraffin wax lying on the bench.

The contact method is very useful when we wish to electrify some conductor or arrangement of conductors which we cannot conveniently rub. For this purpose we merely take an insulated metal ball, rub it, and then touch the conductors with it, repeating the process until we have as strong an electrification as is desired.

The same method is also convenient for examining the electrification of any conductor which cannot readily be moved so as to bring it near an electroscope: in this case we take a small insulated ball, allow it to touch the conductor, and then carry the ball to the electroscope. Sometimes a *proof-plane* is used for this purpose; it is simply a small flat circular piece of brass, about as big as a shilling, mounted on an insulating handle. This instrument has also more elaborate uses, as will be explained in § 123.

80. Electrical Repulsion. We have seen that if we hold an electrified body near the pith-ball electroscope the ball will be attracted. But we cannot work with this instrument without discovering something else, viz. this:—The ball is attracted, and if it be allowed to touch the electrified body, is afterwards driven away from it or *repelled*; and sometimes the repulsion is so strong that the ball will insist on keeping at a distance of several inches from the electrified body. Why is this? In the first place we know that when the ball touches the rod it receives a contact electrification, and as the ball is suspended by a silk thread which is an insulator this electrification must remain on the ball. So that it is evident that the repulsion must be caused by the electrification of the ball. And an additional experiment renders this a certainty, for we have only to discharge the ball when it again becomes *attracted*. We thus learn that an electrified body may, at any rate under some circumstances, *repel* another electrified body.

Exp. 48. Rub a dry glass rod with silk and suspend it as in Fig. 70 or mount it as in Fig. 69. Take another glass rod, rub it with silk and hold it near one end of the mounted rod. Repulsion ensues.

Exp. 49. Repeat Exp. 48, using now ebonite rods rubbed with fur. Again repulsion ensues.

Exp. 50. Now suspend a glass rod rubbed with silk and hold near one end of it an ebonite rod rubbed with fur. Attraction ensues. As a variation mount the ebonite and hold the glass near. Attraction again ensues.

The actions described in Exps. 48-50 are extremely important, and may be summarised thus:

Two glass rods rubbed with silk *repel* one another,
Two ebonite rods rubbed with fur *repel* one another,
A glass rod rubbed with silk and an ebonite rod rubbed with fur *attract* one another.

81. Two Kinds of Electrification. Now let us follow up these experiments and see what we can learn from them. Consider a glass rod rubbed with silk, and call it A. Then a glass rod rubbed with silk *repels* A, while an ebonite rod rubbed with fur *attracts* A. Since, then, the glass rod rubbed with silk and the ebonite rod rubbed with fur behave differently towards one and the same thing, A, they must be in *different electrical conditions*; in other words, the electrifications on them are in some way different, though in *what* way we cannot so far tell. Further, since a pair of glass rods rubbed with silk repel each other, and a pair of ebonite rods rubbed with fur do the same, while one glass rod rubbed with silk and one ebonite rod rubbed with fur attract each other, we learn that bodies in *similar* or *like* electrical states *repel*, and bodies in *dissimilar* or *unlike* electrical states *attract*.

The experiments therefore teach us two things, namely:

(1) *There are two different kinds of electrification.*

(2) *Bodies possessing like electrifications repel, those possessing unlike electrifications attract.*

We must be very careful to note that the experiments teach us *no more than this*; we have no right to infer that there are two kinds *of electricity*—all we can say is that there are two ways in which the electricity can manifest itself.

We can now explain why the pith ball is repelled after touching the electrified rod: it is because it receives by contact an electrification of the *same kind* as the rod. But this only carries us one step, for nobody knows *why* two similarly electrified bodies should repel each other and two dissimilarly electrified bodies attract each other.

The second of the above propositions is frequently enunciated thus:—*Like electrifications (or charges) repel, unlike attract.*

Strictly speaking we have not yet shown that there are *only* two kinds of electrification. It is, however, easy to show experimentally that *all*

electrified bodies can be divided into two classes, A and B. Bodies in class A repel one another but attract neutral bodies or those in class B. Also bodies in class B repel one another but attract neutral bodies or those in class A. This settles the question.

The kind of electrification that exists on a glass rod rubbed with silk is termed a *positive electrification* or a *positive charge*, that on an ebonite rod rubbed with fur a *negative electrification* or *negative charge*. We shall see in § 88 that these terms are very appropriate; for the present we regard them as mere names. The expressions *positive electricity* and *negative electricity* are frequently used, but it is best to avoid their use, as they seem to commit us to the notion that there are two kinds of *electricity*.

Exp. 51. Take some elder pith; cut off two equal sized pieces with a sharp knife. Roll them between the fingers to remove any sharp points or edges, and suspend them by equal lengths of dry silk fibre from the same support.* (1) Rub a glass rod or tumbler† with silk. Bring the glass near the pair of balls: observe they are attracted, and that after contact they are (i) repelled from the glass, and (ii) repel each other; also (iii) they repel each other when the glass is removed. (2) Bring the finger near the balls: they are attracted, and after contact are not repelled by it, nor do they repel each other.

Explanation.—In (1) the glass becomes electrified when rubbed with silk, and the balls become similarly electrified after touching the excited glass: the similarly charged bodies repel. In (2) the balls are attracted by the neutral finger. On contact, the charge is given to the finger, and thus lost.

Exp. 52. Lay two strips (8 inches by two inches) of thin paper† on the table; rub them with the palm of the hand. Observe that the paper clings strongly to the table (§ 67). Lift the strips from the table, one by the right, the other by the left hand; hold the top edges in contact. Observe that the strips do not hang vertically, but at an angle with one another due to their mutual repulsion.

82. Further Study of the Electrified Bodies of § 81. If we take a positively charged glass rod, balance it on a pivot and hold near it a negatively charged ebonite rod, it will be *attracted*. But now, instead of the ebonite rod, let us take a neutral body; then, on holding *it* near the glass rod, the latter will again be *attracted*.

* Such an arrangement constitutes a double pith-ball electroscope. If the support be of ebonite the suspension may be of cotton. Cotton is a conductor, so that the charges on the balls are always shared equally between them.

† Must be warm and dry.

Hence, if we hold near the glass rod a body whose electrical condition we do not know, and find that it is *attracted*, we are left in doubt as to whether the body was originally negatively electrified or neutral. There is nothing surprising in this when we remember that the inductive action (§ 69) of the glass rod modifies the state of the body, so that what it does *when it is near the rod* is no sure test of its state *when it was away from it*. But there is a still more striking instance of this. If we take two glass rods, one very strongly and the other very feebly electrified by rubbing with silk, place one on a pivot and hold the other near it, we shall find that they attract!

Now at first sight this will appear to be a contradiction of the law that similarly electrified bodies repel; but here again we must remember that we have inductive action: the feebly electrified rod is, before the other is brought near, nearly neutral, and the influence of the stronger one so much modifies its state as actually to overcome the effect of its original electrification. So that whenever a positively electrified body attracts one which *before it was brought near* was either neutral or positively electrified, it is because the very fact of bringing it near utterly alters its original condition, in other words *the very test applied alters the character of the thing to be tested*. There is therefore no contradiction.

Of course, all these remarks also apply, *mutatis mutandis*, to negatively electrified rods. And they lead to a very obvious practical conclusion: viz. that the mere attraction of a body by one having known electrification gives us *no reliable information* as to the electrical state of the body we are examining, or, as it is generally expressed, **attraction is no test of electrification.**

But suppose we set on the pivot our glass rod which has been rubbed with silk (and is therefore positively electrified), and hold near it a body whose electrical state we do not know, and find that it is *repelled*. What does *that* tell us? Well, the body under examination clearly cannot before it was brought near the rod have been negatively electrified, for then it would be attracted. Neither can it have been neutral, for then it would also have been attracted. *Therefore it must have been positively electrified.* Similarly if it had repelled an ebonite rod rubbed with fur it must have been negatively electrified. Here, then, we have a sure and certain test: **the repulsion of a body by an electrified rod shows that the body originally possessed the same kind of electrification as the rod.**

83. How to Test a Body. Practically the condition of a body may be tested as follows:—

Take a positively electrified glass rod mounted on a pivot, a negatively electrified ebonite one mounted on another pivot, and a neutral pith-ball electroscope; these three being placed some distance apart to prevent any appreciable inductive action. Then take the body under examination—*e.g.* an insulated metal ball—and bring it near the pith ball: if the latter does not move, the body is neutral, and we need go no farther. But if the pith ball moves the body is electrified, and the two rods must be used to find out the *kind* of electrification: first try one rod and then the other.

The body can never *repel both*: what usually happens is that it repels *one*, and it is then known that its electrification is of the same kind as that of the rod repelled. If it should *attract both*, which does not often occur, it follows that its electrification is weak, but its kind is still doubtful; it is in that case sometimes recommended to bring the body gradually up to first one rod and then the other from a distance, so as to try and catch the repulsive effect before the inductive action becomes strong enough to overcome it, but in practice this does not work well.

Sometimes, in place of the rods, two pith-ball electroscopes are employed of which the balls have been electrified by contact, one positively and the other negatively. But in neither case is the method so delicate as could sometimes be wished, and it is better to use the gold-leaf electroscope as explained in § 99.

84. Electrification of Sundry Bodies. Suppose we now take any body at random, and rub it with any rubber at random, we can by § 83 find out the kind of electrification it acquires. We shall find results such as these:—

Sealing-wax	rubbed with	flannel	becomes	negatively	electrified.
Wood	,, ,,	,,	,,	,,	
Paper	,, ,,	fur	,,	,,	
Glass	,, ,,	,,	,,	,,	
Resin	,, ,,	cloth	,,	,,	,,
Paper	,, ,,	india-rubber	becomes	positively	electrified.

We shall also find that a piece of *rough* or "*ground*" glass rubbed with silk becomes negatively electrified. When we speak of a

"glass rod" we always mean ordinary smooth glass unless the contrary is stated. A kind of rubber sometimes used is made by smearing a piece of silk with a paste consisting of tinfoil, mercury, and resin-ointment ground up together; this paste is called *electric-amalgam*, and a rubber thus prepared is called *amalgamated silk*. Almost anything, even ebonite, when rubbed with this becomes *positively* electrified; and almost anything, even glass, when rubbed with fur becomes negatively electrified.

It is clear from all this that the kind of electrification acquired by a body depends not only upon the body itself, but also upon the rubber.

85. How to Electrify a Conductor. The following case is of great practical importance:—

Any common metal (such as iron, brass, copper, etc.) becomes positively electrified when rubbed with india-rubber, and negatively when rubbed with fur.

We thus have a practical method of electrifying our metal balls, rods, etc., either positively or negatively. The best way is to hold the fur or a sheet of thin india-rubber—a piece of an inner tube of a cycle—in the left hand, and with the right beat the metal sharply against it, taking care, of course, not to earth it by allowing it to touch the fingers. One stroke against the fur or three against the india-rubber are generally enough to produce a fair electrification. If the conductor to be electrified be not conveniently movable, we may electrify one of our balls in this way, and transfer the electrification to it by contact (§ 79).

Exp. 53. Insulate a piece of metal (an iron rod mounted on an ebonite handle or stuck in a glass tube will do) and beat it with india-rubber. Hold the metal near a glass rod which has been rubbed with flannel and suspended as in Fig. 70. Repulsion ensues; hence the metal is positively electrified.

86. Both Kinds of Electrification produced at the Same Time.

Exp. 54. Take a piece of fur, A, (a camel-hair brush is more convenient), and mount it on an ebonite handle, and also take a neutral ebonite rod, B. Have ready a positively and a negatively electrified rod, each balanced on a pivot. Rub one end of the ebonite rod B with the insulated brush A, and afterwards separate them. Hold B near the negative rod on the pivot: it will be repelled, showing that it is negatively charged, as we already knew. Then hold the brush A near the positive rod: it will be repelled, showing that *the brush is positively charged.*

Thus the ebonite and the brush, when rubbed together, both acquire electrification, but of opposite kinds. Now, the same thing happens in every case, as can be shown by similar experiments, using other substances and other rubbers. The only reason for insulating the rubber is because most rubbers belong to the class of partial conductors (§ 77), so that unless insulated most of their electrification passes into the earth and it is difficult to detect it. It should be noted, too, that whenever two *exactly similar* substances are rubbed together, neither of them, as a rule, becomes electrified. The conclusion may therefore be stated thus:—*Whenever by rubbing together two bodies one of them becomes electrified, the other always becomes electrified at the same time but in the opposite way.*

Frictional Order. The following substances are arranged in what is sometimes called *frictional order*; that is, if any two of them be rubbed together, the one *highest* in the list becomes electrified *positively*, and the one *lowest negatively*.

1. Fur.
2. Wool (flannel, cloth, etc.).
3. Wood.
4. Shellac.
5. Resin.
6. Sealing-wax.
7. Glass (smooth).
8. Cotton.
9. Paper.
10. Silk (plain).
11. Glass (rough).
12. The hand.
13. Common metals.
14. India-rubber.
15. Amber.
16. Sulphur.
17. Ebonite.
18. Gutta-percha.
19. Silk (amalgamated).
20. Gun-cotton or Collodion.

It should be remembered, however, that in some cases the results are rather uncertain, on account of variations in different specimens of the materials. But there is no uncertainty about the standard cases of glass rubbed with silk, ebonite with fur, and metal with fur and with india-rubber.

87. The Two Electrifications Neutralise Each Other. We must now resume the consideration of § 86, and we will begin by adding a fresh experiment.

Exp. 55. Take the mounted brush A and the ebonite rod B, and see that everything to begin with is neutral. Hold the ebonite rod by one end and rub the other with the brush, but *do not separate them.* Bring them both together near the pith-ball electroscope: there will be *no* movement.

Now, we know by § 86 that the ebonite is negatively and the brush positively electrified; this new experiment, therefore, tells

us that *the two electrifications neutralise one another*, in other words, the ebonite and brush taken as a whole are *not electrified*, they are *neutral*.

A like result is observed in all cases. If we rub an insulated brass ball with an insulated brush, the two together will not affect the electroscope, though either separately will do so. (See also § 116, Exp. 79.)

88. Electricity. One-Fluid Theory. We have now sufficient experimental evidence to enable us to form *some* idea of the nature of electricity and of what happens to it when bodies are electrified. The evidence in question consists of these three facts :—

1. There are two kinds of electrification.
2. When one is produced (by friction) the other is produced at the same time.
3. They are produced in such a way as when taken together to neutralise one another.

To this may be added that whenever two electrifications of the same kind are given to the same body they increase each other's effects, whereas when two different kinds are given they always neutralise each other, at any rate *partly*. This is a minor point. It may, however, be noticed that the fact that the two electrifications always tend to neutralise one another is sufficient to justify the terms *positive* and *negative* : if we place a positive electrification on a body and then a negative one, the latter act is equivalent to subtracting more or less of the positive one ; hence the terms positive and negative have the same kind of meaning as the signs + and − in algebra.

Now one reasonable way of regarding two things which can neutralise each other is to look upon one as caused by an *excess* of something, and the other by a *deficit* of that same something. Let us picture every body in its ordinary or neutral state as containing a stock amount of something, and let us call that something *electricity* (cf. § 75). Let us further picture a freely and positively electrified body as containing *more* than its stock amount—*i.e.* an *excess* of electricity ; and a freely and negatively electrified one as containing *less* than its stock amount—*i.e.* a *deficit* of electricity. Then we shall have a reasonable view of things, and one that will carry us a long way in the explanation of facts.

At the outset this view enables us to assign a distinct meaning to the term *charge*. By a *positive charge* is to be understood the excess of electricity on a body over and above its stock amount, and by a *negative charge* the deficit of electricity on a body below its stock amount. Thus *electrification* and *charge* do not convey exactly the same idea: the former is a mere name for the condition of an electrified body, while the latter is that to which this condition is due. And to guard against future mistakes it should be noticed that it is only *free* electrifications which can properly be regarded in this way, for, as will be seen in § 106, a body under inductive influence may exhibit electrification simply as the result of some of its electricity being displaced from one part of it to another.

In § 75 we saw that electricity could flow through conductors. And because it can thus flow it is called a *fluid*. Beyond that we know next to nothing about it: it resembles no known material fluid, such as water or air. Most probably it is some form or condition of the *aether** which fills all space and is mixed up with all matter. We can only study its effects and measure its amount indirectly. Moreover, we have no knowledge of the *actual amount* of electricity in any body; all we can deal with is *excesses or deficits*, i.e. *charges*.

The ideas herein set forth constitute what is known as the *one-fluid theory* of electricity. It was originally propounded by Benjamin Franklin about the middle of the eighteenth century, was neglected for a long time in favour of Symmer's *two-fluid theory*, but of recent years has been revived. In its modern form it remains essentially as Franklin gave it; but the view taken of electrical actions is greatly modified, because we now recognise what was so strongly insisted on by Faraday—*viz.* that the insulating medium (air, etc.) which surrounds an electrified body really plays a very important part in its behaviour (see § 91).

It is not claimed for the one-fluid theory that it is *true*: all that is claimed is that it explains a great many electrical phenomena *more simply* than any other theory, and for that reason we shall adopt it.

In accepting for the time being the one-fluid theory there is this which must not be overlooked:—the theory may in the main

* See *Senior Light*, pp. 1, 213.

be true, and yet we may be wrong in ascribing positive electrification to an *excess* of the fluid. It is a mere chance that the word "positive" has been used to denote the electrification of a glass rod rubbed with silk, and we have absolutely no evidence to show that what we call *positive* electrification is due to *excess*, or what we call *negative* to *deficit*. Neither have we evidence to the contrary. We are in absolute ignorance on the point, and we merely assume it to be as we have agreed.

89. Previous Experiments viewed in the light of the One-Fluid Theory. The fundamental facts of electrical attraction and repulsion have not yet been explained either by the one-fluid or any other theory: they are probably due to some action in the insulating medium not thoroughly understood. But apart from this, several other facts which we have learned can be readily accounted for.

Consider for example the following cases:—

1. Take a positively electrified conductor; it contains an excess of electricity which constitutes its positive charge. Now earth it, *the excess escapes to earth* (cf. § 73).

2. Take a negatively charged conductor; it contains a deficit of electricity which constitutes its negative charge. Now earth it, then *electricity runs into it from the earth*, making good the deficit (cf. § 73).

We may therefore regard the earth as a vast storehouse of electricity; whenever an electrified conductor is earthed a small quantity of electricity is imparted to or taken from the earth, but on account of the enormous size of the earth compared with that of any of our conductors, this makes no appreciable difference in the earth's stock.

When electricity passes from a body to the earth this is properly expressed by saying a positive charge passes to earth; and when electricity passes *from the earth to a body*, as in Case 2 above, this is conveniently expressed by saying *a negative charge passes to earth*; it must, however, be remembered that the latter is a mere form of speech, since a negative charge means on the present theory a certain *absence* of electricity: it is as if when we pour water from a jug into a tumbler we should say that the *emptiness of the tumbler has passed into the jug*.

3. Take a negatively charged metal ball, A, and touch a neutral

insulated one, B, with it. Then B becomes negatively electrified. This is usually expressed by saying that part of A's negative charge has passed into B (cf. § 79).

4. Beat an insulated metal ball with india-rubber. Each to begin with contains its stock amount of electricity. The result of the beating is to transfer some of the electricity out of the india-rubber into the ball, so that the latter acquires a positive charge and the former a negative. And whenever we rub two things together any electricity imparted to one must be taken from the other, hence the two electrifications always appear together, one on the body rubbed and the other on the rubber. Moreover, *the amount taken from one must be equal to that given to the other*: in the *two together* there is neither excess nor deficit (cf. § 87).

90. The Corpuscular Theory of Electricity. Recent work has brought new life to the one-fluid theory of electricity. It has been found that the negative charges which can be got by many and varied methods are made up of little corpuscles, each corpuscle carrying a unit of negative electricity. Charges of electricity less than this unit cannot be obtained. All negative charges are made up of an integral number of these units. The name **electron** has been given to these negative corpuscles, and the unit charge is called the *electronic charge*. The electron possesses mass, not in the ordinary material sense, but solely in virtue of its charge, this mass being only about one two-thousandth of that of an atom of the gas hydrogen.

When a body has an excess of electrons it is said to be negatively charged, when a deficit it is positively charged. When we electrify a metal by rubbing it with india-rubber all that we do is remove some of the electrons from the metal and give them to the india-rubber. Thus the metal becomes positively charged and the india-rubber negatively charged. On this theory a passage of electricity from one place to another simply means a transference of electrons, while the difference between a conductor and an insulator is that a conductor allows this transference while an insulator forbids it.

Since the above theory was announced new experimental evidence has pointed to the existence of a positive unit of electricity as well as a negative unit. The mass of the particle bearing this positive unit is also independent of its source, but whereas the electronic mass is only a small fraction of that

of a hydrogen atom the mass of the particle bearing the positive unit of electricity seems to be nearly equal to that of the hydrogen atom. This discovery if substantiated would carry us back to the two-fluid theory again.

91. Dielectric—Electric Field. Consider an electrified body, A, surrounded by air, and let a body, B, be placed near it without touching; then B experiences a force which may be one of attraction or repulsion according to circumstances. In the early days of the science it was customary to regard this force as, so to speak, jumping from A to B with nothing to help it; but modern investigations have shown this view to be wrong, and that *the air plays an important part in the transmission of the force*, though in what way it acts we do not exactly know. If between A and B we put a plate of unelectrified glass, paraffin-wax, ebonite, or any other insulator, B still experiences the attractive or repulsive force, although the magnitude of the force is more or less altered. Thus *all insulators are capable of transmitting electrical force.* And as we shall learn later, it is not only *force, i.e.* mechanical attraction or repulsion, which they can transmit, but the influence termed electrical pressure or potential (Chap. VI.).

Exp. 56. Repeat Exp. 41, this time holding by the hand (or another earthed conductor) large thin sheets of ebonite, mica, glass, tinfoil, etc. (Fig. 74), between the electrified body and the light particles. Observe that the electrical action is transmitted across the ebonite, mica, and glass, but not across the earthed metal foils.

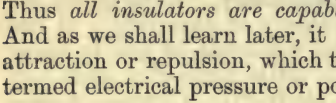

Fig. 74.

When an insulator is regarded as a medium for the transmission of any kind of electrical influence, it is termed a *di-electric*; thus a dielectric is the same thing as an insulator, but the two names refer to different properties of it. The word *insulator* denotes that it *does not allow electricity to flow over or through it*; the word *dielectric* that it *does allow electrical influence to be transmitted across it, and itself plays an actual part in conveying tha influence.*

The whole region round about an electrified body or set of such is called an *electric field.* The strength or "intensity" of the field (§ 146) in general decreases rapidly as we recede from the electrified bodies, and in ordinary cases becomes practically inap-

preciable at a distance of one or two feet. The strength also depends upon the nature of the dielectric, whether air or something else. Clearly, too, it depends upon the amount of electrification of the bodies: a strongly charged body produces a field of greater intensity than one that is feebly charged. (The student may now read a part of § 147 on Lines of Force.)

SUMMARY.—CHAPTER V.

1. Distinction between *Electrification* and *Electricity* (§§ 68, 81).

2. Distinction between *Free* and *Induced* Electrification (§ 69).

3. Distinction between *Conductors* and *Insulators* (§ 72).

4. The charge *on a conductor* does not penetrate into its material (§ 78).

5. Law of attraction and repulsion—viz. *Like electrifications repel, unlike attract* (§ 81).

6. Mode of electrifying conductors practically—viz. by beating with india-rubber for positive, and with fur for negative (§ 85).

7. Simultaneous production of the two kinds, so as to neutralise each other (§§ 86, 87).

8. *The One-fluid Theory.* Every body in a neutral state contains a stock amount of electricity. A positive charge is an excess of electricity, and a negative a deficit. Exact significance of the term *charge* as distinguished from *electrification.* What we *do* and what we *do not* mean by speaking of electricity as a "fluid" (§ 88).

9. The earth a vast storehouse of electricity. What happens when we earth a freely-charged conductor (1) if positive, (2) if negative. Explanation by the one-fluid theory of electrification by contact and by friction (§ 89).

10. The corpuscular theory of electricity: the electron, the unit negative charge (§ 90).

11. A *dielectric* permits of *electrical influence being transmitted across it,* but not of *electricity flowing through it* (§ 91).

EXERCISES V.

1. A pith ball suspended by a silk thread is touched by a negatively charged ebonite rod; the latter is then taken away and a positively charged glass rod held near the ball: state and explain what will happen.

2. An iron rod (mounted on an ebonite handle) is beaten with india-rubber and held near an ebonite rod which has been rubbed with amalgamated silk and which is balanced on a pivot. What will happen?*

3. A negatively electrified ebonite rod is balanced on a pivot, and a neutral brass ball placed near it. What happens?

4. A piece of india-rubber (which is a good insulator) is used to beat an iron rod, and is then held near a pivoted glass rod that has been rubbed with silk. State and explain what happens.

5. A brass rod is balanced on a needle point which is mounted on an ebonite support, and is lightly rubbed with silk. After this the silk is first de-electrified, then insulated, rubbed over a dry glass bottle, and held near the brass rod. State and explain what happens.

6. An ebonite rod is supplied with a flannel cap about an inch long, which fits over one end. Round the flannel is wound a piece of strong silk thread, with about four inches hanging loose. A person holds the rod in his left hand and the loose end of the silk in his right, and turns the flannel cap several times round. The rod with the cap on it is then held near the pith-ball electroscope: what will happen? The flannel cap is then taken off (the silk thread only being touched), and it alone held near: what will happen? The cap is then squeezed in the fingers, again put on the rubbed end of the rod and the two held near: what will now happen?

7. An electrified brass ball is placed in a dry unelectrified glass tube and held near a neutral pith ball suspended by a silk thread. How will the pith ball behave?

8. When a piece of sealing-wax and a piece of dry flannel are rubbed together one becomes positively and the other negatively electrified. When a piece of dry paper and a piece of india-rubber are rubbed together one becomes positively and the other negatively electrified. How could you find out which of the four things, sealing-wax, flannel, paper, india-rubber, are in the same electrical state?

9. Arrange the following substances in the order of their conducting powers for electricity, putting the name of the best conductor first:—air, copper, glass, iron, water, wood, fur, silk.

10. A pith ball is suspended from a metal stand by a fine thread. If you have a strongly electrified glass rod, how can you find out whether the thread is a conductor or non-conductor of electricity?

11. State the disadvantages of glass as an insulator, and describe the best means of overcoming them.

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* In all questions of this kind it is intended that the natural action of each electrified body is strong enough to have, so to speak, its own way, and not to be overcome by induction.

12. Two pairs of light pith balls are hung at the opposite ends of an insulated conductor, one pair being suspended by silk and the other by cotton threads. Describe and explain the behaviour of the balls if the conductor is gradually electrified more and more strongly.

13. State clearly the evidence for the opinion that there are two kinds of electrification.

14. Write a short essay on the one-fluid theory of electricity, and explain in accordance therewith (1) the positive, (2) the negative electrification of a conductor by contact. Explain also the discharge of (1) a positively, (2) a negatively electrified conductor.

CHAPTER VI.

POTENTIAL.

92. Potential. The student must now be introduced to a term which is of great importance in the study of electricity. This term is **potential**, sometimes also called *electric pressure* or *electric tension*. Thus we read of the electrical mains of a town having a "pressure" of 200 volts (the volt being the unit of potential) of a "high tension current" being used to drive electric cars, etc. It is important to get a good grasp of the meaning of potential, and for this purpose two serviceable analogies, viz. those of temperature and level, may be considered first.

(1) *Analogy of temperature.* The *mere temperature* of a body is no criterion of the quantity of heat in it; thus a thimbleful of boiling water has a higher temperature than a bucketful of lukewarm water, although the latter contains more heat. Again, what determines the flow of heat from one body to another is their *difference of temperature*, not the difference of the amount of heat in them; thus if a thimbleful of boiling water is suspended in a bucketful of lukewarm water, heat will flow from the former to the latter, despite the fact that the former already contains less heat. Moreover, heat will continue to flow from the hotter body to the colder *until their temperatures are the same.*

(2) *Analogy of level.* Water always tends to flow from a place of high level to one of low level. If a vessel containing a pint of water stands on the table, and it is connected by a pipe* with another on the floor containing a gallon, water will run from the former into the latter despite the fact that the former already contains less. Moreover, if two or more vessels containing water at different levels be connected by pipes, a flow will take place between them *until the water stands at the same level in each.*

* The pipes are supposed to be connected to the lower parts of the vessels.

Now in the electrical problem potential has much the same meaning as temperature in the thermal problem and level in the gravitational problem. Potential can be defined mathematically, but for elementary students the best explanation of the meaning of the word is to say that *difference of potential is that which produces or tends to produce flow of electricity*, just as difference of temperature produces or tends to produce flow of heat, and of level produces or tends to produce flow of water.

Electricity always tends to flow *from a body at high potential to one at low potential*, and if allowed to flow (*i.e.* if a conducting path be provided between the two bodies) will continue to do so until *the potentials are equalised*. This is **Poisson's principle**, which is the fundamental principle of Electrostatics. More briefly it may be stated thus: *The potential of all parts of a conductor or of any number of conductors in electrical communication is the same*, it being of course understood that we refer to the potential after the flow, if any, is complete, and not while it is actually occurring.*

Poisson's principle always holds. It holds for any number of conductors and also for all parts of a conductor. If a conductor is hollow, like a tin pot, the potential is the same at all points on the inner and outer surfaces and in the interior of the metal.

The earth is so large that the quantities of electricity at our command do not affect the electrical pressure of the earth. *We may therefore regard the potential of the earth as constant, and this constant value is taken to be zero.* Now the literal meaning of "zero" is "nothing," but it is not meant that the earth has no electrical pressure; all that is intended is that the potential of the earth is *our standard of reference*, just as with the Centigrade thermometer we speak of the temperature of melting ice as zero. Moreover, just as temperatures are regarded as positive or negative according as they are above or below this temperature, so are potentials regarded as positive or negative according as they are above or below the potential of the earth.

* The literal meaning of the term Electro*statics* is that branch of the science which deals with electricity *at rest*. Electro*dynamics* (Part III.) deals with electricity in motion, and to this Poisson's Principle applies only in so far that electricity flows from the conductor of higher to the one of lower potential, but for reasons which will be given in § 158 the flow does not stop and the potentials never become equalised.

If an isolated positively charged conductor is earthed, the flow of electricity is from the body to the earth. The body is therefore said to have a positive potential or to be at a potential higher than the earth. Similarly, the potential of an isolated negatively charged body is said to be negative or lower than that of the earth.

Poisson's Principle is in general *not* true for insulators, thus in the case of an electrified ebonite rod it does not follow that the potential is the same at all points of the ebonite. For in the case of non-conductors the electricity can *not* flow from parts at high potential, as would be necessary to equalise the potential.

When two conductors at different potentials are connected by another conductor there is a flow of electricity from the one at the higher potential to the one at the lower potential and the ultimate common potential is intermediate between the initial potentials; an exception to this occurs when one of the conductors is the earth (see above).

There is, however, another important feature in the idea of potential apart from its analogy to temperature and level. A positive charge (even if insulated) tends to produce a positive potential, not only in the charged body, but also throughout the surrounding region, and this without any flow of electricity from the body. Similarly a negative charge tends to produce a negative potential. The potential produced at nearer points is of course more intense than at greater distances. A hot body tends to raise the temperature of adjacent colder bodies, but in this case there *is* a flow of heat from the hot body to the cold body.

93. The Gold-leaf Electroscope.* In Chapter V. we have described a simple form of electroscope, viz. the pith-ball electroscope. We now proceed to describe the gold-leaf electroscope, which is more sensitive and of much greater use. This electroscope consists essentially of a pair of gold-leaves hanging side by side from the same support in a chamber with conducting walls.

Exp. 57. *Make a simple form of gold-leaf electroscope.* Obtain a tin canister C (Fig. 75) of a shape similar to that in the figure. From 6 to 9 inches high is suitable. Solder a binding screw t' to one side. Replace

* Invented by Bennett in 1787.

two opposite sides by glass windows, preferably cutting out the sides so that the edges can be made to act as guides to the glass windows, which may then be easily removed, etc. Obtain a metal rod *ab*, say 6 inches long, and a metal disc D about $1\frac{1}{2}$ inches in diameter (commonly called the "cap"). Bevel off the end of the rod to a narrow V, bore a hole in the disc to receive at a tight fit the other end of the rod, and then solder the rod into D.* Solder also a binding screw *t* to the rod. Obtain or turn a plug P of insulating material—paraffin wax, sulphur, or ebonite—to fit the mouth of the canister. It is convenient to provide this plug with a collar as shown. Make a hole through the plug to take the rod at a tight fit. Now cut, with a razor, two gold leaves 2 inches long and $\frac{1}{4}$ inch broad. Grease ever so little the faces of the wedge-end of *b*, and press them in turn on the ends of the leaves. When the rod is held upright the leaves should hang close together and be of the same length. If not, cut a bit off the longer leaf. Finally, place the plug in position, so that the outward movement of the leaves takes place parallel to the glass windows. When the leaves stretch out horizontally they should just not reach the walls of the canister.

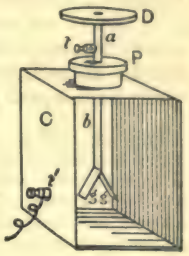

Fig. 75.

Instead of a tin canister a box such as that in which chalk is supplied for blackboard purposes may be used. Knock out the bottom and remove the sliding top. Stand the box on its end. Cut a hole in the top to receive P. Paste tinfoil round the inside of the remainder and the outside of the bottom, the tinfoil continuous throughout in communication with *t'*. Cut a pane of glass to slide in the grooves on the side and fasten a pane to the other side.

Leaves of Dutch metal or thin Aluminium foil may be used instead of gold leaves; also a single leaf resting against a vertical strip of metal is frequently used instead of a pair of leaves.

94. Principle of the Action of the Gold-leaf Electroscope.

Exp. 58. Place the electroscope on an insulator, say a block of ebonite or wax. Connect *t* and *t'* (Fig. 75) by a wire. Beat the cap D with india-rubber; the cap and, therefore, the canister become positively charged, as is evidenced by a pith ball being attracted by the canister and then repelled. *Observe that the leaves do not move.* From this we deduce that when the leaves and the walls have the same potential the leaves do not diverge.

If the cap had been beaten with fur instead of india-rubber the same thing would have happened, the leaves and walls now both

* It is better to screw the rod into the cap. The cap can then be easily removed.

acquiring the same negative potential. The experiment may be made more striking if, instead of merely beating the cap, the terminal S is connected by a wire to a strong electrical machine (Ch. IX.): on working the latter, the leaves and walls acquire a very strong potential, and by holding the knuckle over the cap, sparks an inch or two long may be taken off, but the leaves and walls have equal potentials and not the least divergence occurs.

Exp. 59. Still keeping the electroscope on an insulator, disconnect t and t'. Take two conductors, say A and B, mounted on ebonite supports and join A by a wire to t and B by another wire to t'. Place A and B in contact and charge them. The leaves do not diverge, for the potential is the same all over the combined conductor made of leaves and cap, A, B, and the walls of the electroscope. Now separate A and B and charge A more, leaving B alone. The leaves at once diverge. Charging A has raised its potential above that of B, hence the divergence. Charge A more strongly still, the divergence increases.* The experiment shows that the greater the difference of potential between the leaves and the walls, the greater the divergence.

We may therefore use the divergence† of the leaves as a measure of the potential difference of the leaves and the walls of the electroscope, so that the electroscope acts as an **electrical pressure gauge**.

It should be observed that the divergence of the leaves does not in itself tell us whether their potential be *higher* or *lower* than that of the walls; it merely tells us that there is *a difference* of potential, and indicates the extent of that difference.

95. Electroscope with its Walls Earthed. If the electroscope be placed on an insulating stand and (the leaves and walls being unconnected) the cap being charged, this charge acts across the air in the jar and renders the potential of the walls different from zero, though not equal to that of the leaves. This complicates matters very much; hence, in the vast majority of experiments with the instrument, the walls are kept earthed by allowing it to stand on the table; in that case, by Poisson's Principle *the potential of the walls of the electroscope is always zero, whatever may be done to the cap, leaves, etc.*, and hence the principle of § 94 takes the simpler and more practically useful form: *A divergence*

* Just as when we add heat to a body the temperature rises.

† The divergence of the leaves may be measured by means of a scale fixed on the glass front.

of the leaves indicates that their potential is different from zero, and the amount of divergence indicates the strength of the potential.

Here, again, when we say "the leaves" we, of course, include the cap and any conductor in electrical connection. The divergence does not show whether the potential is positive or negative; this, however, seldom matters, as we generally know beforehand whether we are working with positive or negative potentials, and if not we can—should it be necessary—find out by the method of § 98.

It should be noticed that the principles of this and the preceding sections are *perfectly general*, there are no exceptions to them whatever, and it makes no difference whether the potentials dealt with are due to free charges or to external influences.

In all that we shall in future say about the gold-leaf electroscope it must be distinctly understood, unless the contrary is specified, that *the walls are supposed earthed*; if not, the instrument will in general behave differently from what is stated.

96. To give an Electroscope a Free Charge. It frequently happens that we require the cap and leaves of the electroscope to have a free charge. This is easily done. To charge them positively we beat the cap with india-rubber, to charge them negatively we beat it with fur. As, however, constant beating and rubbing does not improve the instrument it is best to beat an insulated metal ball with these respective substances and transfer a charge by contact. A very feeble charge is sufficient; if too strong the leaves are apt to be wrenched off. A positive free charge gives the leaves a positive potential, a negative free charge gives them a negative potential; in either case their potential becomes different from zero, and they diverge in accordance with the principle of § 95.

The electroscope may also be charged by induction or influence (§ 102).

When the walls are earthed and there is no electrical charge in the neighbourhood except that on the electroscope, the divergence of the leaves, being a measure of the potential of the cap and leaves, is also a measure of the charge on the cap and leaves.

Exp. 60. Stand the electroscope on an insulating stool, and earth the leaves by running a copper wire from the terminal t to a gas-pipe. Charge a metal ball and join it to t'. The potential of the walls of the electroscope then becomes different from zero, while that of the leaves remains zero because they are earthed; hence, in accordance with the principle of § 94, the leaves diverge.

97. Potential of Electroscope due to External Influence.

Exp. 61. Take a neutral electroscope. Its leaves hang together, their Potentials and also that of the vessel being zero. Now rub a glass rod with silk. Hold the rod near the cap of the electroscope. The leaves diverge.

Now the vessel, being earthed, has remained at zero potential: hence this experiment shows clearly that the potential of the cap and leaves has been raised by the proximity of the positively charged body.* Bring the rod nearer; the divergence increases, showing that the potential due to a charged body is greater near the body than far away. Remove the rod. The leaves fall together, showing that their potential or electric pressure has again dropped to zero.

A potential produced by external influence, as in this case, is called an *impressed* or an *induced* potential (see § 100). The electroscope having the induced potential is sometimes loosely said to be "charged" by induction; but it must be carefully borne in mind that, taking the cap, rod, and leaves as a whole, *they are not charged at all*—that is, there is no excess or deficit of electricity in them, for no electricity can flow from the charged body through the air.

98. Simplest use of the Gold-leaf Electroscope. The simplest use of the electroscope is to determine the existence and character (*i.e.* whether positive or negative) of a free charge. It thus serves the same purpose as the pith ball and rods of § 83, but is far more delicate.

To detect the existence of a charge simply hold the body to be examined over the cap: if the leaves diverge, the body is charged; if not, it is neutral. To determine the character of the charge, charge the electroscope positively, thus giving it a free positive potential, and slowly bring the body to be examined up from a distance. If the leaves diverge more, it is clear proof that the body is positively charged; if the leaves diverge less, the body must be negatively charged. A neutral body would also make the leaves diverge less: but we have already found whether the body is charged or neutral.

The reason for bringing the charged body slowly towards the electroscope from a distance is to avoid the risk of a reversal of

* See the last paragraph in § 92.

the potential of the leaves before we have time properly to observe their behaviour. For if a sufficiently strong negative charge is brought slowly towards the electroscope it will first diminish their positive potential, then reduce it to zero (when the leaves collapse), and ultimately produce a negative potential sufficient to make them diverge more than at first. If this is done rapidly the intermediate stages are not noticed and the final stage gives the impression that the charge under examination is positive.

It should be noticed that all these actions primarily depend upon the *potential* of the body. If the body be, as we have supposed, not under the influence of external electrification, the existence and character of its potential depends simply upon its charge (§ 100), so that the method is valid; if there be external influence, the method is valid as respects *potential*, but not as respects *charge*; it is, however, then seldom employed.

In practice the best way is to employ three electroscopes, one neutral, one charged positively, and one negatively, placing them several feet from one another so as to avoid inductive action between them; the body to be examined is then held first over the neutral one, and if the leaves do not diverge there is no need to go further. But if they do it is held in turn over the positive and negative one, and whichever diverges *more* tells us the character of the charge.

99. To test the Strength of the Potential of a Conductor. If a charged conductor is joined by a metal wire to the cap of an electroscope, the leaves diverge. The conductor and the electroscope acquire the same potential (by Poisson's Principle), which is indicated by the relative divergence of the leaves. The electricity which has flowed to the electroscope has, however, weakened the potential of the conductor so that the potential indicated by the divergence is not the original potential of the conductor. To make this decrease as little as possible the instrument should be used without the cap, when the electroscope takes a less charge, and the initial potential is more nearly indicated.*

To test, then, the potential of a conductor remove the cap of the electroscope, fasten to t (Fig. 75) one end of a long thin copper wire to whose other end is soldered a round brass ball or knob about $\frac{1}{4}$ in. diameter. Support the wire (near the knob) by wrapping it tightly for a few turns around a rod of sealing-wax or

* Just as when we wish to take the temperature of a small quantity of water we use a small thermometer.

ebonite,* and bring the knob in contact with the insulated conductor (Fig. 76). The greater or less divergence of the leaves indicates the strength of the potential.

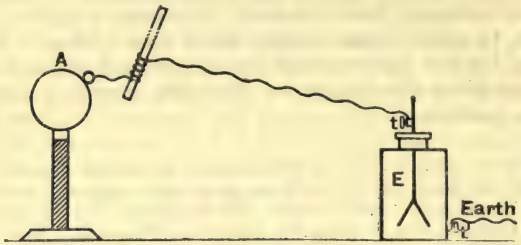

Fig. 76.

Remark.—We have seen that the operation is accompanied by a flow of a charge into the leaves. Both leaves become, therefore, similarly electrified, and their divergence used always to be, and still frequently is, explained as simply due to the repulsion of similarly electrified bodies (§ 81). This view is, however, apt to mislead, and it is best not to dwell too strongly upon it. In any case it represents only *part* of the truth, for the motion of each leaf depends not only on itself and its companion, but upon the *whole electric field within the electroscope*, to the condition of which the walls contribute a very important part (see § 149, and Fig. 129).

100. Free and Induced Potential. Take an insulated conductor, A (Fig. 77),† provided with a binding-screw; place it several feet from the electroscope and run a copper wire from the binding-screw to the terminal *t* of the gold-leaves. Then perform the following experiments:—

Exp. 62. Give a free positive charge to A‡ the leaves diverge. A now has a *positive* potential, and it is a *free* potential—that is, it is due to A's proper charge and not to external influence.

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* Or a pair of insulating tongs may be used. These are simply a small pair of pincers or pliers mounted on ebonite handles.

† The shapes of A and B are immaterial.

‡ B is absent in Exps. 62 and 63.

Exp. 63. Discharge A by earthing: its potential is reduced to zero and the leaves collapse. Repeat Exp. 62, this time giving A a free negative charge: the leaves now diverge at free negative potential.

Exp. 64. Discharge A so as to make a fresh start with everything neutral. Take another insulated conductor B, give it a free positive charge, and bring it gradually towards A, but do not allow it to touch; also do not bring it near enough for a spark to pass. The leaves of the electroscope will diverge and will do so more and more the nearer B approaches A.

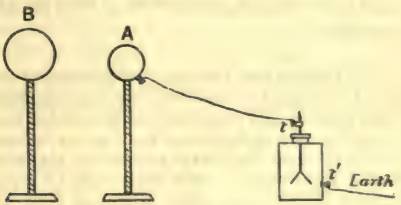

Fig. 77.

This shows that A acquires a potential which becomes stronger the nearer B approaches it. Now remove B: the leaves at once collapse, showing that the potential of A is reduced to zero. It is evident, therefore, that A has acquired no charge, as indeed it could not, because it is separated from B by the air, which is an insulator. *The potential of A was simply due to the external influence of B transmitted across the dielectric;* it is an *impressed* or *induced potential* (cf. § 97). The body B, having a free positive charge, has an electrical pressure above that of the originally neutral conductor A; and, by bringing B near, this increased pressure is transmitted across the dielectric, and falling upon A raises its potential. The potential acquired by A is therefore positive.

Exp. 65. Discharge everything and repeat Exp. 64, only charging B negatively instead of positively; the same results will be observed, this time A acquiring a *negative* induced potential.

The body, B, to whose influence A's induced potential is due, is called the *inducing body.* We have supposed it to be a conductor, but *a non-conductor acts in exactly the same way*: in Exp. 64 we might employ a glass rod rubbed with silk, and in Exp. 65 an ebonite rod rubbed with fur. But the body A must be a conductor, otherwise the action, though essentially similar, is not at all well marked.

In the above experiments all the electroscope does is to serve

as an electrical pressure gauge, whereby we can practically detect the changes of potential; if we do not use the electroscope, these changes will occur just the same.

We thus arrive at the following very important result:—A conductor may acquire a potential in two distinct ways.

1. As the result of its own charge—*i.e.* of an actual excess or deficit of electricity in it: this is called a *free potential.*

2. As the result of external electrical pressure brought to bear upon it across the surrounding dielectric: this is called an *induced* or *impressed potential.*

It will be observed that in the foregoing experiments we have directed that the electroscope be placed several feet from the conductors: the reason is that it may not receive any appreciable potential from the influence of A or B *apart from the wire connection*—that is, no induced potential from the direct action of B or A on the leaves across the air, which would needlessly complicate the conditions. We shall find as we go on that it is very usual to set the electroscope some distance from the bodies under examination.

It must be most carefully noticed that when a conductor has only an induced potential it has *taken, as a whole, no charge whatever*—that is, no excess or deficit of electricity; it simply possesses its stock amount (cf. § 97).

Free and induced potential may exist together. Also potential and charge may or may not go "hand in hand." When a conductor is *not* under external electrical influence its potential and charge go, so to speak, hand in hand—that is, if it has a potential it has a charge, and conversely; also the character of its potential (positive or negative) is the same as that of its charge.

But when a conductor *is* under external electrical influence, its potential and charge do *not* go hand-in-hand. It may have *no charge and yet a potential* (Exps. 64 and 65). Or it may have a charge and yet no potential—that is, be at potential zero when the induced potential just destroys the free one. Or it may have a charge of one kind (say positive) while its potential is of the opposite kind (negative), as in these same experiments when the induced potential has more than destroyed the free one. The student should think all this over very carefully in the light of the experiments referred to, as it is sometimes felt to be perplexing.

101. Use of the Gold-leaf Electroscope to check the Truth of Poisson's Principle. (*a*) *For a free charge.*

Exp. 66. Connect several insulated conductors with copper wire or put them in contact. Give them a free charge. Test the potential as advised in § 99, moving the knob at the end of the wire over all the surface of the conductors. Observe that the divergence remains constant, showing that the potential of the conductors is the same at all points, which is clearly in accord with Poisson's Principle.

This experiment is sometimes said to *prove* Poisson's Principle, but it does not so much do that as check or illustrate it, as we assume that the potentials of the two ends of the wire are equal; it can indeed hardly be *proved* directly, it must rather be looked upon as a fundamental hypothesis whose correctness is established by he readiness with which it explains electrical phenomena.

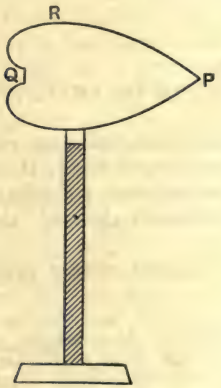

Fig. 78.

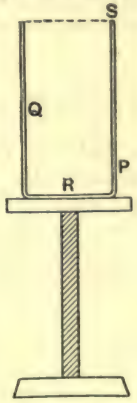

Fig. 79.

It should be observed that it does not in the least matter what *shape* the conductors are. It is very usual to exhibit the experiment with a single conductor of the shape shown in Fig. 78; it is then found that it makes no difference whether the knob be placed at the point P, in the hollow Q, or at any other point R; the divergence of the leaves is precisely the same.

If the conductor is a metal pot, such as that shown mounted in Fig. 79, it makes not the least difference in the divergence of the leaves whether the knob touch the outside of the pot say at P, the inside say at Q or R, or the edge say at S.

(*b*) *For an induced potential.*

Exp. 66 checks Poisson's Principle for *free* potential; it is equally easy to check it for induced potential or for a potential partly free and partly induced.

Exp. 67. Instead of giving the conductors of the last experiment a free charge give them an induced potential, by placing another charged conductor near them, and repeat the last experiment. The same result is obtained.

This should be especially noted; a common mistake is to suppose Poisson's Principle true for free electrification only, and to think that when a conductor, A, is under the influence of some other electrified body, B, the potential of A is stronger at the end near B than at the end farther away, which is not the case—it is the same all over and through A.

ELECTRIFICATION BY INDUCTION OR INFLUENCE.

102. Induced Charge. When an insulated and originally neutral conductor, A, is near a freely charged body, B, then, as we have seen (§ 100), A acquires an induced potential, and is sometimes loosely said to have an "induced charge," though it really has no charge at all.

We now proceed to consider an *induced charge properly so called.*

Exp. 68. Take an insulated neutral conductor, A, and bring near it a freely and positively charged body, B. While B is near, earth A (by touching it with the finger), then *break the earth connection* (by taking the finger away), and *lastly remove* B. Then test A, either by the pith-ball electroscope, or, still better, by the gold-leaf electroscope, as in § 98: it will be found to be *negatively charged.*

Explanation.—When B is brought near the insulated conductor A, the latter acquires a positive potential. When A is earthed its potential necessarily becomes equal to that of the earth, that is *its potential is reduced to zero.* But (since no further change has been effected in A's electrical surroundings) this reduction from a positive to a zero potential can only have happened as the result of some electricity having flowed out of A into the earth, A thus acquiring a deficit—that is, a

negative charge. After earthing, and before removing B, the potential of A is zero, its positive induced potential being exactly destroyed by the negative free potential due to its negative charge. Merely breaking the earth connection makes no difference. But as soon as B is removed the induced potential of A disappears, and there remains nothing but the free negative potential due to A's negative charge. In fact, *as soon as* B *is removed* the negative charge on A becomes a free one, just as surely as if it had been imparted by beating with fur, and accordingly affects the electroscope as explained in § 97.

It should be carefully noted that the negative charge is *acquired by* A *at the moment of earthing*: removing B in no way alters the charge, but simply leaves it free to produce its own potential unmolested by external influence. The negative charge on A is called an *induced charge*, in allusion not to its character (for it is really free as soon as B is removed), but to the method whereby it has been produced. The term is therefore not so appropriate as the phrase "induced potential," which we have previously used, since the latter denotes a potential due to external influence then and there in operation.

It should be carefully observed that in the foregoing process of charging A inductively the amount of A's induced charge is determined by the fact that the free negative potential due to it must be equal and of opposite sign to the induced positive potential due to B's influence; in other words, *so much electricity must flow out of* A *when it is earthed as shall ease the total electrical pressure of* A *down to that of the earth.* Other things being the same, the nearer B is to A the higher is A's induced potential; hence the greater must be the negative induced charge in order to counteract it—thus the induced charge is greater the nearer B is to A. Similarly also, other things being the same, the induced charge is greater the greater the inducing charge.

In what has been hitherto said, we have supposed the inducing body B to be positive. The induced charge on A is then negative. Had B been negative the induced charge would have been positive: the student should think out carefully the details of this case. It will thus be observed that the **induced charge is always opposite to that of the inducing body**; in this respect it is the reverse of a contact charge (§ 79), which is always of the same kind as that of the body which imparts it.

As pointed out in § 100, A will derive an induced potential from B equally well whether B be a conductor or an insulator; hence we can impart an efficient charge from a non-conductor to a conductor by induction, though not by contact.

This was the old-fashioned way of charging conductors. If one wished to charge a metal ball negatively, it was the custom, instead of beating it with fur (which takes about a second), to dry a glass rod before the fire, do the same with a piece of silk, rub the rod vigorously with the silk, hold the rod near the ball, touch the ball and then withdraw the rod, the whole process, including the drying, taking about ten minutes.

The method, however, is well adapted for charging the gold-leaf electroscope. Thus to charge the electroscope positively rub a piece of sealing-wax or ebonite with flannel (ordinary cloth will do), hold it near the cap, touch the cap for a moment, and then withdraw the sealing-wax or ebonite (cf. § 96).

It will be noticed that in all the foregoing discussion we have supposed the body A *acted upon* to be a conductor; the case where it is an insulator is unimportant.

SUMMARY.—CHAPTER VI.

1. *Electrical Pressure* or *Potential* and its analogy to temperature and level. In what respect it differs from these. *The electrical pressure of the earth is constant and taken as zero* (§ 92).

2. *Poisson's Principle* in all its aspects as embodied in the following statement: "*The Potential at all parts of a conductor, or of any number of conductors in electrical communication, is the same however they may be electrified or under whatever circumstances they may be placed, provided always it be understood that the electricity in them is at rest—that is, that they are in an electrostatic condition. Also if any two conductors at different potentials be electrically connected, electricity flows from the one of higher to the one of lower potential until their potentials are equalised; while if they were at equal potentials to begin with, connection produces no effect whatever.*" Particular case of the above when one of the conductors is the earth (§ 92).

3. Principle of the action of the Gold-leaf Electroscope, especially with the walls earthed (§§ 93, 94, 95).

4. Use of the electroscope to test the strength of the potential of a conductor (§ 98) and to check Poisson's Principle (§ 101).

5. Free and Induced Potential (§§ 96, 97), and their coexistence (§ 100).

6. Induced Charge (§ 102).

NOTE ON CHAPTERS V. AND VI.

In Chapter V. an account is given of the attractions and repulsions which electric charges exert upon one another. In Chapter VI. an account is given of potential as determining the flow of electricity in conductors. It is most important that the student should understand that the theory of potential is not a new principle quite distinct from the theory of attractions and repulsions: it is merely *a different aspect of the same phenomenon.*

Consider for a moment the analogous problem in Hydrostatics. Water tends to flow from a high level to a low level; and many problems with regard to the behaviour of bodies of water are most easily solved by this principle. But, as a matter of fact, the real cause of the flow of water is the weight of the water, *i.e.* the earth's attractive force, and in the mathematical theory of the subject *the laws of level are deduced entirely from the laws of the earth's attraction.*

In exactly the same way the movement of electrically charged bodies and the flow and distribution of electric charges in conductors may be regarded as entirely due to the attractions or repulsions which exist between the different charges and between different portions of the same charge, and in the mathematical theory of the subject *the laws of potential are deduced entirely from the laws of attraction and repulsion.* In the simplest cases it will be quite easy for the student to see how the laws of potential give the same results as the laws of attraction and repulsion.

Take, for instance, the case of two positively charged conductors. Each produces a positive potential in the surrounding region, the potential being lower at greater distances from the charge (see § 92, last paragraph). Also each tends to move into regions of lower potential, taking the shortest route available. Hence each tends to move directly away from the other: or, in other words, the two charges repel one another.

EXERCISES VI.

1. Explain in the light of Poisson's Principle what happens in the following cases:—

(i) A neutral conductor is earthed.
(ii) Two neutral conductors are connected by an insulated wire.
(iii) A negatively charged conductor is earthed.
(iv) A positively charged conductor is connected with a neutral one.
(v) A negatively charged conductor is connected with a neutral one.
(vi) Two conductors having equal positive charges but unequal potentials are connected.
(vii) Two conductors having unequal positive charges but equal potentials are connected.

2. How would you show that a piece of metal when rubbed with flannel is charged with electricity, and how would you test whether the charge is positive or negative?

3. An electroscope is standing on the table and its leaves are diverging. Describe and explain what experiment or experiments you would perform in order to ascertain whether their potential was positive or negative.

4. A charged conductor B is used to induce a charge on a conductor A.

(*a*) What is the reason for breaking the earth contact *before removing the inducing body* B? What would happen supposing B were removed first?

(*b*) Will it make any difference in the final state of A if it had originally a charge of its own?

(*c*) Will it make any difference in the final state of A if it is earthed *before* bringing B near it?

5. A neutral conductor, A, is connected with the leaves of a gold-leaf electroscope, by means of a wire running from it to the terminal *t* (Fig. 75). A positively charged conductor, B, is placed near A: how will the leaves behave? A is then touched by the finger: how will they now behave? The finger is now removed and B discharged: how will they now behave? Give reasons for your answers.

6. An insulated conductor A is neutral. Near to it is brought a positively charged conductor, B. A is then momentarily touched by the finger, and afterwards B is momentarily touched by the finger; B is then taken away and tested: what will its electrical state be? Also was the potential of A after B was touched the same as if B had been not earthed but taken away?

7. You are provided with an insulated metal ball, an ebonite rod, and a catskin: how would you charge the ball *positively*? Explain what happens at the several stages of the process.

8. You are provided with a stick of sealing-wax and a piece of flannel: how would you, by means of these, charge a gold-leaf electroscope positively?

9. A gold-leaf electroscope is placed on an insulating stand and *the cap is earthed.* The netting is placed in electrical connection with a neutral conductor, A, mounted on an ebonite support and standing on the table. A positively charged conductor, B, is held near A: how do the leaves behave? A is then earthed: how do the leaves behave? The earth connexion of A is next broken and B is taken away: again how do the leaves behave? Explain all these actions.

10. Given two insulated metal spheres, A and B, of which A is positively charged and B is neutral, show how a positive charge may be communicated to a gold-leaf electroscope by means of A and B without A losing any of its charge.

11. Two precisely similar gold-leaf electroscopes are placed a long way apart and their caps connected by a wire. A positively charged sphere is then brought near one of them: will their indications in any way differ, and if so which will show the greater divergence? Also what effect will be produced if either of the electroscopes be momentarily touched by the finger and the charged sphere then removed?

CHAPTER VII.

INDUCTION OR INFLUENCE. HOLLOW CONDUCTORS.

103. Condition of the Dielectric surrounding a freely charged Conductor. Consider a freely charged conductor surrounded by a dielectric—say air (cf. § 91). The electrical pressure of the conductor makes itself felt at every point in the field; in other words each point of the field acquires a potential which is positive or negative according as the charge on the conductor is positive or negative. If the charge be positive, the potential becomes *lower* as we recede from the conductor; if negative, it becomes *higher*. In either case it becomes *weaker*, and under ordinary circumstances is inappreciable at a distance of one or two feet.

Exp. 69. Place a gold-leaf electroscope two or three yards from the charged conductor and attach to it a long copper wire with a small brass knob at the farther end, as in § 99; then, holding the wire, as in § 99, move the knob to and fro in the field, when it will be found that the nearer it approaches the conductor the more the leaves diverge. This shows that the potential of the dielectric weakens as the distance increases.

Every one knows the meaning of the term *slope* or *gradient* as applied to a hill, and in the same way we may speak of the **potential-gradient** in an electric field. If, as we travel in imagination through the field, we pass from a place of higher to one of lower potential, we are on a down gradient, and *vice versa*; while if the potential is uniform throughout any part of the field, we may speak of that portion as at a dead level of potential. Poisson's Principle tells us that all parts of a conductor are at a level in this sense.

What has been said above may be expressed thus:—There is a potential-gradient in the dielectric surrounding a freely charged conductor, *the down-gradient being away from the conductor if the latter be positive and towards it if negative.*

If we knew exactly how the potential varied with the divergence of the leaves, we could, by measuring this divergence, obtain an estimate of the potential at points in the field.* The results may be conveniently exhibited by a curve. Let distances

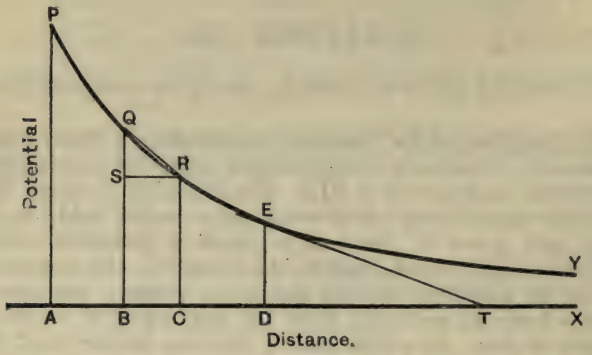

Fig. 80.

along A X (Fig. 80) represent distances along a straight line drawn through the dielectric from a point on the surface of the conductor, and the perpendicular heights of points on the curve P Y from A X represent the potentials of the points at the foot of these perpendiculars. Thus if A represent the point on the conductor from which the line is drawn, the potential of A, *i.e.* of the conductor, is given by A P. The potential of a point B is B Q, that of a point C, C R, and so on, the potential gradually decreasing as the distance of the point from the conductor increases.

Obviously the mean value of the potential gradient between two points B, C,

$$= \frac{\text{Potential of B} - \text{Potential of C}}{\text{BC}} = \frac{\text{BQ} - \text{CR}}{\text{BC}}$$

$$= \frac{\text{QS}}{\text{SR}} = \tan \angle \text{QRS}.$$

* The potential of the ball will not be exactly the same as the potential of the neighbouring air.

This, in the limit when B and C are close together, is equal to the slope of the potential-distance curve. For example, the potential-gradient at D = tan ∠ E T D where E is the top of the perpendicular drawn from D to the curve and E T is the tangent at the point E.

It should be noted in Exp. 69 that the gradient becomes less steep the further we recede from the surface of the conductor, and also that close to that surface it is not in general the same all over; it is, except in special cases, steeper in some cases than in others; this fact will receive important applications in the next chapter in connection with electric surface density.

All the remarks of this article apply when the charged body is a non-conductor—only of course the potential of the body itself is now in general not the same throughout.

104. Condition of the Dielectric inside a freely charged Hollow Conductor. This is a very important case not included in § 103. Consider a completely closed hollow conductor—*e.g.* a tin pot with the lid on—and let it be insulated and freely charged. Then the air (or other dielectric) inside will experience electrical pressure and therefore acquire a potential. But how does this potential vary from point to point in the air? Now it can be proved by mathematical reasoning that *it does not vary at all; it is the same at all points, and, moreover, is the same as that of the actual conductor.*

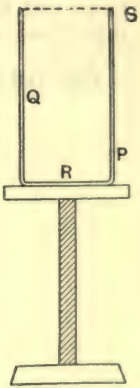

Fig. 81.

The study of hollow conductors is important. The form of hollow conductor commonly employed is known as **Faraday's ice-pail**; it is simply a tin pot (Fig. 81) *without* a lid, and should be fairly deep in comparison with its diameter,* in which case the potential of the air is practically uniform as soon as we get well inside the pot. Near the opening the potential is weaker.

Indirect experimental proofs of the above theorem will be found in the succeeding sections, but no satisfactory *direct* one can be devised.

* A depth equal to about twice the diameter answers very well.

It should be noted, in order for the theorem to be true, it is not necessary for the conductor to be of continuous metal plate; wire gauze answers just as well, provided that the meshes be not extremely coarse, say not more than $\frac{1}{4}$ inch square.

105. Case of a Hollow Conductor *not* freely charged. In § 109 it will also be shown that the theorem of the preceding article is equally true, if instead of, or in addition to, the conductor possessing a free charge, there be charged bodies *outside it*, provided that if it be in the form of an open pot, such bodies are placed well below the level of the mouth, so that the electrical pressure due to them may fall on the metal and not on the internal dielectric.

If, however, there be charged bodies *inside* the conductor, the theorem is emphatically *not* true; this case is in fact precisely similar to that of § 103 when the experiment is performed in an ordinary room, the sides and bottom of the pot taking the place of the walls and floor of the room, *e.g.* if the ball have a + charge there will be a down-gradient of potential from the ball in all directions towards the pot.

106. Inductive Displacement. In § 102 we have explained

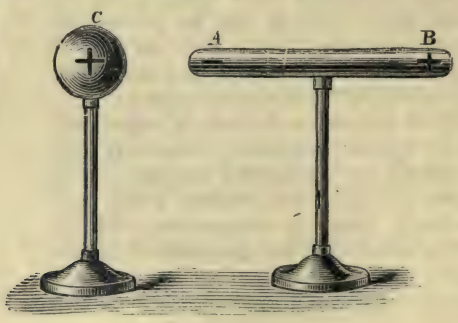

Fig. 82.

how by earthing a conductor when under external electrical influence an induced charge is imparted to it.

Let us now consider what happens when the conductor is

insulated. In Fig. 82 let a positively and freely charged conductor, C, be brought near an insulated and originally neutral conductor A N B. Then it is found by experiment that the portion N B becomes positively charged and the portion N A

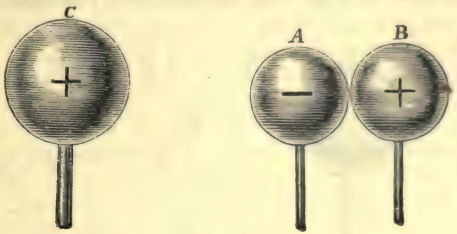

Fig. 83.

negatively. The best way to prove this is to have another conductor, A B, made in two approximately equal parts (mounted on separate supports) and allow them to rest against one another; if while under the influence of C these parts be separated and then removed from C, and tested as in § 98, the

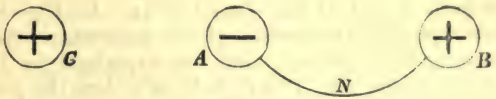

Fig. 84.

end B will indicate a positive and the end A a negative charge.* If now (while still away from C) the two parts be made to touch each other and again tested either separately or together, they will be found perfectly neutral.

The conductor A B may be of any shape, and it is frequently convenient to employ two insulated metal balls, A and B (not necessarily of the same size), which may either rest in contact (Fig. 83) or be connected by a wire (Fig. 84). When C is placed as shown, the near one A becomes negatively and the far one

* This can also be tested by means of a proof plane (§ 123).

B positively charged, while the two together are not charged at all.

This is readily explained as the result of electrical pressure acting from C across the dielectric. To make it clear, let us (taking the case of Fig. 83) consider the potential of C's field

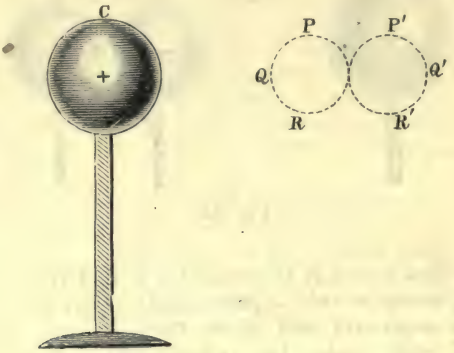

Fig. 85.

before the balls A and B are placed in it. In Fig. 85 let the dotted lines represent the portion of the field where the balls A and B are *going to be* placed; then, as explained in § 103, the potential on the near side, P Q R, is higher than the potential on the far side, P′ Q′ R′. Consequently when the balls are introduced we are applying greater electrical pressure to A than to B; the result is to force electricity out of A intc B; that is A acquires the negative and B the positive charge.

It is clear that these charges must be equal, since B merely acquires what A loses. Electricity is simply *displaced* from A into B (or in the case of a single conductor from the side near C to the side away from it), and the phenomenon is therefore spoken of as *inductive displacement*, or simply *displacement*. When the balls are separated and C removed, each retains the charge thus acquired, and when (C being away) they are brought together again, there being now no external influence to maintain the displacement, the excess from B re-enters A, making good the deficit, and both become neutral.

It is very important to observe that the two balls *taken together* have no charge, and also that when a single insulated conductor (as A B, Fig. 82) is brought under inductive influence the conductor *as a whole* receives no charge.

Another point should be very carefully noted. *Before* the conductor A N B (Fig. 82) or balls (Figs. 83 and 84) are placed in the field the potential near C is higher than that farther away. But as soon as conducting material takes the place of the dielectric no such difference of potential can exist (Poisson's Principle), so that the potential on the near side of A is just the same as on the far side of B; indeed the potential is the same throughout the entire material. The displacement of electricity from A to B may be regarded as easing the electrical pressure on A and increasing that on B, so that whereas *before* the flow there was a difference of pressure, *after* the flow there is none, the potential becomes uniform throughout and of some value intermediate between those of the dielectric at Q and Q′ (Fig. 85), prior to the introduction of the balls. It is this uniform potential which constitutes the induced potential (§ 100) of the balls (see also § 153).

If the inducing body, C, be negatively charged, all the foregoing remarks apply with the necessary changes. By § 103 the potential of the dielectric on the far side P′ Q′ R′ (Fig. 85) is now greater than on the near side P Q R; hence when the balls are introduced the displacement of electricity occurs from B to A, so that A acquires a positive and B a negative charge, while both acquire a uniform induced negative potential.

If, in Fig. 84, B be the rod and leaves of an electroscope instead of a ball, the divergence of the leaves indicates the common potential of A and B. But, as above explained, electricity has been displaced from A to B, thus easing off the electric pressure of A and making it slightly lower than that of the dielectric which originally occupied A's place, hence the divergence does not *accurately* measure the potential of the dielectric at that place.

Taking the case of a single conductor, as A B, Fig. 82, then while insulated and under the influence of a + inducing charge on C it has a + induced potential. If it be then earthed sufficient electricity runs out of it to reduce its potential to zero, thus leaving a deficit. This becomes the case already contemplated (§ 102), the deficit being the negative induced charge.

It should be noticed that in no case is it necessary for the *inducing* body C to be a conductor. It is sometimes said that the body A B *acted upon*

must be, but this is not the case. If A B be an insulator, inductive displacement occurs in the same way as for a conductor, but to a less extent. Moreover, *earthing* an insulator does not reduce its potential to zero, though it may more or less weaken it; the induced charge is therefore much less.

In reference to Fig. 82 we have said that the portion N B receives a positive and N A a negative charge, electricity being displaced from the latter into the former. The boundary N between the positively and negatively charged portions has *no* charge and is called the *neutral line*,* its exact position is difficult to find: it is somewhere about the middle of the conductor. In Fig. 83 the neutral line is somewhere about where the balls touch. It is manifest that if separation were effected *exactly* at the neutral line one portion would contain the entire excess and the other the entire deficit.

In the case (Fig. 84) of two conductors connected by a fairly long fine rod or wire, the whole rod or wire is practically on the neutral line, so that one conductor may be regarded as containing the entire excess and the other the entire deficit. This is a rather important point.

The words "induction" or "influence" were originally employed to denote the phenomenon of inductive displacement; it is, however, best to extend it to include all actions taking place across the dielectric, more especially the transmission of electrical pressure and the consequent production of "induced potential"; inductive displacement is thus merely *one of the effects* of induction.

The principle of inductive displacement affords a partial explanation of the attraction between charged and neutral bodies (cf. § 69). Thus consider a positively charged body A with an insulated pith ball near it. Though B was originally neutral it is now no longer so; by induction the side near A becomes negatively, and the far side positively charged. Hence, by the principle of § 81, A attracts the charge on the near side and repels that on the far side of B, but the latter being farther away the force of repulsion is less than the force of attraction, so that on the whole there is attraction.

If B is earthed instead of insulated its entire charge will be negative; there is now no repulsive force, and the actual attraction is much stronger.

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* The student should remember at this point (cf. §§ 78, 109) that all charges reside on the *external surface* of conductors. It is the *surfaces* of N B and N A that acquire the respective excess and deficit, not their interior; and the neutral line is a line *round* the conductor rather than a section *through* it. The interior of the section is neutral wherever it is situate, whether through the neutral line or elsewhere.

This explanation is clearly incomplete, for it throws no light whatever upon what is after all the main question—viz. *Why* do bodies having like charges repel, and those with unlike charges attract?

107. Old View of Induction. So-called "Bound" and "Free" Charges. We have just seen that, taking the case of a single conductor A B under the influence of a free + ve inducing body C, the potential of A B while insulated is + ve, and that if it then be earthed sufficient electricity runs out of it to reduce its potential to zero, the deficit constituting its induced (negative) charge.

Now the phenomena of inductive displacement and of induced charge were among the very earliest discovered, being known long before the conception of potential had entered men's minds, or the dielectric was supposed to play any part in electrical actions. An explanation of these phenomena was consequently made out which was quite in accord with what was then known, but whose unfortunate retention in many books at the present day is fruitful of manifold misconceptions in the minds of students. The explanation runs thus :—

"When the positively charged body C (Fig. 82) is brought near the neutral conductor A B, then, since like electricities repel and unlike attract, the near part A of the conductor becomes negatively and the far part B positively charged. The negative charge on A is bound by C's attraction, while the positive on B is free. Hence when the conductor is earthed the free positive charge escapes while the bound negative remains."

There are numerous errors not so much palpably expressed as incidentally implied in this explanation : the most serious of these is that *it assumes the amount of electricity which is left in the conductor on earthing to be equal to that originally displaced into the portion A N on the near side of the neutral line* : this in general is not true.

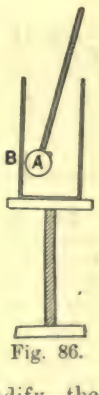

Fig. 86.

108. No Inductive Displacement in a Field of Uniform Potential. The argument of § 106 shows that inductive displacement is due to one part of the conductor occupying a portion of the field P Q R (Fig. 85), which was originally at *different potential* from the part P′ Q′ R′ occupied by another portion. If, therefore, the original field be one of uniform potential, there will be no inductive displacement. Thus in Fig. 86, when an insulated uncharged metal ball A is introduced into a charged metal pot B, the ball experiences the same electrical pressure all over, so that there is no influence tending to drive electricity from one part of it to another, and no part of it acquires any kind of charge. Moreover, there being no displacement, there is nothing tending to modify the electrical pressure, so that *the potential of the ball is precisely the*

same as that of the air before its insertion, and therefore the same as that of the pot.

Exp. 70. Place a tin can B (Fig. 86) on an insulating support and charge it. Tie a metal ball A (a door handle) to a piece of white silk thread, or stick it to a rod of ebonite or sealing-wax. Place A inside B without making contact, then withdraw it and touch the cap of an electroscope with it. No divergence occurs. Test again, placing A in another part of the pot. Again no divergence results.

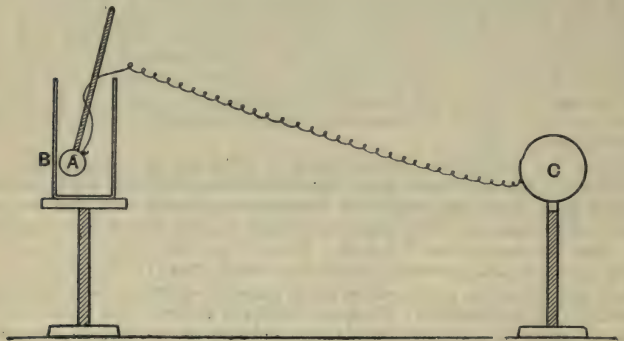

Fig. 87.

Exp. 71. Use the apparatus of Exp. 70, and this time, while A is within B (but not in contact with it), connect A by a wire to a neutral conductor C (Fig. 87) placed outside the point, and therefore in a region of different potential. In this case inductive displacement should occur. To test this, disconnect the wire from A while A is still in position, and then take A out and test for a charge. You will find that A has a charge of an opposite sign to that of the pot, thus showing that when the two ends of a conductor are in regions of differential potential inductive displacement occurs.

Exp. 72. Place a large biscuit tin, H K (Fig. 88), on its side on insulating supports. Take two insulated conductors A and B and furnish one of them, say B, with an insulating handle R, say an ebonite rod fastened to B's ebonite support. Place A and B within the tin and in contact with each other and such that R projects through a small hole cut in the side of the tin. Fit the cover to the tin. Strongly charge the tin. Now by manipulating R separate A and B, taking care not to discharge the tin. Remove A and B and test them separately by a neutral electroscope. You will find that in whatever position they were placed inside the tin neither exhibits the least trace of charge. The inference is that the field in which they are placed is one of uniform potential.

The following is a standard experiment in illustration of the principle of this article.

Exp. 73. Take a large gauze cage provided with a door and put a gold-leaf electroscope inside it so that the metal base of the electroscope stands on the metal work of the cage while the cap does not touch the cage, but is simply surrounded by the enclosed air. Shut the door, place the whole on an insulating stool, and charge the cage strongly by means of an electrical machine (Ch. IX.); *the leaves of the electroscope do not diverge.*

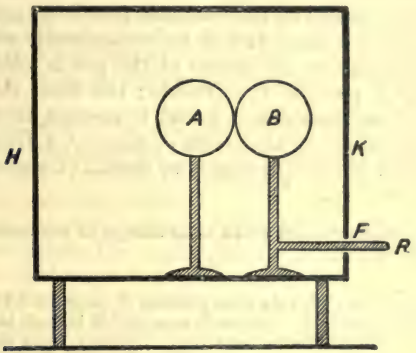

Fig. 88.

The reason is as follows:—The cap and leaves are surrounded by the air within the cage, and therefore, just like the ball A in Fig. 86, they acquire the potential of the cage. Also the walls of the electroscope, being in conducting communication with the cage, acquire, by Poisson's Principle, the same potential. Hence there is no divergence (Exp. 58).

In practice it is sometimes advisable to screen off an electroscope from the action of electrified bodies in the neighbourhood. To do this it suffices to cover the instrument with a metal cage whose lower edge rests on the table on which the electroscope stands; supposing the electroscope uncharged, the inside of the cage is at uniform potential (zero) and the walls of the electroscope are the same, so that no divergence occurs. Even if the electroscope be charged the *external bodies* produce no effect upon it.

109. No Charge on the Inner Surface of a Freely Electrified Hollow Conductor: Emptying out. When an uncharged ball, A, is held inside a freely charged pot, B, as in Fig. 86, we have found by Exp. 73 that the potential of the ball is equal to that of the pot. If, therefore, we make the ball touch the inside of the pot we shall be merely connecting two conductors which to begin with are at equal potentials, and therefore, by Poisson's Principle, nothing whatever will happen: there will be no flow of electricity either from the pot to the ball or *vice versa*, and, accordingly, if we take the ball out and test it by a neutral gold-leaf electroscope, we shall find it to be *perfectly neutral*. Now the ball while touching the inside of the pot is, electrically considered, simply a part of that inside; the fact, therefore, that when taken out and tested it is found to possess no charge proves that there is *no charge on the inside surface of a freely electrified hollow conductor*: the entire excess or deficit of electricity resides on the *outside surface*.

We may here mention that there is no charge in the actual material of any conductor (cf. § 122).

Exp. 74. Repeat Exp. 70, this time placing A in contact with the bottom of B. Test A, it is neutral. Before it touched B it had the potential of the internal field. If the latter had been different from that of the pot, there would on contact have been a flow of electricity one way or the other (by Poisson's Principle). The fact that there was no flow proves that the potential of the dielectric within the hollow conductor is the same as that of the hollow conductor.

The fact that all the charge resides on the outer surface of a conductor, which is of great importance, is one of several which were originally discovered by Faraday in respect of hollow conductors. It is scarcely needful to point out that it is only true for places well inside the pot, it is not true near the edge; if, however, the pot were furnished with a metal lid, so as to form a completely closed conductor, it would be true throughout the entire interior. The reason is that *the different portions of the charge are repelling one another* and therefore tend to separate mutually.

It should be carefully noted that if the ball be charged to begin with, and then made to touch the inside of the pot, it makes no difference in the final result. The charge on the ball is **completely transferred to the pot**, passing through to its outer

surface, and this is true even if the pot be charged at the outset, no matter how strongly. This process is known as **emptying out the charge** from the ball; it is the only known method whereby a charge can be *completely* transferred from a conductor to another insulated conductor: if the ball be made to touch the outside of the pot, or of another ball, the charge is *shared between them*. This emptying-out process will play an important part in some subsequent sections.

Exp. 75. Using again the apparatus of Exp. 70, first strongly charge A. (It makes no difference whether B is charged or not.) Place A in contact with the bottom of B, then remove A and test it. It is neutral, thus showing that on contact with the inner surface of B all its charge left it.

On reviewing the foregoing arguments and experiments, it will be seen that the absence of charge on the inner surface of the conductor is a necessary consequence of the uniformity of potential of the internal dielectric.

Now in § 105 we have pointed out that this internal potential is also uniform when the conductor is *not* freely charged, *provided there are no charged bodies inside it*: hence *in the latter case also there will be no charge on the inner surface.* This can easily be proved experimentally, first of all placing some charged bodies in the neighbourhood of the can.

It often greatly puzzles a student to reconcile the fact that there is *no charge* on the inner surface of the pot with the one pointed out in § 101, that the potential of the pot is the same inside as outside: this is part of a larger question which cannot be discussed here.

110. Proof of the Theorems of §§ 104, 105. The theorem of § 104 in reality consists of two, viz. that the potential of the dielectric inside a freely charged hollow conductor is—

(1) Uniform.

(2) Equal to that of the conductor.

These two require separate proofs. (1) was proved by Exp. 72 and (2) by Exp. 74.

The theorems of §§ 104, 105 are very important and the reader should not proceed further until they are well understood. It should be remembered, too, that they are equally true if there are charged bodies outside the conductor.

111. Hollow Conductor with a charged body inside it: Faraday's Ice-Pail Experiment. In this case (see § 105) the potential of the internal dielectric is not uniform and the arguments of the preceding articles do not apply. The case must therefore be examined separately: its investigation constitutes the most important of Faraday's "ice-pail" experiments, and is generally known as *the* ice-pail experiment. It is as follows:—

Exp. 76. On the cap of a neutral electroscope (Fig. 89) place a neutral metal pot A. Take a brass ball mounted on a long ebonite handle, give it a charge which we will suppose positive and equal to Q, and hold it in the pot as shown: the pot will then acquire an induced potential (§ 100) and the leaves will consequently diverge. Now move the ball about inside the pot (taking care that it does not touch) and it will be found that (so long as we keep it well below the level of the mouth) the divergence remains quite unchanged, thus showing that the induced potential of the pot is independent of the position of the ball within it. This is an important preliminary, but now comes the *main* part of the experiment:—Having noticed the divergence just mentioned, make the ball *touch* the inside of the pot so as (§ 109) to empty the charge, Q, completely into the latter: *the divergence of the leaves will not in the least alter.* Now when Q is emptied into the pot the latter of course acquires a free potential in place of the original induced one, and since the ball is completely discharged it may be removed from the pot without making any difference.

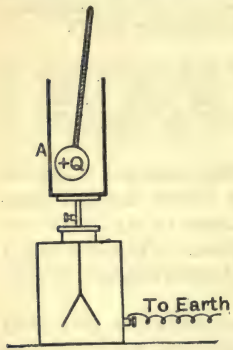

Fig. 89.

Our final conclusion, therefore, is that *when a ball having a charge, Q, is held within a metal pot, the potential of the latter is the same as if the ball were absent and the charge, Q, belonged to the pot itself.* There is, however, another aspect of the case to be considered—*viz.* the *charges* on the inner and outer surfaces of the pot when the ball is held within it before touching. These charges are, of course, due to inductive displacement, the outside of the pot acquiring a positive charge $(+x)$ and inside an equal negative charge $(-x)$, and the question is what is the relation between these local charges and the charge, Q, on the ball. To settle this it is best to resort to another experiment:—

Exp. 77. Take *two* metal pots, A and B (Fig. 90), of which B just fits inside A, and place them on an insulating stand as shown. Let the inner one, B, be provided with a white silk thread, CC, by means of which it can be lifted out of A. Hold the ball with the charge, + Q, inside B, then B acts as the inside surface and acquires the charge $-x$, while A acts as the outer surface and acquires the charge $+x$. Now, still holding the ball inside B, lift B out of A by the silk thread, so that we have B with the charge $-x$ and inside it the ball with the charge + Q. Then make the ball *touch* B, and afterwards test the pair by an electroscope, *when they will be found perfectly neutral.* This shows that + Q has neutralised $-x$ and therefore *Q and x must be equal.*

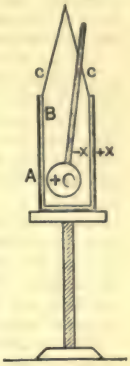

Fig. 90.

We therefore learn that *when a ball having a charge + Q is held inside a metal pot, it produces by inductive displacement an equal charge + Q on the outer surface of the pot and a corresponding charge — Q on the inner surface.*

One more experiment in conclusion.

Exp. 78. Place a metal pot on an insulating stand, hold within it the ball having a charge + Q, and touch the pot with the finger so as to get an induced charge on the pot as in § 102. Then remove the finger, make the ball touch the pot, and test the pair by an electroscope: this will be found perfectly neutral, hence the induced charge on the pot must have been − Q. *Earthing the pot in fact simply removes the external charge, leaving the internal unaffected.*

This is precisely what would happen according to the old "bound" and "free" theory (§ 107); it appears therefore that *in this particular case* the theory holds—as, however, it is in general invalid it is best to abandon it altogether.

It should be noted also that the case of the charged ball well inside the hollow pot is the only case in which the **induced and inducing charges are *equal***; if instead of the pot we use another ball, or if we hold the charged ball *outside* the pot, the induced charge is *less* than the inducing.

It should further be noted that if a ball with a charge + Q be held inside a pot, and the pot itself has an independent charge, q, the charges on the pot *so far as the influence of the ball is concerned* are just the same as if q were absent, *viz.* + Q on the outer surface and − Q on the inner, but to the former the charge q proper to the pot is *superadded* so that its *actual* charges are + (Q + q) on the outer surface and − Q on the inner surface.

112. Another Theorem concerning a Hollow Conductor with a charged body inside. In §§ 104, 108, we have seen that when a hollow pot is freely charged, the potential of the internal dielectric is uniform and equal to that of the pot, and that an *uncharged* conductor placed anywhere within the pot has impressed upon it the same potential.

Let us now consider the case where the internal conductor, as well as the pot, is charged. To begin with let the pot be insulated but *not* charged, and let a ball with a (positive) charge, Q, be held inside; the ball then has a certain potential, V, and the pot a lower (impressed) potential, x. Now give the pot a (positive) charge, q; this raises its potential by a certain amount, say v. The potential at all points within the pot so far as due to *its charge alone* is the same as if the ball were absent, it is uniform and equal to v. But to this must be superadded the potential due to the charge on the ball. The *actual* potential of the ball is therefore $V + v$, and that of the pot $x + v$.

Thus if a charged body be inside a hollow conductor, the effect of giving the latter a charge is simply *to raise the potential of the hollow conductor and the enclosed body by the same amount*, so that *their potential difference remains unaltered.*

Clearly also, since all points of the internal dielectric have their potential equally raised, *the potential-gradient at any assigned point inside a hollow conductor is independent of any charge the conductor itself may possess.*

113. Electric Screens and Shadows. If a charged body is placed inside an earthed metal pot, the potential of the pot is of course zero, and it is therefore incapable of applying electrical pressure to the external dielectric or to any external body. In other words, the pot completely screens the external region from the influence of internal electrified bodies, thus casting, so to speak, an electrical shadow in much the same way as a lighted candle placed within the pot would give no light outside.*

If instead of a pot we employ a broad flat metal plate, and place the charged body on one side of it, not near the edge, the action is very similar; there is a good electrical shadow on the other side of the plate. (See also Fig. 133 B.)

114. Electric Tension: Electric Spark. Consider any portion of a dielectric wherein the potential is not uniform: then in general a body placed in that region will experience a mechanical force. Now the particles of the dielectric itself are bodies placed in that region, and accordingly these experience a mechanical force. This force is commonly termed *electric tension*; its magnitude depends upon the potential-gradient of the

* Of course the light from the candle would shine upwards through the mouth of the pot, so that unless the latter were provided with a lid there would be a limited region above the mouth *not* in shadow: *it is much the same with the electrical action.*

portion of the dielectric considered. The general tendency of this tension is to rupture the dielectric, though whether it actually does so will depend, of course, upon the strength of the tension and the "rigidity" of the dielectric, that is, its power of resisting mechanical force. The rupture of the dielectric is accompanied by a spark, and an actual transference of electricity. The action may be readily followed by considering the following experiment:—

Exp. 79. Take a metal ball surrounded by air. Charge it highly positive with a machine (Ch. IX.). Then there is a down-gradient of potential as we travel through the air away from the ball, but it is not steep enough to produce any great tension, and the air is not ruptured. Now bring the end of an earthed wire towards the ball. The wire is of course at zero-potential, and as it gets nearer and nearer to the ball the potential gradient in the air-space between the two gets steeper.* Hence the tension in this region increases. At the same time the layer of air between the wire and ball becomes thinner, and therefore less and less able to resist the tension; and presently, when the wire is perhaps within about half an inch of the ball, the air momentarily breaks down and a spark passes, partly discharging the ball. If instead of the earthed wire we employ an insulated knob, then as the knob approaches the ball its potential will not be zero, but will rise, owing to electrical pressure transmitted to it across the air; for a given distance apart, therefore, the gradient will be less than in the case of the finger, and the knob will have to be brought nearer before a spark passes: when at length a spark does pass, the knob will receive what is practically a contact charge from the ball.

From Exp. 79 it will be gathered that, in all experiments on induction, care must be taken not to bring the inducing body near enough to the conductor we wish it to act upon for a spark to pass between them.

It should be noticed that when a neutral conductor B is charged by a spark from an electrified conductor A, the potential acquired by B is not so strong as if the two were made to touch. Suppose the spark to pass at a distance of $\frac{1}{2}$ inch, then, after passing, A's potential still remains stronger than B's, and the air between them is still subject to tension, though not sufficient to produce a second spark *at that distance*. But if they now be brought nearer another spark will pass, B will receive a little more charge, and the potentials will become more nearly equal, though they can never be *absolutely* so until they are in actual contact.

Of course, after air is ruptured it mends itself directly, but this is not the case with solid dielectrics. Thus, if we take a thin glass jar coated inside and out with tinfoil, earth the outer coat and charge the inner strongly, there will be a very steep potential-gradient between the inner and outer surfaces of the glass, and consequently a strong tension, so that the glass is very likely to become pierced or cracked, and of course permanently damaged. The arrangement referred to is practically a Leyden jar (§ 138), and in this way the not uncommon breakage of such jars is explained.

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* In § 135 we shall see that the approach of the wire lowers the potential of the ball. We neglect this in order to simplify the reasoning; the action is too small to affect the general conclusion.

It is scarcely needful to state that since glass, ebonite, etc., are mechanically far more rigid than air, it requires much greater tension to rupture them, so that two conductors at given different potentials can be brought much nearer together without a spark passing when separated, say by glass, than when there is nothing between them but air.

115. Further Study of Charge and Potential. We have already learned (§ 100) that a conductor may have a potential due to its own charge (*i.e.* a free potential), or one due to the influence of other electrified bodies (*i.e.* an induced potential), or both combined.

There are some further points to study in connection with the free potential of a conductor, and we shall deal with these in the next few sections. It must be understood throughout that the conductor is, unless the contrary is stated, supposed free from independent inductive influences; it simply has a free charge and a free potential due to that charge.

The subject of the next section is very important, both in itself and as a preliminary to the further studies proposed.

116. To Test the Amount of Charge of a Freely Electrified Conductor. In § 99 we have explained how to test the *strength of the potential* of a conductor. But this does not test the *amount of its charge* any more than the temperature of a body tests the amount of heat in that body. But if we perform different experiments with the *same body*, varying only the amount of heat, then the temperature depends only on the said amount of heat—the greater the one the greater the other. In like manner if we perform different experiments with the *same identical conductor*, varying only the charge, the strength of the potential will depend only on the amount of the charge—the greater the one the greater the other. Hence we have the following method for comparing the amounts of charge on two freely electrified conductors A and B whatever their size and shape, etc.

Take a tin pot and set it on the cap of a gold-leaf electroscope. Empty out the charge of A into the pot, as in § 109, and note the divergence of the leaves. Then discharge the pot and empty out the charge of B into it, again noting the divergence. By §§ 95, 99, the divergence in each case indicates the potential of the conductor, made up of the pot, cap, rod, and leaves. But, since the conductor is *identically the same in both cases*, the stronger

potential corresponds to the greater charge. *Hence the conductor which, under these circumstances, gives the greater divergence has the greater charge.*

For practical purposes there is no necessity to make either conductor *touch* the pot; for, as pointed out in § 111, the potential of the pot is the same when either conductor is *merely held well inside it.* This is a great advantage, because we can test the amount of charge without allowing any of it to leave the conductor; all we have to do is to *hold the conductors one after another well within the pot, and the one that produces the greater divergence has the greater charge.*

With the knowledge acquired above we can settle the following problem:—

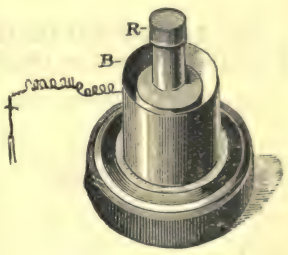

Fig. 91.

Exp. 80. *Show that when a body is electrified by friction the body and rubber acquire opposite charges of equal amount.* Two metal cans, A and B, are arranged as shown in Fig. 91 and insulated from each other. The inner, A, is lined with fur, and an ebonite rod, R, fits closely into it. The outer can, B, is connected to an electroscope by a wire as shown, or may simply stand on the cap. Twist the rod R. The leaves do not diverge, showing that the total charge inside the can B—that is, the charge on the ebonite and fur together—is nil; in other words, that the negative and positive charges are equal. To make sure that charges are actually present withdraw the ebonite, when any free charge on the fur will at once affect the electroscope.

117. Do Potential and Charge always go hand in hand?

Exp. 81. *To show that two conductors may have the same potential but different charges.* Take two insulated conductors not exactly alike, say two brass spheres of different sizes, give one of them a charge, place them some distance apart, and connect them by a thin metal wire held in an insulating clamp, so that the charge is divided between them. Then by Poisson's Principle they have the same potential, and of course if tested, as in § 99, this will be found the case. Moreover, since they are a long way apart, neither receives any appreciable impressed potential from the other; each has simply the free potential due to the charge on it. Now remove the wire, then each retains its potential and charge unaltered. Next hold them one after another in a metal pot standing on the electroscope, as described in § 116. *The bigger sphere will be found to produce the greater divergence, showing that it has the greater charge.*

Exp. 82. *To show that two conductors may have the same charge but different potentials.* This of course is a necessary consequence of the last experiment, but the following experiment shows it *directly* :—Take two precisely similar gold-leaf electroscopes, place on one of them a small tin pot and on the other a large one. Take two metal spheres of exactly the same size, make them touch, and give them a charge. Since they are both exactly alike, their charges must be equal. Empty the charge from one into the small pot, and that from the other into the large one, or (§ 116) merely hold one in one pot and one in the other without touching : *the electroscope in connection with the small pot will show the greater divergence. Here the two pots have equal charges, but the small one has the stronger potential* (Exp. 85 proves the same thing).

118. To show that the Charge on a given Conductor at given Potential depends on the Nature of the Surrounding Dielectric.

Exp. 83. Take two precisely similar conductors (Fig. 92) mounted on ebonite supports, each conductor consisting of a brass ball (A, B) surmounted by a brass rod (P, Q). Of these conductors let one, A P, be surrounded simply by air, while the other, B Q, is embedded up to nearly the top of the rod in a thick layer, D D, of paraffin-wax. Give the conductor, A P, a charge and then place it some distance from B Q and connect the ends, P, Q, by a thin insulated metal wire, so that the charge may be divided between the conductors and each acquire the same potential, as in Exp. 81. Then separate them by removing the wire, and hold them one after the other in a metal pot standing on an electroscope, when it will be found that *the one embedded in the paraffin-wax produces a greater divergence and therefore has a greater charge.*

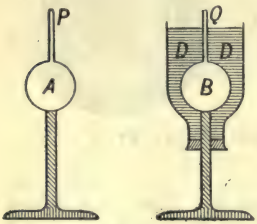

Fig. 92.

This is commonly said to be because the "specific inductivity" or "specific inductive capacity" or "dielectric constant" of paraffin-wax is greater than that of air, but this is merely taking refuge in words and explains nothing until we know to what actual property of the dielectric the so-called "specific inductivity" corresponds : this is discussed in the more advanced parts of the subject.

The specific inductivity, not only of paraffin-wax but of all solid dielectric such as glass, shellac, ebonite, etc., is considerably greater than that of air. The figures are glass 6, ebonite 3, paraffin-wax 2·3, shellac 2·8, petroleum 2·0, air being taken as unity.

119. To show that the Potential of a freely charged Conductor is weakened by bringing an Earthed Conductor near it.

Exp. 84. Take an insulated conductor, A, provided with a binding-screw, and connect it by a copper wire to the terminals of a distant electroscope (Fig. 93). Give it a free charge: the leaves diverge, the amount of the divergence indicating the potential. Now bring near the conductor A

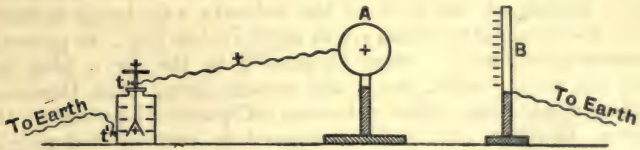

Fig. 93.

an earthed conductor B, say a metal ball or sheet, or simply the hand: the leaves of the electroscope will be seen to diverge less, and the divergence will diminish the nearer B approaches to A, thus showing a gradual weakening of A's potential. Of course the *charge* on A is not altered in amount by the process. We hence learn *the potential of a given conductor having a given charge is weakened by the presence of an earthed conductor in its neighbourhood.*

The principle of Exp. 84 is most important, and, as we shall see in § 135, is that on which the action of Leyden jars and other condensers depends.

The explanation is as follows:—Let us suppose A's charge positive, so that the leaves diverge at positive potential. Then, as explained in § 102, the earthed conductor B acquires a negative induced charge. The latter acting across the dielectric between A and B imparts to A a negative induced potential. The total potential of A is now its original free potential *diminished by the induced one*, so that its actual potential is lower (and weaker) than when B was absent. The nearer B is to A the greater is the induced negative charge and the greater the weakening of A's potential.

If B be insulated instead of earthed (and uncharged), it still weakens A's potential, though in general to a much smaller extent. The explanation is similar. Here, however, we have inductive displacement of B instead of an induced charge, and the weakening effect on A's potential of the negative charge on the near side of B is in a great measure counteracted by the

opposing influence of the positive charge on the far side. It is clear from this that in general the bigger B is the more marked will be its weakening effect upon the potential of A, and that in the special case where B is a very thin plate, so that the near and far sides are close together, the weakening effect will be practically *nil*.

It should be incidentally noticed that when a charged conductor stands on the table in the ordinary way, being mounted on an insulating stem some six or eight inches high, its potential is not so strong as if it were suspended in the air a long way from the table, walls, etc.; the reason being that these are earthed conductors. The reduction of potential owing to this cause is, however, very slight compared to that of bringing an earthed conductor very near the charged one, and is commonly disregarded.

A conductor insulated and surrounded to an infinite distance on all sides by a particular dielectric is said to be *isolated* in that dielectric; and when no other particular dielectric is referred to, air is understood; it is obvious that such a condition cannot be realised in practice, though a ball suspended by a white silk thread in the middle of a large room would be a fair approximation to it.

120. Electrostatic Capacity. Consider now a freely electrified conductor A, and if there are any other conductors in its neighbourhood let them be maintained at zero petential, say by being earthed. Collecting the several results of §§ 117-119, we see that the potential of A depends upon

(1) The amount of its charge,

(2) Its size and shape.

(3) The nature of the surrounding dielectric as expressed by its specific inductivity.

(4) The position, shape, etc., of the surrounding conductors, if there are any.

Now it can be proved that when items (2), (3), (4) are given the potential of A is proportional to its charge, in other words *the ratio of its charge to its potential is constant.* This ratio is called the *electrostatic capacity*, or briefly the *capacity* of the conductor A; its value depends upon the latter three of the fore-

going items.* For a conductor of given shape the capacity is in general greater the larger it is, the more effectually it is surrounded by earthed conductors and the greater the specific inductivity of the dielectric; as pointed out in § 118, the specific inductivity of all solid dielectrics—*e.g.* glass, shellac, ebonite, etc., is considerably greater than that of air, so that the effect of surrounding a conductor with any of these is to increase its capacity.

Exp. 85. Cut a piece of tinfoil, A (Fig. 94), 1 foot long 6 inches wide. Attach one narrow end by paste or gum to an ebonite or varnished glass rod, B, and the opposite edge to a piece of wood, C, just heavy enough to keep A taut. Connect A by a wire to a distant small electroscope (cap removed). Charge A till the leaves have a moderate divergence. Then rotate B, winding up A like a blind. Note that the divergence of the leaves increases, thus showing that the potential of A is rising. Since the charge on A is practically constant it follows that the capacity of A decreases as its area decreases.

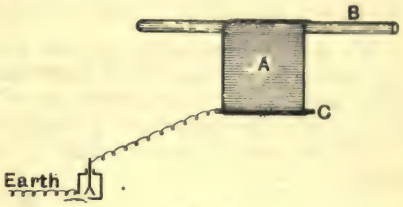

Fig. 94.

If a conductor is not freely charged—*i.e.* if it is under the inductive influence of some independent electrification, say an excited ebonite rod, or if there are any neighbouring conductors not at zero-potential, the ratio of its charge to its potential is in general *not constant* and is *not* called its capacity; the proper definition of the capacity of a conductor is the ratio of its charge to its potential *when it is not influenced by any independent electrification and neighbouring conductors, if any, are maintained at potential zero*:† a loose employment of the term in any other sense is apt to lead to serious confusion.

The definition of capacity may appear somewhat arbitrary to the student. He may be tempted to ask why could we not define the capacity of a con-

* Its value in terms of these items can in many cases be determined mathematically.

† Of course the charge induced on neighbouring earthed conductors is not *independent* electrification.

ductor as the quantity of electricity which it will contain. The answer is that the quantity of electricity which a conductor can contain is almost indefinite under favourable circumstances. If our ordinary vessels for containing liquid, instead of being of definite volume, were made of some material such as india-rubber, which would yield to pressure, then it would no longer be possible to measure the capacity of such a vessel by the quantity of fluid that it contained.

Or, taking a vessel of definite size, we may consider the quantity of gas it may contain. This quantity clearly depends on the "pressure" to which the gas is subjected. At a double pressure the vessel will contain double the mass of gas, and we might define the capacity of the vessel as the mass of a known gas which exerts a given pressure on the inside of the vessel. So we define the capacity of a conductor by the quantity of electricity which raises its potential by a given amount, say from zero to unity.

We shall subsequently (§§ 150, 155) explain how both charge and potential are capable of mathematical estimation in terms of their respective "electrostatic units"; if then Q and V denote respectively the charge and potential of a conductor under the conditions above named, and C its capacity, we have by definition—

$$C = \frac{Q}{V}$$

or

$$Q = CV \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

121. Energy of a Freely Charged Conductor. In any process of charging a conductor work has to be done. If it be charged by friction the greater part of the work expended in rubbing is wasted in heating the muscles of the arm, etc.; but a portion is stored up as energy in the conductor, and when the latter is afterwards discharged this store is transformed into heat, although the amount is generally very small. This is the source of the heat of an electric spark. That even a small spark *is* hot can be shown by briskly rubbing an insulated metal ball with fur and holding it over a gas-burner, when the gas will be ignited.

The energy of a charged conductor is an instance of *potential energy* (see Appendix, § 242).

If a charged body is held over light substances these are lifted; the conductor thus does work by virtue of its potential energy.

The energy of a charged conductor depends jointly upon its charge and its potential. If Q and V denote the charge and

potential respectively, estimated in their corresponding electrostatic units, and E the energy estimated in ergs, then it can be proved that

$$E = \tfrac{1}{2} Q V. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Moreover, if C denote the capacity of the conductor, then (§ 120) we have $Q = C V$, whence (1) may be written in either of the forms

$$E = \tfrac{1}{2} C V^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or

$$E = \tfrac{1}{2} \frac{Q^2}{C} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Care should be taken to avoid the popular error that electricity is a form of energy. *Heat*, as explained in treatises on that subject, *really is* energy, but electricity is no more energy than is air or water: the reasoning whereby the truth of this assertion is established is, however, beyond our province.

It is important to observe that, just as in order to confer energy upon water we must have some portion at higher (or lower) pressure than its surroundings, so in order to confer energy upon electricity we must have some portion at higher (or lower) potential than its surroundings: we cannot get work out of water at a dead level or electricity at uniform potential.

It should also be noticed that all energy is of the same *essential nature* whatever form it takes or whatever its source. Electricity is something quite different from matter, and potential is something utterly different from mechanical pressure; but the energy of a charged conductor is as truly mechanical as that of a horse. The phrase "electrical energy," sometimes employed, simply refers to the *way in which the energy manifests itself*, not to its *nature*.

Reconsideration of the Analogies to Potential. The analogies mentioned in § 93 are useful up to a certain step, but after that they are apt to mislead. One defect in both of them is that they present nothing whatever to correspond to the dielectric, which is the essential thing in electrical action (as the student will learn as he progresses with the subject).

The second analogy lends itself well to the conception of potential-gradients. Also just as the quantity of water in a

tank is equal to the cross-section of the tank multiplied by the head of water, so is the quantity of electricity on a conductor equal to the capacity of the conductor multiplied by the electrical head or potential.

SUMMARY.—CHAPTER VII.

1. A freely charged body produces a potential (of the same sign as its own) in the dielectric all round it, which becomes weaker as the distance from the body increases (§ 103).

2. **Potential-Gradient.** The dielectric *outside* a freely charged conductor is a region of potential-gradient, which in the case of a *positive* charge is a down-gradient as we *recede from* the conductor, and in the case of a negative charge is a down-gradient as we *approach* it (§ 103).

3. The interior of a hollow conductor is a region of *uniform potential*, in other words of *nil-gradient*, however the conductor be charged, always provided there is no electrified body *inside* it, the potential of the entire internal dielectric being equal to that of the conductor itself (§§ 104, 105, 110).

4. Inductive displacement (§ 106).

5. Fallacy of the old "bound" and "free" theory (§ 107).

6. No inductive displacement in a field of uniform potential (§ 108).

7. There is *no charge* on the inner surface of a hollow conductor under whatever circumstances it is placed, provided there be no electrified body *inside* it—but its *potential* inside is the same as outside (§ 109). **Principle of emptying out** (§ 109).

8. When a conductor having a (positive) charge is inside but not touching an insulated hollow conductor, it imparts to the latter an induced potential equal to the free one it would have if the inside conductor were absent and itself possessed the said charge. Also by inductive displacement its outer surface acquires a (positive) charge equal to that on the internal conductor, and its inner an equal (negative) charge (§ 111).

9. Electric screens and shadows (§ 113).

10. Electric tension and sparks (§ 114).

11. Method of testing the amount of charge (§ 116); compare it with the method for testing potential (§ 99).

12. Two freely charged conductors may have the same potential but different charges (or the same charge but different potentials) (§ 117).

13. Specific Inductivity (§ 118).

14. Influence of a neighbouring conductor; meaning of an *isolated* conductor (§ 119).

15. Meaning of **Electrostatic Capacity.** The capacity of a conductor depends upon *its size and shape, the specific inductivity of the surrounding dielectric, and the position, shape, etc., of surrounding (earthed) conductors* (§ 120).
Fundamental formula : $Q = CV$ (§ 120).

16. Work and Energy. The energy of a freely charged conductor is proportional to the product of its charge and its potential. Fundamental formulae :

$$E = \tfrac{1}{2} QV = \tfrac{1}{2} CV^2 = \tfrac{1}{2} \frac{Q^2}{C} \text{ (§ 121).}$$

EXERCISES VII.

1. An open tin pot about 4 inches broad and 9 inches high is insulated and charged positively. Trace the changes of potential at a point conceived to start outside, pass through the metal near the bottom, travel up through the middle of the pot, and come out at the mouth.

2. If the electroscope is in the condition explained in Exp. 73, and we push a long brass rod with a brass knob at the outer end through one of the meshes of the cage (taking care that it does not touch the latter), and make it touch the cap of the electroscope, how will the leaves behave (i) when the rod is insulated, (ii) when earthed?

3. In the preceding question, if the rod be afterwards removed, will the leaves of the electroscope be charged, and if so in what way?

4. A metal funnel rests with its upper edge in contact with that of a hollow metal cylinder of such length that the end of the stem of the funnel is near the middle of the cylinder, which is supported by an insulating clamp. Some shot charged positively are poured through the funnel into an insulated metal vessel some distance below the cylinder. If the cylinder and vessel are uncharged at the start, what will be their final charges?

5. A deep metal pot is stood on the cap of an electroscope and charged : the leaves diverge. A neutral insulated brass ball is then (1) introduced well into the pot without touching, (2) made to touch the bottom of the pot, (3) taken away. Will any difference of divergence be observed in any of these observations?

6. A charged insulated conductor A is surrounded by a closed metallic box. What experiment would you make to show that the charge on the inside of the box is equal and opposite to that on the conductor?

7. A metal pot is placed on the cap of an electroscope. The following operations are then successively performed; state and explain the final behaviour of the leaves in each case:—(i) + Charged ball held inside pot and then withdrawn; (ii) + Charged ball made to touch inside of pot and afterwards withdrawn; (iii) Pot touched by finger, finger removed, + charged ball introduced, pot again touched by finger, finger again removed, and lastly ball withdrawn.

8. Given two similar insulated metal pots and a small charged metal ball suspended by a fine silk ribbon, how would you charge the pots (*a*) with equal charges of the same sign, (*b*) with equal charges of opposite sign?

9. A gold-leaf electroscope stands on the table and a charge is given to the cap. The instrument is then shut up in a gauze cage standing on an insulating stool and the cage charged: how will the leaves behave?

10. A charged metal ball, A, is connected by a wire to the cap of an electroscope and the leaves diverge. The following operations are then performed in succession; state and explain the behaviour of the leaves in each case:—(i) A large uncharged insulated ball, B, is placed near A; (ii) B is touched by the finger; (iii) B is taken away.

11. A +ve charge is given to the cap and leaves of an electroscope and the leaves diverge. The hand is then held close over the top. What happens and why?

12. An insulated conductor, A, is brought near the cap of a gold-leaf electroscope which has been charged positively. State and explain what will happen (1) if A is unelectrified, (2) if A is charged positively, (3) if it is charged negatively.
Also state and explain what will happen if A be earthed.

13. Describe an experiment whereby you could accurately prove that when one kind of electrification is produced the opposite kind is also produced in equal quantity.

14. An electrified metal ball is introduced into a dry glass tube closed at one end, and then the tube, being held in the hand, is brought near to the cap of an electroscope. What will be the effect on the leaves if the exterior of the tube (1) is, (2) is not, covered with tinfoil?

15. The extremity B of a wire A B is attached to the cap of a gold-leaf electroscope. By means of an insulating handle the other end, A, is placed in contact, first with the blunt and then with the more pointed end of a pear-shaped insulated and electrified conductor. Describe and explain the movement of the leaves of the electroscope.

16. An electroscope is surrounded by a cylinder of wire gauze which is put to earth. If an electrified body is brought near it, how will the leaves behave? Give reasons for your answer.

17. Under what circumstances is it possible to transfer *the whole* of the charge on a conductor to another insulated conductor?

18. Describe an experiment to prove that two points may have the same potential, though one is positively charged and the other is either uncharged or charged negatively.

19. How could you show experimentally that when a charged conductor is introduced into another conductor without touching it the charge induced on the external conductor is equal to the inducing charge?

20. Discuss the analogies between differences of level, temperature, and electrical potential respectively.

21. Two equal insulated uncharged metal spheres, B and C, are placed at opposite sides of and at equal distances from a charged sphere A. What is the electrical state of B and of C, and what will happen if the part of B nearest to A is connected by a fine wire with the part of C farthest from A?

22. The case surrounding the gold leaves of an electroscope is made of metal with glass windows. When the instrument is placed on an insulating stand and the case charged with electricity, is there any divergence of the leaves? When the instrument is placed inside an insulated and charged metal vessel, is there any divergence? Give reasons for your answer.

23. Two equal conducting spheres are charged, one with positive and the other with an equal quantity of negative electricity, and are placed a certain distance apart. A third conducting sphere, which is insulated but uncharged, is placed exactly half-way between them. What is (i) the potential, (ii) the state of electrification of this last sphere? Give reasons for your answer.

24. Two gold leaves attached to the end of a glass rod are positively charged and diverge from one another. What effect if any is produced upon the divergence (1) when a spherical metal pot connected to earth is placed so that the leaves are at its centre, (2) when after insulating the pot a gradually increasing positive charge is given to it? Give reasons.

25. A stream of water falls from a vertical nozzle directed downward into a small aperture in the cover of an insulated metal pot about four feet below. Near the nozzle is an insulated electrified sphere. What electrical changes take place (i) when the stream breaks into drops immediately on leaving the nozzle, (ii) when it falls as a continuous stream into the pot below? If the stream entering the pot were electrified, how might the water be allowed to flow from the pot unelectrified? Adopting this arrangement, what would be the effect of continuing the above processes for an indefinite time?

26. Give the line of reasoning by which you would *prove* on experimental grounds that the potential of the dielectric inside a freely charged hollow conductor is uniform and equal to that of the conductor.

CHAPTER VIII.

DISTRIBUTION OF ELECTRICITY ON CONDUCTORS.

122. The Charge resides entirely on the Outer Surface of a Conductor: Biot's Theorem.—In § 109 we have seen that in the case of a freely charged hollow pot none of the charge resides on the *inner* surface, and that the same is true even when the pot is under inductive influence, provided there be no charged bodies inside the pot.

Now consider a freely charged *solid* conductor. Does the charge in this case reside on the surface, or does some of it penetrate the conducting material? To answer this question we must resort to experiment.

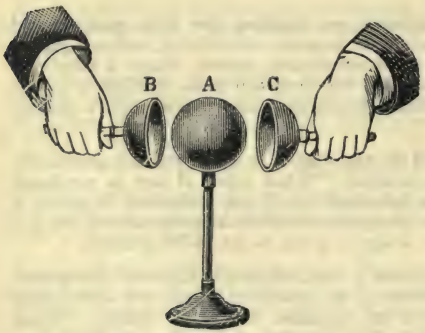

Fig. 95.

Exp. 86. In Fig. 95, A is a solid insulated conductor (which may be of any shape), and B and C are two metal covers mounted on insulating handles which fit it exactly, and meet so that when placed over A the whole is practically a solid conductor with its outer surface movable.* Now fit B and C over A and give the whole thing a charge, say by touching B with a charged ball or the disc of an electrophorus (§ 133). Then remove the covers B and C and test each by a neutral gold-leaf electroscope: they will be found charged. Again, test A by the electroscope: *it will be found perfectly neutral.* Hence the charge was entirely on the outer surface. As previously explained this result is due to the mutual repulsions of the different portions of the charge.

* If B is suspended by a white silk thread instead of being fixed to the top of a rod, B and C will fit over it and enclose it much better.

The experiment may be varied thus:—

Exp. 87. Charge A *before* placing the covers over it. In this case the charge to begin with is of course on A, which if tested by a neutral electroscope will make the leaves diverge. Now put the covers on, and afterwards remove them and test them and A: it will be found that A is completely neutral, all its original charge having passed on to the covers.

Clearly the only use of the covers in any case is to furnish us with an outer surface which can be removed, so that the exterior and interior may be separately tested; but the *fact* must be the same even when there are no movable covers.

The experiments above described are known as Biot's or Cavendish's experiments. Combining the result with the result of § 109, we learn that *in any freely charged conductor, whether solid or hollow, the charge resides entirely on the outside surface.* We shall refer to this result as **Biot's Theorem.**

By a modification of the experiments, Biot's theorem may be shown to be true of induced charges or of the local charges due to inductive displacement, always provided (§ 109) that if the conductor be hollow there must be no charged body inside.

It should be observed that Biot's theorem is *not true of insulators* under any circumstances, for if instead of a metal ball and covers we employ ebonite, glass, etc., a charge in general remains on the ball after the covers are removed.

Students generally experience great difficulty in reconciling the fact that the *charge* on a conductor resides exclusively on the outer surface with the fact that its *potential* is the same throughout the entire material; it must, however, be remembered that electrical pressure *primarily applied* to the surface of a conductor is *transmitted* throughout its entire material in much the same way as, if we have a vessel of air or water fitted with a piston and apply mechanical pressure to the piston, that pressure is transmitted through the entire mass of air or water.

An historical piece of apparatus for the further confirmation of Biot's Theorem is depicted in Fig. 96, and is known as "Faraday's butterfly net." It consists of a net of linen gauze (a fairly good conductor) mounted on an insulating stem with a heavy base. It is furnished with silk threads, S, S, attached to the apex, by pulling which it may be turned inside out. Now suppose it set as in the figure, and a charge given to it. If an insulated ball be made to touch the outer surface and then carried to a

neutral electroscope, the leaves will diverge, but if it be made to touch the inner they will not: this shows again that the charge is all on the outer

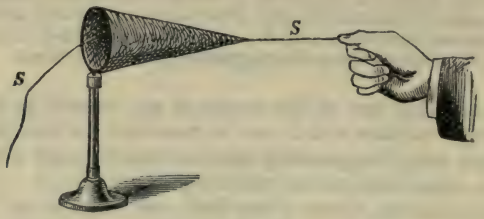

Fig. 96.

surface. Now pull the left-hand thread so as to turn the net inside out, and again test: the charge will be found all on the *new* outer surface, it having passed through when the net was reversed.

The following experiment illustrating the theorem in the case of an induced charge is instructive:—

Exp. 88. Set a hollow tin pot, A, on an insulating stand, and place a smaller one, B, inside it, the two being insulated from one another by ebonite or paraffin wax. Give the outer one, A, a (positive) charge. Of course this will raise B's potential, but will neither charge it nor produce any inductive displacement (cf. § 108). Now earth the inner pot, B: this will give it a negative charge (cf. § 108). Test the *inside* of B by touching it with an insulated brass ball and carrying the latter to a neutral electroscope: the leaves *will not* diverge. Then test the outside of B in the same way: the leaves *will* diverge. It is scarcely necessary to observe that, since there is now a body inside A bearing a charge of unlike sign to that of A, the charge on A does *not* reside entirely on the outer surface (cf. § 111).

123. How to Test the Amount of Charge on a given Small Area of a Conductor. Consider now a charged conductor, and let P be a certain point on its surface. Consider a given small area, say a square centimetre, of its surface immediately round P. On this area there will exist a certain portion of the charge. Now suppose we wish to measure this portion practically. To do so we must have a means of taking it off and examining it separately. This is done by means of a *proof-plane*.

Exp. 89. Make a proof-plane (Fig. 97). Fix a small brass disc *d*, or a penny piece, to a handle H of sealing-wax, or by means of a little sealing-wax or a screw to a handle of ebonite about nine inches long.

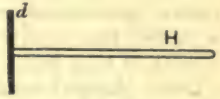

Fig. 97.

The proof-plane is placed flat on the area to be examined; it thus for the moment becomes the outside surface of that portion of the conductor, and by Biot's Theorem the charge originally on the area in question is now on the little piece of brass. We now lift the latter off, being careful not to tilt or drag it, and it carries the charge with it. To test the amount of the charge we adopt the method of § 116. A *very small** tin pot (about 1 inch diameter and 3 inches deep) is placed on the cap of a neutral electroscope and the proof-plane carefully lowered into it without making contact: *the amount of divergence of the leaves indicates the amount of charge on the small area of the conductor under examination.*

Theoretically the brass disc *d* should accurately fit the part of the conductor on which it is placed, for otherwise it will slightly alter the shape of the conductor and cause a modified distribution of the charge. As, however, we require to test different parts of the surface some of which may be flatter than others, this is impracticable, and it is usual to make the brass flat. Since it is very small, the fitting will be *very nearly accurate*, and quite good enough for all practical purposes, unless it happens that the part of the conductor to be tested is very pointed.

124. Distribution of Charge: Surface Density.

Exp. 90. Take a freely charged conductor of an irregular shape (Fig. 78), and by the method of the preceding section compare the charges per given small area at different parts of its surface. Place the proof-plane on the flattest part of the surface and then carry it to the pot; the leaves of the electroscope will diverge but slightly. Now try the curved part: they diverge more; now try the aperture: they will not diverge at all; and now try the point: they diverge most of all.

From this experiment we learn that the charge per given small area is greatest round the point, less on the curved part, still less

* We say small because under ordinary circumstances the charge on the proof-plane will be small.

on the flat, and practically nil inside the aperture—the latter, indeed, being practically the inside of a hollow conductor.

The charge per given small area (generally per square centimetre) immediately round any point on the surface of a conductor is called the *electric density* or **surface density**, or, briefly, the *density* at that point; what we have learned, therefore, is that in a freely charged conductor *the density is greatest at the pointed portions and least at the flat, while if there be any distinct hollows the density in them is nil.*

In the case of a rod the density would be found considerable at and near the ends and very small in the middle; in a flat circular plate it is great at the edges and small in the middle; in a

Fig. 98.

rectangular plate it is great along the edges, greater still at the corners, and small at the middle; and in a sphere it is uniform all over. Fig. 98 exhibits diagrammatically the density at different parts of sundry conductors; the inner continuous line shows the conductor, while the distance of the dotted line from the adjacent points thereof indicates the density at those respective points.

Students are often liable to confuse the results of this section with that of § 101, and to think that, because the *density* varies from place to place, therefore the potential does so: this is not true—the potential is, of course, the same all over, in accordance with Poisson's Principle. As already seen (cf. §§ 106, 109), we must *not* attribute the potential at a point on the conductor merely to the *local charge* at the point in question.

The following is an instructive experiment on potential, charge, and density.

Exp. 91. Place two insulated metal spheres, one large and one small, some distance apart, connect them by a thin insulated metal wire, and give them a charge. Then we have seen (§ 117) that both are at the same potential while the big one has the greater charge. Now test their

electric densities by the method of Exp. 90, and it will be found that *the density on the small sphere is the greater*: this we should expect, seeing that the bigger a sphere is, the flatter is its surface.

In this experiment, since the spheres are a long way apart, they do not produce any appreciable inductive displacement on one another and the density on each is uniform. If instead of being placed at a distance and joined by a wire they be made *to touch*, the same facts will be broadly true, but the density on each sphere is now no longer uniform, and the comparison must be made at places remote from the niche between them; *within* this niche the density on both is very small, the niche resembling the hollow, Q, in Fig. 78.

The conception of electric density and the mode of testing it is precisely the same for induced charges and for the local charges due to inductive displacement as for free charges, but the actual value of the density at any point will in general be different. Thus in a freely charged sphere (not near other conductors and surrounded by nothing but dry air) the density is uniform all over, while when earthed and charged by induction the density is greatest on the side nearest the inducing body, and least on the side directly opposite.

125. Relation between Surface Density of Electricity and Potential Gradient.

Exp. 92. Charge an insulated sphere and place it in a position well away from other conductors. Charge it strongly (use an electrophorus or a machine [Ch. IX.]). Test the distribution by the method of Exp. 90. It is uniform. Now bring close to it an earthed metal plate, C D (Fig. 99), and test the distribution again. You will find the density at the part, A, away from the plane is much less than that at the part, B, near the plane. If we travel from A outwards through the medium we should have to go one or two feet before we came to a place of nearly zero potential, whereas if we travel outwards from B we strike zero potential as soon as we reach the plate C D. We thus have a steeper potential-gradient on the right hand side.

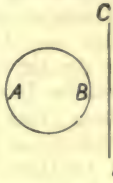

Fig. 99.

Thus a large surface-density is associated with a steep potential-gradient, and in fact can be mathematically proved to be proportional to it.

From the foregoing it will be readily seen that the density at a given point of a conductor having a given charge depends not only on the shape of the conductor, but also upon the circumstances under which it is placed, especially as respects other conductors adjacent to it. It will also be seen that the fact of a conductor possessing a sharp point does not *by itself* ensure that the density at that point will be great, because it is quite possible

to place the point in such a way that the potential-gradient in the dielectric adjoining it shall be small. For example, if the conductor be a deep metal pot and a pin be soldered on the bottom of it inside, the density at the pin-point will be *nil*.

Again consider a pear-shaped conductor with a metal plate in front of the point, the two being connected by a wire (as in Fig. 100). The portion of the dielectric between the point and the plate may be roughly regarded as the interior of a hollow conductor, and therefore (§ 104) the potential-gradient in it is small, so that the density at the point is small—it may in fact, if the plate is sufficiently near the point, be smaller than at any other part of the conductor.

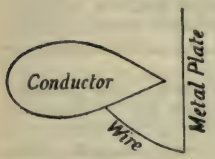

Fig. 100.

126. Electric Convection from Points. Consider any small area of a charged conductor. The particles* of air adjoining that area receive a contact charge and undergo repulsion the amount of which is greater the greater the electric density. Hence, at sharp points when the density is very great, the repulsion may become sufficiently strong to overcome the pressure of the air behind the charged particles, in which case the latter will be driven away carrying the charge of the conductor with them: this phenomenon is known as *electric convection*, or the *discharging power of points*.

Another closely related action is their *collecting power*. If we take a neutral conductor A with a sharp point, and hold a positively charged conductor B a few inches in front of the point, it is found that A acquires a positive charge just as if there had been actual contact. The explanation is clearly as follows:—B acts inductively on A, and by the principle of inductive displacement the sharp point acquires a negative charge, while the other side of A acquires a positive one. The (negative) density at the point is very great, and negatively charged air particles are carried across to B. These take with them negative electricity, so that A is left positively charged. Moreover, the negative charge on the air particles neutralises some of the positive electricity on B,

* Other particles taking shares in this process are dust particles and charged gaseous particles. Some of these latter are always present.

so that the positive charge on the latter is decreased, and the final effect is precisely the same as if there had been a direct transference of electricity from B to A.

The action of points may be shown by numerous experiments.

Exp. 93. Solder a needle into a small brass base, leaving the point sticking up, and stand it on the cap of an electroscope. Charge the latter: the leaves will diverge, *but will soon fall to again*, the instrument discharging itself by convection from the point. The discharge is accelerated by holding the hand (or any earthed conductor) over the needle point, because this steepens the gradient, as explained in §§ 114, 125.

Exp. 94. Having set the needle on the cap (the whole being neutral) hold a charged conductor a few inches above it: the leaves will diverge. Then lift off the needle with a pair of insulating tongs and remove the conductor: they will remain permanently diverged, showing that the cap, rod, leaves, etc., have received an actual charge, and not merely an induced potential, as would have been the case without the needle. Show that the sign of the charge on the electroscope is the same as that on the conductor. Do this by holding the conductor over the cap. Increased divergence results.

It is clear from what has been said in § 125 that a sharp point will have no special discharging (or collecting) power unless it is in a part of the dielectric where there is a steep potential-gradient: for example, a needle point soldered well inside a tin pot will not help to discharge the pot.

The discharging action of points and sharp edges renders it very necessary that the edges of the conducting parts of electrostatic instruments should be nicely rounded off. Thus, in an electroscope the edges of the cap, binding-screws, etc., must all be rounded, otherwise the instrument will not retain any charge that may be given to it. In like manner the edges of our tin pots must be turned down or have wires soldered to them. For the same reason all instruments for electrostatic experiments must be kept very clean, for particles of dust and dirt act as discharging points.

127. Lightning and Lightning Conductors. A flash of lightning is simply a huge electric spark. During a thunderstorm the clouds, for reasons into which we cannot enter, become highly charged with electricity and acquire a very strong potential, as the result of which a steep potential-gradient and consequent strong tension (§ 114) is set up in the air between them and the earth, and should this tension become sufficiently great a flash passes downwards. Moreover, since the clouds are nearer to

the tops of trees and buildings than to the ground, there is a steeper potential-gradient immediately over these objects than elsewhere, and they are therefore more liable to be struck.

A *lightning conductor* is a device whereby the "collecting power" of points (§ 126) is utilised to help discharge the clouds by the process of electric convection, and so diminish their tendency to produce flashes. It consists of an iron rod* run up the side of the building to be protected and insulated from it, the lower end of the rod being well embedded in the ground, and its upper terminating above the top of the building in numerous sharp points. It is best to have several such conductors one up each corner of the building.

Lightning conductors occasionally fail, and in place of them it was proposed by Maxwell to cover the building with a network of wires connected to the earth so that it might be screened from the electric influence in the same way as an electroscope covered with a wire cage (§ 108). A modification of this system has recently been introduced in which, in addition to the usual vertical conductors, a horizontal one connected to them runs along the roof and down to the eaves, which it also encircles. The metal rain-pipes are joined to the conductors, and a special contrivance is employed for making good earth contact; this consists of one or more hollow perforated metal tubes filled with charcoal and kept wet by communication with the rain-pipe: these tubes are well embedded in the ground and the conductors connected to them.

SUMMARY.—CHAPTER VIII.

1. Biot's Theorem (§ 122).

2. Use of proof-plane and proof-cone in conjunction with small pot for testing *density* (§§ 123, 124); compare this carefully with the method of § 99 for testing *potential*.

3. Relation between electric density and potential-gradient: the electric density at a sharp point is *not necessarily* very great; it all depends upon whether there is a steep gradient in the dielectric near the point (§ 125).

4. Discharging and collecting power of points (§ 126).

5. Action of Lightning Conductors (§ 127).

EXERCISES VIII.

1. In electrical experiments it is not unusual to employ balls, etc., made of wood coated with tinfoil, and they act precisely as if they were of metal throughout. Why is this? Would it matter whether the balls were solid or hollow?

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* Formerly copper was employed as being a better conductor, but iron is more efficient, besides being cheaper; ordinary "galvanised" iron telegraph wire does very well.

2. Two pear-shaped conductors are placed near together with their points facing one another. They are then connected by a copper wire and charged. Discuss roughly how the charge distributes itself over them.

The wire is now removed and the conductors charged, the one to a + ve and the other to a − ve potential. How will the charges be distributed?

3. A stout piece of copper is bent into the form of a ring with an opening, and into one side of the opening is soldered a needle, so that its point faces the opposite side, and is distant about $\frac{1}{8}$ inch from it; the whole is mounted on a brass stand and placed on the cap of an electroscope, which is then charged. State and explain how the leaves will behave.

4. It is often loosely asserted that "the density on a charged sphere is uniform"; state exactly under what conditions this is true.

5. An orange, into which a sewing-needle has been stuck point outwards, is suspended by a dry silk thread. A charged body is brought near it (1) opposite the point of the needle, (2) opposite the side remote from the needle. State and explain the electrical effect in each case.

6. A sharp point attached to a conductor, A, is held near an insulated charged conductor, B. What will be the effect on B if A is (1) insulated, (2) uninsulated?

7. Two gold-leaf electroscopes similar in all respects, except that a needle projects from the cap of one of them, are placed at equal distances from a charged conductor, and the leaves of both diverge. When the conductor is removed, one pair of leaves collapses suddenly, and the other slowly. Explain the difference in their behaviour.

8. A hemispherical metal bowl to which a short metal point is attached is charged with electricity. What difference, if any, in the rate of loss of charge will there be according as the point is fastened to the concave or convex side of the bowl?

9. A flexible conducting thread to the end of which a needle is attached lies at the bottom of an insulated charged iron pail. One end of the thread always remains in contact with the pail, while the needle is lifted out of the pail (without touching it) by a silk thread. Describe and explain any change which takes place in the charge of the pail during this process.

10. Two metal spheres are placed in contact and charged. On testing with the proof-plane it is found that the density in the niche between them is much less than at the far ends. Account for this.

11. Two equal metal spheres are placed close together but not in contact. Equal charges of electricity are given to the spheres. Give a sketch showing the general character of the distribution of electricity over the spheres when the charges are (*a*) of the same sign, (*b*) of opposite sign.

12. A positively charged small body is placed in the neighbourhood of a metal sphere; give diagrams roughly indicating the distribution of electricity over the sphere (1) when the sphere is insulated and uncharged, (2) when it is earthed. Diagrams should be given for two positions of the charged body, one of these being near the sphere and the other a considerable distance away.

CHAPTER IX.

ELECTRICAL MACHINES AND CONDENSERS.

128. The Frictional Electrical Machine. A frictional electrical machine is a mechanical arrangement for generating a charge by the friction of two suitable substances, and for collecting and storing the charge so produced. Such a machine comprises essentially three parts: the *generator*, consisting of the body

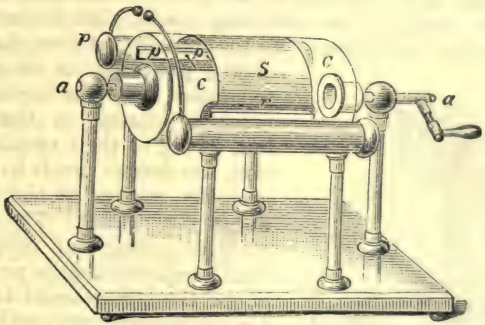

Fig. 101.

rubbed and the rubbers, the *collecting combs*, and the *prime conductor*, in which the charge collected by the combs accumulates. A very simple form of machine is shown in Fig. 101, which represents the essential parts of the ordinary cylinder electrical machine. The generator consists of a glass cylinder,* C, capable of rotation about its axis, *a a*, and a rubber, *r*, made of a leather-covered pad coated with a mixture of an amalgam of zinc and mercury with grease (§ 84).

* A large bottle—if carefully made and mounted—answers the purpose well.

The rubber presses against the surface of the cylinder, and on rotation the friction between the glass and the amalgam causes the surface of the former to become positively electrified, and that of the latter negatively. As the cylinder rotates the electrified portions leave the rubber behind, and other portions previously unelectrified come under its influence. Before again returning to the rubber the electrified surface of the glass passes close to the row of sharp-pointed wires which constitutes the collecting comb, *p p*. These points, acting in the way explained in § 126, "collect" the charge from the surface of the glass, which is thus rendered neutral and ready to be again electrified by the friction of the rubber. Connected with the comb is the prime conductor, P, an insulated conductor on which the charge collected from the cylinder accumulates.

S is a silk flap attached to the rubber, and extending over the cylinder nearly to the collecting comb. Its use may be explained roughly as follows. As the cylinder revolves the positive charge produced on its surface is separated from the corresponding negative charge on the rubber; but, owing to their mutual attraction, the former tends to slip back over the surface of the glass towards the rubber. The silk flap, however, acting as a rubber, has a small negative charge produced in it, and the attraction of this charge for the positive one on the glass is just sufficient to prevent any appreciable loss by reunion of the separated charges.

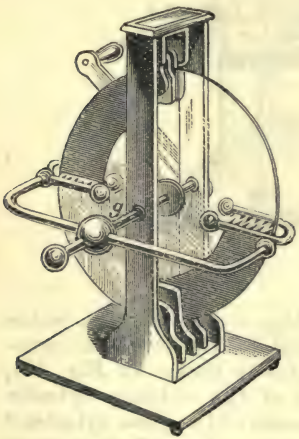

Fig. 102.

Fig. 102 illustrates a more usual form of electrical machine. It is known as the **Plate Electrical Machine**, and is in principle exactly similar to the one just described. A circular plate of glass takes the place of the cylinder. The rubbers, two in number, are placed at the upper and lower extremities of the vertical diameter of the plate, each one being made up of two pads so arranged that the plate runs between them, and

consequently has both sides excited. The combs, having to collect from both sides of the plate, are bent round it in the way shown in the figure, and are placed at the extremities of its horizontal diameter. The silk flaps and prime conductor* perform the same function as in the cylinder machine.

The action of the machine is readily understood. Following a portion of the surface of the plate from the lower rubber round through a complete revolution, we notice that as it leaves the rubber the glass is positively charged, and comes round to the right-hand comb, where its charge is collected by the points; after being thus rendered neutral, it next comes under the influence of the top rubber, again becomes positively charged, and in passing the next comb is again neutralised, ready to be again excited by the friction of the bottom rubber. In this way the prime conductor acquires a positive charge whose magnitude increases as the machine is worked until the loss by defective insulation and convective discharge from its surface is equal to the gain from the combs. To reduce this source of loss as much as possible every part of the conductor is carefully rounded and polished, and the machine should be thoroughly clean and dry.

As generally worked the rubbers of the machine are earth-connected, that is, they are not insulated, but directly connected with the framework of the machine.† In this way no charge accumulates on the rubbers. If, however, they are insulated, then the negative charge accumulates, and finally becomes sufficiently great to prevent the positive charge produced on the glass from reaching the combs, and thus the machine ceases to work. If rubber and prime conductor are both insulated, but connected together by a conducting wire, then the negative of the rubber neutralises the positive charge induced in the prime conductor, and, however hard the machine may be worked, no sign of electrification can be detected in either rubber or prime conductor, thus proving that the quantities of electricity produced by friction are equal, but of opposite sign (cf. Exp. 80).

129. Experiments with the Electrical Machine. Work an electrical machine until the prime conductor is charged. Then perform the following experiments.

* The prime conductor is here attached to the frame of the machine, and is insulated by the glass stems g, which should be well varnished and kept perfectly clean and dry.

† In Fig. 101 the rubber r is attached to the insulated conductor shown in the front of the figure.

Exp. 95. Present the knuckle or any earthed conducting body to the prime conductor. A spark at once passes and the prime conductor is momentarily discharged.

Exp. 96. Attach a tassel of light thread or strips of paper or hair to the prime conductor. When the prime conductor is charged the tassel opens out, each individual thread or hair repelling and being repelled by every other one.

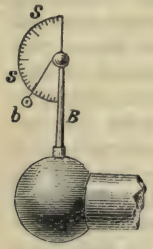

Fig. 103.

Henley's quadrant indicator is a simple application of this effect. As shown in Fig. 103, it is often attached to the prime conductor of electrical machines, to indicate the magnitude of the charge here collected. The pith ball, *b*, carried by a light fibre, is repelled from the similarly charged brass rod, B, to which it is attached; and the angular distance through which it is repelled may be read off on the circular scale, *s s*, and roughly indicates the state of electrification of the conductor.

Exp. 97. *Study the action of points.* Work the machine in a dark room.

(*a*) Note the action on the points of the collecting combs. A bright

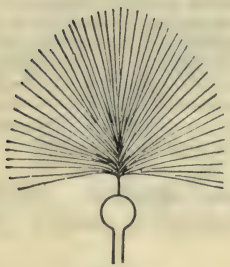
Fig. 104.

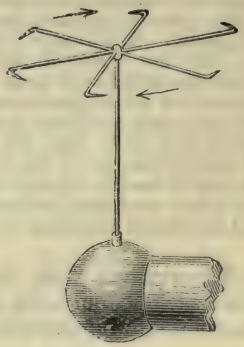
Fig. 105.

luminous glow or brush will be seen to diverge from each point, indicating that there is a continuous convection discharge from the points on to the surface of the glass.

(*b*) Attach a sharp-pointed rod or wire to the prime conductor; you will now find it impossible to get more than a tiny spark from the machine, but you will observe a bright brush discharge something like Fig. 104 coming from the pointed end of the wire.

(*c*) Hold a sewing-needle in the hand and present the point to the charged prime conductor of the machine. A silent discharge occurs from the point neutralising the charge on the conductor.

(*d*) Direct the points of either (*b*) or (*c*) towards a candle-flame. The "wind" of the discharge blows the flame aside.

(*e*) Mount an electric "whirl" (Fig. 105) on the prime conductor. The arms carrying the points are lightly pivoted on a central axis. When the machine is worked the wind from the points gives a backward reaction which drives the whirl around in an opposite direction to that in which the pointed ends of the wire are bent.

Exp. 98. Place a piece of wood on four blocks of paraffin wax or on two glass tubes lying on the floor. This constitutes an *insulating stool*. Stand on the stool and grasp the prime conductor of the machine. You are now practically a part of the prime conductor, and will become gradually charged as the machine is worked. Put your finger near any conductor such as a gas bracket or another person. Sparks fly just as they did from the prime conductor. Turn on the gas and light it with a spark. Your hair, if dry, will stand on end, so also will the fibres of your clothing. Stop the machine, you quickly lose the charge, for electricity escapes from the point of every hair and fibre.

Exp. 99. Soak a piece of filter paper in a solution of potassium iodide and starch in water. Spread the wet paper on a glass plate. Hold the glass plate in the hand and let the end of a wire connected to the prime conductor rest somewhere on the paper. Work the machine. A blue colouration appears on the paper around the end of the wire. This proves that the discharge passing between the end of the wire and the hand has liberated iodine in the neighbourhood of the wire (cf. Exp. 110).

130. Ebonite Plate Machine. On account of the persistence with which moisture adheres to glass, any frictional machine wherein that material is employed is troublesome to manage. It needs to be stood for some time before a fire so as to get thoroughly dried, and even then the results are apt to be disappointing. Besides this, the constant heating and cooling, and the mechanical strain on the glass due to the irregular potential it acquires on electrification (§ 114), generally cause it to crack after a few months' use.

These troubles are largely avoided by employing a thick ebonite plate instead of a glass one. Unless the weather is very damp, such a machine works *fairly* well without any drying at all, while

if it be taken to pieces and the rubbers and plate both dried it gives admirable results, sparks two or three inches long being obtained by a couple of turns of the handle. The rubber for such a machine should consist of well "amalgamated" leather or silk, which imparts a *positive* charge to the ebonite (§ 84), and therefore to the prime conductor; it is best also to *abolish the flaps*, which are found a hindrance rather than a help.

131. Limit to the Action of an Electrical Machine. One would at first sight think that no matter how long we went on working a frictional machine we should continually impart more charge to the prime conductor, so that there would be no limit to the charge it could acquire.

This is by no means the case. In the ebonite plate machine (§ 130) it is practically found that when in good order a couple of turns of the handle will give a spark say two inches long, while twenty turns will not give an appreciably better spark. Now, how is this explained? *When two things are rubbed together each can acquire a potential of a certain strength, depending upon the nature of the things; the potential actually acquired may be weaker than this, but it cannot be stronger.* For example, if we beat a copper ball with india-rubber the ball can, and may, attain a certain potential V, called the *limiting potential*, but as soon as this is reached it is useless to beat any more; no further charge is produced on the ball. If now we discharge the latter its potential becomes zero; and if we then renew the beating we can generate more charge until the potential of the ball again becomes V, and so on. This is a *fact*: we do not propose to explain it theoretically.

Now let us apply this fact to a frictional machine. By friction against the rubber the highest potential which any part of the plate can acquire is the limiting potential, V, say. The prime conductor starts at zero potential, and as it becomes charged its potential rises. *But it can never exceed V*; indeed, it can never quite reach V, for when the difference of potential between the prime conductor and the part of the plate opposite the collecting combs becomes small the combs will (cf. § 114) cease to "collect," and the potential of the prime conductor will have attained its limit: after this it is useless to go on turning the handle. Of course, if the conductor be now discharged wholly (by earthing), or partially (by connecting to some other conductor), its potential will be lowered, and on again turning the handle a fresh charge will be generated until the potential again reaches its limiting value. We have, then, this principle:—*The prime conductor of a given machine (and of any conductor in connection with it) can be raised to a certain definite limiting value, but no further.*

The theoretical value of this limiting potential depends purely on the nature of the plate and rubber and the distance between the plate and collecting points, but its practical value also depends upon whether the machine be in good order: if, for example, it be damp the charge will leak away and the actual potential will never reach its theoretical limit. The limiting potential does not depend on the *size* of the machine, but a big machine when at the limiting potential can furnish a greater *charge* than a

small one, and as the chief use of such machines is to furnish a charge to condensers (§ 135) (whose capacity is large) this is a manifest advantage.*

It should be observed too that, comparing a good electrical machine with, say, a copper ball beaten with india-rubber, though the machine doubtless affords a stronger potential, this is not its main advantage, which consists rather in the fact *that it supplies a charge rapidly*. To charge a condenser from the machine may be likened to filling a pail from a pump; to charge it from the beaten ball may be compared to filling the pail by ladling water into it with a spoon.

132. Spark from Machine and Other Conductors. It has been found that the *length* of the spark which passes between two conductors is roughly proportional to their difference of potential,† a spark 1 inch long corresponding to a potential-difference of about 100,000 volts (equivalent to about 300 electrostatic units of potential).‡

If one of the conductors be earthed its potential is zero, and the length of spark is then simply proportional to the potential of the other. The *greatest* distance across which in any assigned instance a spark will jump is called the *striking distance*. Thus consider a charged frictional machine; bring the knuckle gradually near the prime conductor, and suppose when it comes to a distance of $1\frac{1}{2}$ inch a spark passes: then the striking distance is $1\frac{1}{2}$ inch, and indicates that the potential of the prime conductor is (about) 150,000 volts. Suppose now the knuckle brought to a distance of $\frac{1}{2}$ inch, and the machine worked: a spark will now pass when the potential reaches (about) 50,000 volts, and it will be impossible to raise the potential above this until the knuckle is taken away.

It should be carefully noted that when a charged conductor is brought near an earthed one it is the *length* of the spark that indicates the *potential* of the conductor. The *brightness* of the spark (as also the noise it makes and the heat it emits) depends upon the *electrical energy* stored in the conductor, and this

* It is sometimes roughly asserted that with a given machine every turn of the handle produces a given charge: this is approximately true provided the prime conductor be a good way off its limiting potential, but not otherwise: as the limiting potential is approached the charge produced at each turn grows less and less.

† This does not hold for pointed conductors, but only for those with fairly flat or rounded surfaces.

‡ A volt is the unit of potential employed for practical purposes; it is defined in § 164. An electrostatic unit of potential is about 300 volts.

depends both on *the potential and the charge*. Thus consider two spheres, one small, the other big, and both at the same potential. They will both give sparks of the same length. But the electrostatic capacity of the big sphere is the greater, and it has a greater charge and greater energy (§ 121), hence the spark from it will be brighter and emit more heat than that from the small sphere.

133. The Electrophorus. This is a little instrument for generating electricity whose main action is due to induction. It is shown in Fig. 106. It consists essentially of two parts: a circular brass plate B, generally called the *disc*, to which is attached an insulating handle (ebonite is preferable to glass), and an ebonite plate E, called the *cake* (the ebonite is frequently replaced by some resinous substance). The lower side of the cake must be earthed (for the reason set forth in small print at the end of this section), either by simply resting it on the table, or still better by having it fitted into a shallow earthed metal base called the *sole*.

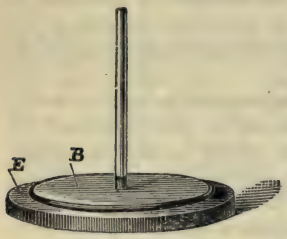

Fig. 106.

In order to work the instrument we commence by beating the cake with fur, which gives it a negative charge and produces a field of negative potential in the air above. We then hold the disc by the insulating handle, and set it on the cake as shown in the diagram; it then partakes of the negative potential of the field, or to put it in other but equivalent words, the negative charge of the cake acts upon it inductively, giving it a negative induced potential* (cf. § 100).

* There is practically no *contact* charge conveyed from the cake to the disc. Since the cake is a non-conductor there would be very little even if the two fitted accurately over one another, but as a matter of fact the surface of neither is truly plane, and the contact occurs only at a few points, so that there is a thin layer of air between. The inductive action of the cake takes place through this layer. But that is not all; for the charge on the cake does not lie entirely on the surface, as it would if it were a conductor, but penetrates the interior, and the internal part of the charge acts inductively on the disc across the upper layers of the cake itself.

The disc B is now momentarily earthed by touching it with the finger, when electricity flows into it from the earth in sufficient quantity to bring its potential up to zero, so that it acquires a positive charge. The disc is lastly lifted (by the handle), when, the inductive influence of the cake being removed, the positive charge confers upon it a positive potential, and if it be made to touch an insulated neutral conductor part of its charge will flow into the same in obedience to Poisson's Principle. Generally a spark passes at the same time. If instead of an insulated conductor we employ an earthed one (*e.g.* the knuckle) the disc is completely discharged. After discharging it may be replaced on the cake, touched and lifted, and the same thing occurs again. We may thus charge any conductor by instalments; but if the conductor be of large capacity (*e.g.* a Leyden jar) the process is very tedious on account of the small capacity of the disc, and the consequent small charge it acquires each time.

Since, except for the original electrification of the cake, the whole process is one of induction, the charging of the disc in no way diminishes the charge on the cake, and were it not for accidental leakage the cake when once electrified would charge the disc an indefinite number of times. The electrical energy furnished by the instrument is simply the equivalent of the work done by the operator in separating the disc and cake against the mutual attraction between the positive charge on the former and the negative on the latter; it has nothing to do with the work expended in beating or rubbing the cake.

It should be observed that it is not necessary to delay earthing the disc until after it is on the cake: it may be earthed before or simultaneously; in any case the final charge on the disc will be the same. Instead, therefore, of using the finger, we may allow the edge of the disc to touch the flange of the sole in the act of lifting or setting down. Sometimes the instrument is made with a small brass peg through the ebonite cake, its lower end being soldered to the sole and its upper lying flush with the top of the ebonite; this earths the disc, at the instant it is put on the cake, and saves time.

As regards the necessity of earthing the lower side of the cake, the explanation usually given is as follows, though really there is more in the matter than therein appears:—When the lower side of the cake is earthed the negative charge produced on the upper side by the fur induces a positive

charge on the sole (or on the part of the table underneath the cake), and the attraction of this on the negative charge causes the latter to penetrate the cake, thus rendering its electrification more thorough and permanent. If, on the other hand, the lower side be insulated, only inductive *displacement* can occur, there is practically no influence tending to produce penetration, and the instrument works but poorly.

Exp. 100. *Make a paper electrophorus.* (*a*) Group four tumblers (mouth upwards) on the table in order to support and insulate a metal tray (bottom upwards). Put a sheet of brown or note paper on the tray, pinch it to the tray between the forefinger and thumb of the left hand, and rub the paper with the palm of the right hand or a dry clothes-brush.

(*b*) Lift the paper from the tray. Present a knuckle to the latter : an electric spark is obtained.

(*c*) Replace the paper on the tray, taking care that the fingers do not touch the metal, then present the knuckle to the tray : a spark is obtained.

(*d*), etc. Repeat operation (*b*), then (*c*), several times, a spark is obtained on each occasion.

Note.—The tumblers and paper (use three or four sheets of foolscap, some to be warming while one is being used) must be well dried before a fire or in an oven. *To lift the paper* bring together two opposite corners, and pinch between finger and thumb. The paper may be *re-electrified* by rubbing with the hand as in (*a*). (When this fails substitute a freshly warmed piece and rub it by the hand.)

Explanation.—The friction with the hand and brush electrifies the paper negatively. *The tray* being earthed by the finger acquires a + charge. The spark in (*b*) is due to this + charge. The upper surface of the tray gets a + charge, the lower one a − charge. When the tray is earthed the − charge goes to earth. This gives the spark in (*c*). The + charge is left behind. When the paper is lifted away this + charge becomes free and then gives the spark in (*b*) and (*d*). In this experiment the cake (electrified paper) is lifted from the cover (tray).

Exp. 101. Use an electrophorus to charge an insulated conductor.

134. Influence Machines. In machines of the type described in § 128 the charge is generated *entirely by friction*, while in the electrophorus (§ 133) an *initial* frictional charge is made to give an unlimited number of small *induced* charges. In the more modern Influence or Induction Machines a *small* initial friction charge is made to yield a *very large* induced charge, and, for strong electrostatic effects, these are the best machines hitherto devised. The earliest of them was Lord Kelvin's *Replenisher*, and this was succeeded in turn by the *Holtz*, the *Voss*, and the *Wimshurst* Machines.

The following simple experiment will introduce the principle upon which all these machines work :—Take two tin pots, A and B (Fig. 107), on insulating stands, place them some distance apart, and give one of them, A, a *very small* charge which, we will suppose, is negative ; the smallness of the charge should be checked by touching the outside of the pot with an

insulated brass ball and carrying the latter to a neutral electroscope, when it should produce a very slight divergence. Then take a brass ball, mounted on an ebonite handle, and perform the following series of operations:—

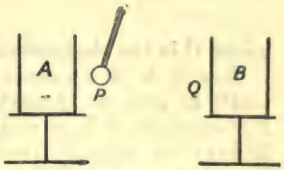

Fig. 107.

(i) Hold the ball near A, say in the position P, and momentarily earth it by touching it with the finger; it thus acquires a very small induced positive charge.

(ii) Insert the ball in the pot B, making it touch the bottom; its charge is thus (§ 109) completely emptied into B, so that *B acquires a very small positive charge.*

(iii) Remove the ball (which, of course, is now destitute of charge) from the pot and hold it near the outside in some such position as Q, again touching it with the finger; it thus acquires a very small induced negative charge.

(iv) Empty this small negative charge into the pot A, as in (ii); it thus becomes added to the charge already there, and *A acquires a greater negative charge* than at first.

(v) Repeat operations (i) and (ii); since A has a greater negative charge, the positive charge induced in the ball in operation (i) will be greater, and in operation (ii) this greater charge will be added to the charge already in B.

(vi) Repeat operations (iii) and (iv); this reproduces their previous effects on an augmented scale.

Then repeat operation (i) again, and so on until all the operations have been repeated several dozen times; at each emptying a larger and larger charge of one kind is acquired by A and of the opposite kind by B, until finally, if an insulated ball be made to touch the outside of either of them and then carried to an electroscope, a great divergence of the leaves will be produced; sometimes even as the operations are continued the pots crackle and discharge sparks.

The student will see that the way in which the large charges grow out of the initial small one is similar to the way money accumulates at compound interest; the principle is, in fact, often referred to as the "**compound interest principle.**"

135. Condensers. It frequently happens in electrical experiments that we require a conductor to have a very large charge. Now suppose we are furnished with a good electrical machine, the limiting potential (§ 131) of whose prime conductor is V, and with a conductor A; by connecting A to the prime conductor of the machine it can be made to acquire a potential equal to V, but no greater; it will then have a certain charge, Q, which

will be the greatest possible under the given circumstances, the amount of this charge being given (§ 120) by the equation

$$Q = CV \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

where C is the electrostatic capacity of the conductor A.

Now if A be a simple conductor of manageable size, C is too small to give to Q anything like so great a value as required. For example, if it be a sphere of radius 10 centimetres (about 4 inches) its capacity (see § 154) is 10 electrostatic units; if, then, the machine will work up to a potential of 1000 electrostatic units, the greatest charge the ball could acquire would be 10,000 electrostatic units, a quantity which really is very small. To get a charge of say 1,000,000 electrostatic units (which is quite moderate) we should require a ball 1000 centimetres in radius (about 70 feet diameter).

If, then, we are to get a big charge on a conductor of reasonable size, we must have some means of increasing its electrostatic capacity, in other words, of *weakening the potential produced in it by a given charge*. To effect this we avail ourselves of the principle of § 119: we place the conductor A in the vicinity of an earthed conductor, B. Moreover, as we require a very decided weakening of the potential of A, we must bring all parts of B as near to A as possible. An instrument constructed on this principle is called a *condenser* or (electrostatic) *accumulator*.

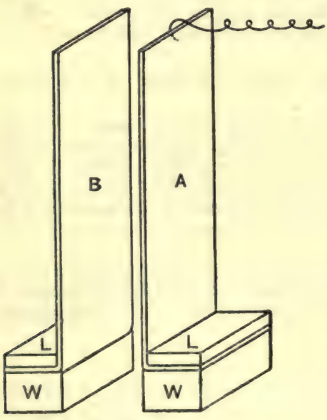

Fig. 108.

Exp. 102. *Make and use a simple condenser.* Cut out two pieces of tinplate, A, B (Fig. 108), 1 foot square. Bend a piece on each 2 ins. wide at right angles to the remaining portion. Procure two blocks of paraffin wax, W, W, 1 ft. long, 1 in. deep, and not less than 2 ins. wide. Warm the bent portions of the tinplate in the Bunsen flame and then stick them to the wax blocks. When cold stand the blocks up and place blocks of lead, L, L, on each block to keep the large portions of the tinplate upright.

Stand A and B on the bench about 2 inches apart (Fig. 109). Attach a wire to the prime conductor P of an electrical machine and let it rest on A. Work the machine. Set up a neutral electroscope some distance off, attach a wire to the binding screw t (Fig. 75), and supporting the

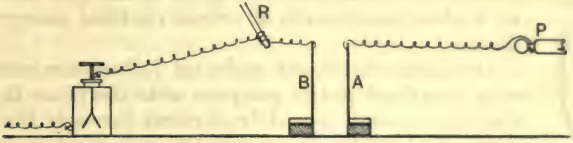

Fig. 109.

other end by an ebonite or sealing-wax rod, R, test the potentials of B and A as the machine is worked. Both potentials are rising, but B is always less than A. The potential of A is free, that of B is induced.

When the divergence caused by the potential of A is as large as convenient, remove the wire from A to P and connect A permanently to the electroscope by slipping the wire from t under the block of lead on the base of A. Earth B. Note the partial collapse of the leaves. Why is this? Evidently to keep the potential of A up to its former mark a larger charge is necessary. If the fall of potential of A was very large it may now be necessary to give A a further charge from the machine. Keeping B earthed, move it first nearer to A, then away from A. Observe what happens to the electroscope. When B approaches A, the potential of A falls. When B retreats from A the potential of A rises.

We have defined the capacity of a conductor as the ratio of its charge to the potential produced by the charge. Evidently the capacity of A depends very much on the proximity of B. When B is near A the capacity of A is large. When B is far away it is small.

The combination of an insulated plate and an earthed plate is called a condenser. When as in the case above the plates are parallel the arrangement is called a parallel plate condenser or an **Æpinus condenser**, from the name of the inventor.

Exp. 103. Place B as close to A as possible. Charge A to the limiting potential of the machine. Take a wire connected to earth, hold it close to A and observe the spark. Repeat the experiment, B being now far away from A. The spark is by no means so intense. In each case A was at the same potential; from the formula $Q = CV$ it follows that the charge on A is proportional to the capacity of A. The experiment therefore shows that the capacity of A is greater when B is nearer than when B is far away.

In the foregoing method of discharging A we have used a metal wire earthed to the gas-pipe; this ensures good earth contact, the discharge is sudden, and the spark instantaneous. If, instead of this, we make the earth contact through the woodwork of the table, etc., it is less efficient, and the spark is more of the nature of a slow fizz, though of course its total energy is the same.

Fig. 110.

It is clear that it can make no difference whether the wire is earthed to the gas-pipe or to the plate B, for in the latter case it would be earthed through B's earth-wire. Indeed, the more usual way of discharging any condenser is to make direct contact between the two plates by means of an instrument (Fig. 110) called a *discharging tongs*, which is simply a pair of brass rods terminating in brass knobs and working stiffly on a pivot, the whole being mounted on an ebonite handle: the pivot-joint permits the knobs being placed at a convenient distance for reaching the plates of the condenser.

To discharge the Æpinus condenser by the tongs, we should hold the latter by the insulating handle, place one knob on B, and bring the other gradually towards A, when a spark would pass between A and the knob, and the charge would run from A to earth *viâ* the tongs, the plate B, and the wire Y. Or we may equally well *first touch the plate* A with one knob of the tongs, and then bring the other gradually towards B; the same result ensues, the only difference being that the spark now passes between B and the tongs.

If the tongs have *no insulating handle*, then, *provided we first touch the earth plate* B, it makes not the least difference: the charge will spark across from A and run through the tongs to earth, practically none travelling *viâ* the body and floor, because these offer far greater resistance to the flow than do the tongs; but if we first touch the insulated plate A then A will discharge itself through the body and floor before there is time to bring the other knob up to B.

136. Condition of the Earth-plate of a Condenser. Of course the earth-plate of a condenser is at zero *potential*. But it has an induced *charge* (§ 102) of the opposite kind to that imparted to the insulated plate; thus in Fig. 109, if A be charged positively B becomes charged negatively by induction. The amount of this

induced charge is in general less than that on the insulated plate, though they become more nearly equal the nearer the plates are together; and hence the view sometimes set forth that when the two plates are connected by the discharging tongs the two opposite charges neutralise one another can scarcely be regarded as correct. Indeed, in the Æpinus condenser, if we put the plates say 2 inches apart, charge A positively, and after insulating B connect A and B by the insulated discharging tongs, and afterwards take them away and examine them separately, we shall find each to have a positive charge.

It seems scarcely needful to point out that in charging one plate, A, of a condenser, the other, B, must be kept earthed *while the process is going on*, for otherwise (§ 119) its presence does not materially affect the potential of A. But *after* charging we may if we please insulate B, and then, provided we do not alter the relative position, etc., of the plates, the whole condenser remains in just the same state as while earthed.

137. The Capacity of a Condenser. For the sake of brevity it is usual to speak of the capacity of the plate A of a condenser when in presence of the earth-plate B as the *capacity of the condenser*.

In like manner when we charge the plate A we are generally said to "charge the condenser." But it should be noted that since, when a charge is imparted to A, a more or less nearly equal charge of opposite kind is induced on B, the *condenser as a whole* has very little charge; the instrument enables us, so to speak, to charge one plate strongly at the expense of the other; but we get *a big charge on one plate*, and that is what we want.

When B *completely* surrounds A, we have what may be termed an *enclosed condenser*; a metal ball immersed in a deep (earthed) tin pot is practically such. In that case, by the principle of § 111, the charge induced on the outer conductor is *exactly equal* (and of opposite kind) to that on the inner, so that an enclosed condenser taken as a whole has absolutely no charge, and if, after charging, the previously earthed coat be insulated and the two coats connected by the tongs *each becomes perfectly neutral*.

Exp. 104. *To show that the capacity of a condenser is increased when the air between the plates is wholly or partially replaced by a medium of higher specific inductivity than air.*

Take the condenser of Exp. 102, earth B, connect A to a gold-leaf electroscope and charge A until the leaves diverge widely. Now insert a sheet D of ebonite, paraffin wax or glass between A and B (Fig. 111). Note what

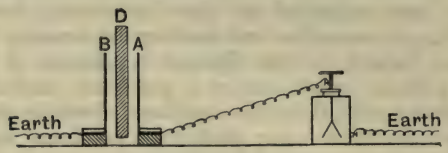

Fig. 111.

happens to the leaves of the electroscope. A partial collapse indicates that the potential of A is lowered. Since $Q = CV$ and Q is unaltered it follows that if the potential of A falls the capacity of A must have been increased.

The effect of placing a sheet of dielectric between the plates of a condenser is to increase the capacity of the condenser. It was by measuring this increase of capacity that Cavendish and Faraday first measured the specific inductivity of dielectrics.

138. Practical Forms of Condenser. The Æpinus condenser, and indeed any condenser in which the dielectric between the two plates is *air*, is not of much service for collecting a really big charge, for the following reason. In order that *any condenser* may do this we require its electrostatic capacity to be very big, for which purpose the plates must be pretty close together. Now, if this be the case and we raise the insulated plate A (Fig. 111) to a strong potential, then, since the potential of the earthed plate is zero, there is a rapid weakening of potential, or in other words a steep potential-gradient in the part of the field between the two plates, and the tension (§ 114) on the dielectric becomes great; unless then its mechanical rigidity be sufficient to enable it to resist this tension it gives way, a spark passes between the plates, and the condenser in great measure discharges itself.

The mechanical rigidity of air is very small, so that with an air condenser we must either keep the plates a considerable distance apart, in which case the electrostatic capacity is too small, or we must be content with a weak potential in the insulated plate: in either case the charge acquired by the latter is below our requirements.

The difficulty is remedied by employing a *solid dielectric* between the plates, which has also a secondary advantage, because its specific inductivity is greater than that of air, so that, other things being the same, the capacity is increased. A simple condenser of this kind is the so-called *fulminating pane* (Fig. 112) invented by Franklin; it consists of two sheets of tinfoil pasted one on each side of a glass or preferably ebonite plate. The tinfoil coats play the part of the plates A and B of the Æpinus condenser, while the ebonite takes the place of the intervening air.

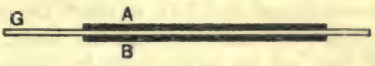

Fig. 112.

But by far the most convenient form of condenser is the Leyden jar. Its ordinary form is shown in Fig. 113. It consists of a wide-mouthed glass jar the sides and bottom of which are coated both inside and outside to within a short distance of the neck with tinfoil. The mouth is fitted with a bung of "baked" wood, which is a fairly good insulator; and through this passes a brass rod, which touches directly, or through a chain, the inner tinfoil coat at the bottom of the jar, and on the top of which is fixed a brass knob. When the jar stands on the table its outer coat is earthed *viâ* the wood, etc., and corresponds to the plate B, Fig. 111, the inner coat corresponding to the plate A, while the glass constitutes the intervening dielectric D.

Fig. 113.

It is very essential that the glass employed in the manufacture of these jars should be *good insulating glass*, otherwise they are practically useless. Also an india-rubber bung is preferable to a wooden one.

139. Study of the Leyden Jar. We have employed the Æpinus condenser to illustrate some of the points regarding condensers generally; others can be more conveniently studied by means of the Leyden jar.

The ordinary method of charging the jar is to stand it on the table and place the knob in contact with the prime conductor of a machine, either directly or by means of a metal wire. The inner coat then acquires the potential of the machine, and since

the electrostatic capacity of the jar is large the charge acquired is large. To discharge the jar we connect the outer coat and knob by the discharging tongs, when a bright spark passes with a sharp click. If the tongs are not insulated care must be taken, as explained in § 135, to *touch the earthed coat first*, otherwise the charge will pass through the body, producing a "shock" which, if the jar be large, may be very dangerous. To get the full advantage (or disadvantage) of the shock, the outer coat should first be grasped in one hand, and the knob then touched with the other; the whole energy of the discharge is then concentrated on the body.

It should be noted, by the way, that the severity of a shock received from any conductor will depend upon the electrical energy stored in the conductor—that is (§ 121) jointly upon its charge and potential (cf. § 132).

It should be observed that on account of the closeness of the coats of a Leyden jar, and the fact that one completely surrounds the other (except for the protruding rod and knob), the (negative) charge induced on the outer is very nearly indeed equal to the (positive) charge imparted to the inner. In fact, a Leyden jar is to all intents and purposes an "enclosed condenser" (§ 137). If, then, the jar be charged, and then placed on an insulating stool and its coats connected by insulated discharging tongs, both coats will become practically neutral, so that the discharging is as effective as if the outer coat had remained earthed.

140. How to Charge a Leyden Jar Negatively by means of a Machine yielding a Positive Charge. A Leyden jar is commonly said to be charged positively or negatively according to the character of the charge on its *inner* coat. Now, if our machine yields a positive charge, as most machines do, and we charge the jar in the ordinary way by standing it on the table and placing the knob against the prime conductor, the jar of course becomes "positively charged." Suppose we wish, by means of the same machine, to "charge it negatively," we proceed thus:—Set the jar on an insulating stool, earth the knob, join the prime conductor of the machine to the outer coat by a metal wire, and work the handle: the outer coat receives a positive and the inner a corresponding induced negative charge, and the desired result is achieved. If the knob be now un-earthed and the machine removed, the jar can be lifted by its outer coat and set on the table. Touching the outer coat in the act of lifting causes a minute positive charge to flow to earth, and renders the potential of the inner coat negative, so that after the jar is placed on the table its condition is in all respects the same as if it had been charged in the ordinary way from a machine yielding a *negative* charge.

SUMMARY.—CHAPTER IX.

1. Description of Plate Machine (§§ 128, 130). A given machine can produce a certain definite *limiting potential* (§ 131).

2. Roughly the *length* of the spark obtained from a machine or any conductor or condenser indicates its *potential*, while the *brightness and noise* of the spark indicate its *energy* (§ 132).

3. The *Electrophorus* and theory of its action (§ 133).

4. Principle of the action of *Influence Machines* (§ 134).

5. General principle of condensers (§ 135), viz. *to increase the capacity of a conductor by putting an earth plate near it.*

6. The earth plate of a condenser is at potential *zero*, but it has a *charge* of opposite sign to that on the insulated plate. If the condenser be an *enclosed* one the charge on the earth plate is *equal* (and opposite) to that on the insulated plate, otherwise it is *less* (§ 136).

7. Leyden jar (§ 138) and modes of charging and discharging it (§§ 139, 140). A Leyden jar is practically an enclosed condenser (§ 137).

EXERCISES IX.

1. In an experiment with an Æpinus condenser the plate B is first put a long way off, A is worked up to the limiting potential of the machine, and is then removed and held inside a large tin pot standing on the cap of a gold-leaf electroscope. The experiment is then repeated (with the same pot and electroscope), the plate B standing fairly close to A. Will there be any difference in the divergence of the leaves in the two cases, and, if so, in which will it be the greater? Give reasons for your answer.

2. In an experiment with the Æpinus condenser the machine is not used but the plate A is connected with the cap of an electroscope. To begin with, B is a long way off, a charge is given to A, and the leaves diverge. What difference, if any, will be observed in the divergence as B is gradually brought nearer A? Give reasons.

3. In an Æpinus condenser the plate B is not earthed, but is connected by a copper wire to a large insulated metal ball. Will this enable us to charge A more effectively than if the ball were absent?

4. The plate B of an Æpinus condenser is earthed and also connected with the cap of a gold-leaf electroscope. The plate A is then positively charged and B afterwards insulated. Supposing there to be no leakage or other disturbing influence, how will the leaves behave?

The plate A is now moved a little farther away: how will the leaves behave? Lastly, an ebonite rod rubbed with fur is gradually brought over the cap of the electroscope: what further will happen? Give reasons for your answers.

5. An electrical machine stands on a glass table, and you have no means of earthing it. What would you do to charge a Leyden jar by this machine?

6. The inner coating of a Leyden jar is connected by a wire with the prime conductor of an electrical machine and also with a gold-leaf electroscope. If the jar rests upon a sheet of glass, a quarter of a turn of the machine produces a large divergence of the leaves of the electroscope. If the glass be removed ten turns of the handle are required to produce the same deflection. Explain this.

7. Describe some apparatus by which an indefinitely large quantity of electricity may be obtained by means of electrostatic induction from a minute initial charge.

8. How do you explain the fact that in the ordinary process of charging a Leyden jar it is necessary to connect its outer coating to earth?

9. A sheet of tinfoil is suspended by a dry silk thread and charged as highly as possible by an electrical machine, but on discharging it only a slight spark is obtained. If the tinfoil is placed on a sheet of dry glass lying on the table and charged by the same machine a bright spark can be obtained. Explain the cause of the difference.

10. If a Leyden jar, A, stand on an insulating stool, and its knob be connected to the prime conductor of an electrical machine, it cannot be charged. But if, standing on the stool, its outer coat be connected to the knob of another jar, B, whose outer coat is earthed, then on working the machine both A and B become charged. Explain this.

11. Two Leyden jars of different sizes have the outer coatings earthed, and *equal charges* are given to the inner ones. Their knobs are now connected by discharging tongs, and a spark is observed to pass. Explain this.

12. An electrical machine is placed inside an insulated chamber lined with tinfoil. The rubber of the machine is connected with the tinfoil. What will be the effect upon an electroscope placed outside and connected with the chamber when the machine is in action? Explain your answer.

13. A and B are two condensers of the same form and dimensions, A having glass and B air for the insulator between the plates. If they were charged simultaneously from the prime conductor of an electrical machine,

what difference would be observed in the sparks obtained by discharging them separately after removal from the machine?

14. An electric machine in action and a Leyden jar are placed upon insulated supports and the positive terminal of the machine is connected to the inner coating of the jar. Describe fully what differences, if any, in the electric state of the jar are produced by each of the following operations respectively:—(1) The outer coating of the jar is connected to earth. (2) The negative terminal of the machine is connected to earth. (3) The outer coating of the jar and the negative terminal of the machine are connected together. (4) The outer coating of the jar and the negative terminal of the machine are each connected to earth.

CHAPTER X.

ELEMENTARY MATHEMATICAL THEORY.

141. In this chapter we shall deal with some of the mathematical aspects of electrostatics. The law of electric force between charged bodies will be stated and proved, and a definition will be given of the intensity of an electric field. Faraday's important conception of lines of electric force will be described, and maps will be given of various electric fields illustrating the great use of these lines of force. Potential will be defined mathematically and its values in some important cases given. The conception of equipotential surfaces will also be described and their positions and use indicated in certain fields. Finally we shall come to the mathematical definition of electrostatic capacity and shall show how the capacity may be evaluated in the simple case of an isolated sphere.

142. Point-charge. Law of Force. Electrostatic Unit of Quantity. An electrical charge concentrated at a mere point is called a *point-charge*. The conception is of frequent service, though it is impossible in practice to realise it; an approximation is afforded by a very small charged sphere, and moreover the charge on *any* body or system may be regarded as an *assemblage* of an indefinitely great number of point-charges.

Consider now two point-charges, say both positive. The force of repulsion which either exerts on the other acts along the line joining them, and its magnitude clearly depends in some way upon the charges themselves and their distance apart. It also depends on the nature of the enveloping dielectric.

We will first suppose the dielectric to be air. The exact law then is that *the force is proportional to the product of the charges, and inversely proportional to the square of the distance between them*; or, in algebraical form:—

$$\mathrm{F} \propto \frac{q\,q'}{r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

where q, q' are the charges, r the distance between them, and F the force.

The *proof* of this law rests upon two items of evidence which will be given in § 145. The first is direct, but only approximate, and is furnished by the experimental researches of Coulomb, who, by means of his "torsion balance," proved it to be very nearly true for two small charged spheres. The second is indirect but rigorous, and consists in deducing the law as a mathematical consequence of the fact that the potential inside a freely charged hollow conductor is uniform (§ 110).

The relation (1) is a mere *proportion*. It may, however, be written in the form of an *equation*, viz.:—

$$F = c\frac{q\,q'}{r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

when c is a "constant," *i.e.* a quantity depending *only on the units of measurement*, not on the actual charges, distance, etc. Let us then agree to estimate r in centimetres and F in dynes, and let us take as our unit a charge *of such magnitude that when concentrated at a point and placed one centimetre from another equal point-charge (both being entirely surrounded by air), it exerts upon it a force of one dyne:* this unit is called the *Electrostatic Unit* of Charge or Quantity. If, then, we put $r = 1$, *i.e.* 1 centimetre, and q and q' each $= 1$, *i.e.* 1 electrostatic unit, we must, in accordance with this definition, have $F = 1$, *i.e.* 1 dyne. Hence from (2) we have

$$1 = c.\frac{1.1}{1^2}, \text{ i.e. } c = 1;$$

whence (2) takes the simple form

$$\mathbf{F} = \frac{\mathbf{q}\,\mathbf{q}'}{\mathbf{r}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

which is exactly similar to the corresponding formula in magnetism (cf. § 36).

If the charges are immersed in some dielectric other than air, the *proportional* law (1) remains unchanged, but the *actual force* becomes divided by K, the specific inductivity of the dielectric, so that in place of (3) we have:

$$\mathbf{F} = \frac{1}{\mathbf{K}}\frac{\mathbf{q}\,\mathbf{q}'}{\mathbf{r}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

It is comparatively seldom that (4) is required, and whenever not otherwise stated, the dielectric is understood to be air. If in

(3) or (4) q and q' be of the same sign, *i.e.* the charges of like kind, the expression for F is *positive*, this indicates *repulsion*; if they are of opposite sign, *i.e.* the charges of unlike kind, the expression is *negative*, which indicates *attraction*.

EXAMPLES—1. In one case two point-charges of 2 and 3 units respectively are placed 4 inches apart, and in another case two of 5 and 6 units are placed 9 inches apart; compare the forces.

Since this is a mere question of proportion, and not of absolute measurement, it does not matter what the several units are—those of charge may be quite arbitrary, and inches will do as well as centimetres. We merely employ equation (1), thus:

$$\frac{\text{Force in 1st case}}{\text{Force in 2nd case}} = \frac{\frac{2 \times 3}{4^2}}{\frac{5 \times 6}{9^2}} = \frac{81}{80}.$$

2. Two small spheres, having charges of 15 and 40 electrostatic units, are placed 10 centimetres apart. Find the force between them.

[N.B.—In all questions of this kind the "small spheres" are to be treated as points.]

This requires equation (3), which gives

$$\text{Force} = \frac{15 \times 40}{10^2} = 6 \text{ dynes.}$$

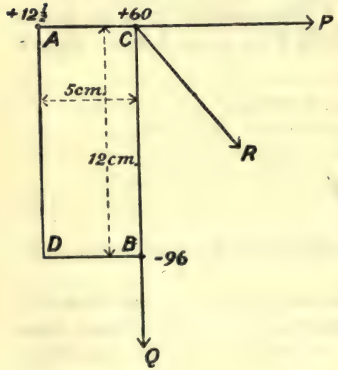

Fig. 114.

3. A C B D (Fig. 114) is a rectangle whose sides are 5 and 12 centimetres respectively. At C is placed a positive charge of 60, at A a positive charge of $12\frac{1}{2}$, and at B a negative charge of 96 electrostatic units. Find the resultant force on C.

We must first find the forces exerted on C by A and B separately, and then combine them to find the resultant (see Appendix, § 245). Now A exerts a force of repulsion, P, whose magnitude by 3 is

$$\frac{12\frac{1}{2} \times 60}{5^2}, \text{ i.e. } 30 \text{ dynes.}$$

Also B exerts a force of attraction, Q, of magnitude

$$\frac{96 \times 60}{12^2}, \text{ i.e. } 40 \text{ dynes.}$$

These two forces, P and Q, act at right angles.

Hence, if R denote the required resultant, we have

$$R^2 = (30)^2 + (40)^2 = 2500;$$

$$\therefore R = 50 \text{ dynes.}$$

The *direction* of R is approximately as in the diagram; the student familiar with the parallelogram of forces can calculate it *exactly*.

143. Force between two Electrified Spheres, or between an Electrified Sphere and a Point-charge. True and Nominal Force. Consider any two charged spherical conductors, A and B (Fig. 115), external to one another. Then it can be proved, as a mathematical consequence of the law of § 142, *that if the*

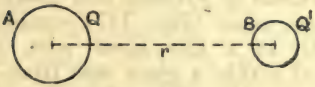

Fig. 115.

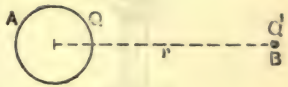

Fig. 116.

charge on each were uniformly distributed the force between them would be the same, both in magnitude and direction, as if their entire charges were concentrated at their respective centres; denoting then their charges by Q, Q', and the distance between their centres by *r*, the force F would be given by

$$F = \frac{Q\,Q'}{r^2} \quad \ldots\ldots\ldots \quad (1),$$

and the same would be true in the simple case where one of the spheres, say B, is replaced by a point (Fig. 116).

It must, however, be remembered that in Fig. 115 each sphere by its inductive action *disturbs the uniformity of distribution on the other*, and in Fig. 116 the point-charge Q' similarly disturbs the uniformity on the sphere, the consequence being that in *neither* case is (1) the *true* value of the force.

It appears from the above that in every case where two electrified bodies or systems act on one another, it is necessary to distinguish between the *true or actual* force between them and what may be called the *nominal* force, *i.e.* the force they *would* exert on one another if neither of them disturbed the distribution of electricity which would exist on the other in its absence; frequently, however, the distinction is overlooked, thus creating confusion.

It may be shown mathematically that *if the distance between two electrified spheres be great compared with their radii*, the true and nominal forces are practically equal, so that *each sphere may be regarded as a point-charge concentrated at its centre.* In all elementary problems this condition is tacitly supposed, and hereafter we shall make no difference between true and nominal force.

144. The Torsion Balance. The Torsion Balance was invented by Coulomb in the year 1785 for the purpose of comparing the forces of repulsion exerted between two similarly charged bodies under different conditions of distance and charge. The instrument (Fig. 117) consists essentially of a fine silver wire, *w*, suspended within a glass case from a brass head, B, and carrying at its lower extremity a light lever, *ab*, made of shellac or some light insulating substance. This lever moves in a horizontal plane, and is furnished at one extremity, *b*, with a gilt pith ball. The brass suspension head, B, fits into a short tube in which it can be easily rotated, and carries a pointer which moves over a brass disc D on the edge of which is engraved a circular scale divided into degrees. This arrangement for the suspension and control of the wire is called the *Torsion head.*

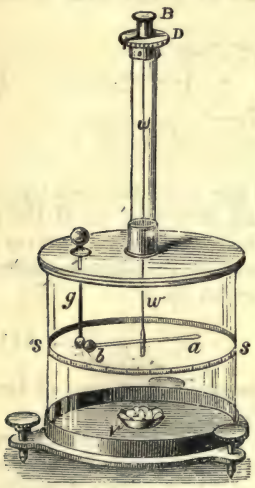

Fig. 117.

On that portion of the glass case which encloses the lever a scale, *s s*, is etched around the circumference at the level of the horizontal axis of the lever; and through an aperture in the cover a second gilt pith ball, *c*, attached to the end of a glass rod, *g*, can be introduced in the position shown in the figure. To absorb moisture and thus improve the insulation of *g* and *a b* a small vessel, V, containing calcium chloride or pumice-stone soaked in strong sulphuric acid is placed in the bottom of the case.

The principle of the instrument consists in balancing the

force of repulsion between the two charged balls, b and c, against the torsion (*i.e.* the twist) of the wire. The torsion head is adjusted until the balls b and c are lightly in contact. The pointer of the torsion head should then be at the zero of its scale and the ball b should be opposite the zero of the scale ss. The ball c is now removed and charged; on being replaced it shares its charge with b and mutual repulsion occurs. As a result b is repelled until the moment* of the forces of torsion of the wire balances the moment of the force of repulsion between the balls about the axis of the wire.

As the instrument is used to make comparative measurements of electrical force, it is evidently essential that the law connecting the angle of torsion with the moment of the resisting forces of torsion should be known. The law is known from experiment, and it is found that the angle of torsion is proportional to the moment of the forces producing the torsion. Thus, for example, the couple necessary to twist the end of a given wire through 20° is twice as great as that necessary to twist it through 10°. Hence, if in one experiment the force of repulsion between the charged balls is sufficient to repel through 10°, and in a second experiment to repel through 20°, then the force of repulsion is twice as great in the second case as before.

We will first show how to use this instrument in order to compare the charges on the pith balls in two different experiments. To do this we must first eliminate the effect due to change of distance between the balls. This can be done by adjusting the torsion head so that the distance between the balls shall be the same in each experiment.

Thus in the example which has been given above the distance between the balls in the second experiment is twice as great as in the first experiment, but by turning the torsion head so as to bring b nearer to c, the ball b may be brought back to its position in the first experiment, *i.e.* 10° from c. Let it be necessary in order to effect this adjustment to turn the torsion head through 80°. The twist on the wire is now 80° due to the torsion head above and 10° due to the deflection of the lever below, that is 90° in all. In the first experiment the torsion head was untouched and the twist on the wire was due only to the 10° deflection of the lever below. Hence at the *same* distance apart

* For definition of moment of a force see Appendix, § 247.

the force of repulsion between the balls is 9 times as great in the second case as in the first—that is the charge on each ball is $\sqrt{9}$ or 3* times as great in the second experiment as in the first.

145. Proof of the Law of Inverse Squares.

I. The Torsion Balance Method. If, using the Torsion Balance, we keep the charges on the balls constant, it is easy to show that the force between the balls is inversely proportional to the square of the distance between them. In an experiment of Coulomb's he charged c, placed it in contact with b and observed a deflection of the lever of 36°. The torsion head was then turned to reduce this deflection to one-half, *i.e.* to 18°. To do this Coulomb found it was necessary to turn the torsion head through 126°.

Since the angles of deflection in the two cases are small the linear distances between the balls are proportional to these deflections. By halving the deflection the distance between the balls has been halved. But it will be observed that the total twist on the wire is now 126° + 18°, or 144°; which is four times the initial twist, *i.e.* the force of repulsion now is four times the original force of repulsion. So that by halving the distance the force of repulsion has been quadrupled, which is the inverse square law.

Coulomb verified the law for different distances, and also investigated the force of attraction when b and c were oppositely charged. The above method of calculation is only correct when the angular distances are small. If they are larger the calculation is more complicated, but the law $F = \frac{Q Q'}{r^2}$ can still be proved.

Owing to various defects the experimental results obtained with the Torsion Balance are rough. More accurate instruments now exist for proving the same theorems, but the Torsion Balance will always be of great historical interest.

* From the formula $F = \frac{Q Q'}{r^2}$, F is evidently proportional to $Q Q'$ when r is constant. Also if the balls b and c are equal in size their charges will be equal, therefore F is proportional to Q^2, *i.e.* $Q \propto \sqrt{F}$.

II. **The Concentric Spheres Method.** Assuming the inverse square law as true, Cavendish proved mathematically that there should be no electric charge in the interior of a charged conductor. He confirmed this experimentally in 1772, and in this way verified the truth of the law on which his calculations were based. In his experiment he supported a conducting sphere G within a large hollow conducting sphere H made in two halves, so that it could be opened out like a book. He then strongly charged H and made electric contact between G and H by an insulated wire. The wire was then withdrawn and H opened out and discharged. A double pith-ball electroscope was then brought up to G. No divergence resulted; hence G was neutral, *i.e.* it had received no charge from H. The experiment has since been repeated with great refinements by Maxwell: the same result was obtained.

It is also possible to prove by the inverse square law that the electric intensity inside a charged conductor is zero.

It should be remembered that we only know that the inverse square holds as far as we have tested it. It may not be true for very minute distances.

146. Force and Intensity in any Electric Field; Influence of Specific Inductivity on Intensity. Let there be any system of electrified bodies. The air-space around them constitutes an electric field, and if a point-charge, Q, be introduced at any point, P, of this field it will experience a certain force in a certain direction.

Now the whole system of electrified bodies to which the field is due may be regarded as made up of a number of point-charges more or less connected. Let q be any one of them, and r its distance from Q. The force exerted by q on Q is

$$\frac{qQ}{r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the force of the entire field on Q is the resultant of all the elements of which (1) is the type. Now since we are disregarding the disturbing action of Q on the system, *the value of q is independent of Q*, hence the force (1) is, for a given system of electrified bodies, proportional to Q.

From this it may be shown on mechanical principles that if F be the force of the entire field on Q, the line along which F acts

is independent of Q, and F itself is proportional to Q—*i.e.* the ratio

$$\frac{F}{Q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

is independent of Q; it depends only on the system of electrified bodies, the position of P, and the units of measurement. This ratio is called *the strength or intensity of the electric field at P.*

Since the intensity is independent of Q we may put $Q = +1$; (2) then tells us that *the intensity of a field at any point is the force which the field would exert on a positive electrostatic unit placed at that point:* this is the definition commonly given. It being agreed to estimate our forces in dynes, the intensity clearly represents so many *dynes per electrostatic unit of charge.*

Intensity thus being really a force belongs to the class of quantities known as *vectors*, that is, it possesses not only magnitude, *but also direction*, and can be resolved by the ordinary methods of Mechanics, the component of an intensity I, in a direction making an angle θ with its own, being $I \cos \theta$ (cf. § 48).

If F is the mechanical force on a charge, Q, situate at a point of a field where the intensity is I, it is obvious from (2) that

$$F = IQ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

If in (3) we make Q negative, F becomes negative, indicating that, although F acts along the same line as when Q is positive, it does so *in the opposite direction*; it is for this reason that in the last definition of intensity we imagine a *positive* electrostatic unit placed at the point; this specifies *the direction* as well as the magnitude of the intensity.

The simplest field is that due to a charged point or uniformly charged sphere; in that case we have by (1), § 143,

$$\text{I (at an external point)} = \frac{Q}{r^2} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where Q is the magnitude the point-charge or the charge on the sphere and r is the distance of the point considered from the point-charge or the centre of the sphere (r being greater than the radius of the sphere).

The intensity of the electric field *inside* a charged conductor, *e.g.* a charged sphere, is zero. This follows at once from the law of inverse squares and the fact that all the charge is on the outer surface. It also follows from the fact that there is no potential gradient within a conductor (cp. § 151).

When a point-charge, Q, is surrounded entirely by air, the intensity due to it at a distance r cm. is $\frac{Q}{r^2}$. If it be surrounded entirely by a medium of dielectric constant, K, the intensity at the same distance is $\frac{Q}{Kr^2}$; the effect, therefore, of replacing the air by another medium is to divide the former intensity by K. Now any electrified system may be regarded as made up of point-charges: hence we learn *if we have any electrified system and replace the air surrounding it by a medium of specific inductivity, K, the intensity at every point of the field becomes divided by K*, its direction remaining unaltered.

It should be noted that, in calculating the force on a charge, Q, situated at a point, P, of a field, neither Q itself nor any other charge existing at P is to be reckoned as one of the factors of the field, the reason being that a point-charge exerts no force *on itself* or on any other charge *superadded to the same point*; were this otherwise such equations as (1) and (2), § 143, would not hold, but an additional term would make its appearance. Failure to realise this sometimes leads a student into difficulties.

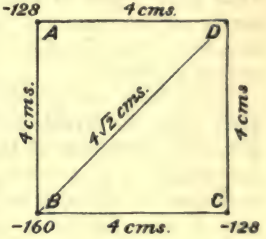

Fig. 118.

EXAMPLE.—A B C D (Fig. 118) is a square whose side is 4 cm., and negative charges of 128, 160, and 128 electrostatic units are placed at A, B, C, as shown. Find the intensity of the field at D and at B.

First consider D. We must imagine a + ve charge of 1 electrostatic unit placed at D, and then calculate the force on it due to the charges at A, B, and C: this will be the required intensity. Now the force due to A is $\frac{-128 \times 1}{4^2}$, *i.e.* -8; the negative sign merely indicates attraction (§ 142), the force thus being 8 dynes *along* D A. Similarly, the force due to C is 8 dynes along D C. The resultant of these is, by the parallelogram of forces (§ 245),

$$8\sqrt{2} \text{ dynes along D B} \quad \ldots\ldots\ldots \text{(i)}.$$

Again, the force due to B is $\frac{-160 \times 1}{(4\sqrt{2})^2}$,

$$\textit{i.e.}\ 5 \text{ dynes along B D} \quad \ldots\ldots\ldots \text{(ii)}.$$

Hence by (i) and (ii) the intensity at D is $8\sqrt{2} + 5$ "dynes per unit pole," and is directed along D B.

Next consider B. We must now imagine + 1 electrostatic unit placed there and calculate the force on it due to the charges at A and C, those at B, as above explained, counting for nothing ; proceeding as in the previous case we find the intensity at B to be $8\sqrt{2}$ dynes directed along B D.

147. I. Lines of Force. We saw, in Part I. Magnetism, that a magnetic field could be mapped out by lines of magnetic force, the positive direction of a line at any point being the direction in which an isolated N-pole would be urged.

Now consider a charged body surrounded by an unlimited insulating medium. If another similarly charged body be brought near the first, it will experience a force of repulsion. The whole space surrounding the charged body in which this electrical force is exerted is called the **field of force** due to the charge of that body. In any field of force the direction of the force at any point may be indicated by drawing a short line at that point; and in this way the field can be mapped out by a series of continuous lines and curves, each line indicating accurately the direction of the force at each of the points through which it passes.

These lines are called **lines of force, a line of force being a line such that its direction at any point through which it passes gives the direction of the electric force at that point.** If, then, we imagine a point-charge placed at any point P on one of them and free to move, it will travel along that particular one,* one way if the charge is positive and the opposite way if it is negative. To avoid confusion it is usual to understand by "the direction" of a line of force the one in which a *positive* point-charge tends to move; this is indicated in diagrams by an arrow, and sometimes, for emphasis, spoken of as the *positive direction.* Since (§ 92) a + ve charge always tends to move from a place of higher to one of lower potential, *the positive direction of a line of force is the direction of the down-gradient of potential along it*; we may thus conveniently speak of travelling "up" or "down" the line.

The simplest field of force is that due to an isolated point-charge or isolated sphere; the lines of force are all straight

* If the line of force is curved the motion of the charge should be suitably retarded if it becomes too fast, else the momentum of the particle will carry it off its own line, just as an express train will leave the rails at a curve unless the brakes are applied.

lines through the point or centre of the sphere respectively. The direction of the force for all points lying on any given line is therefore the same. Fig. 119 shows the lines for a positive point-charge and Fig. 120 for a negatively charged sphere.

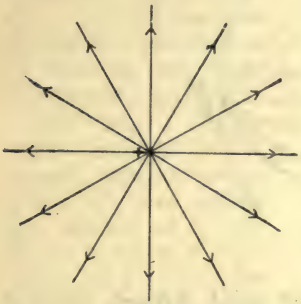

Fig. 119.

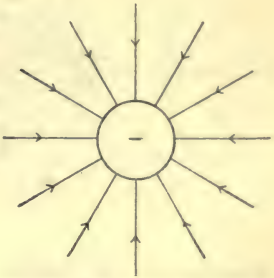

Fig. 120.

If, however, the field is due to two or more charged bodies the forces at a point due to the separate charges must be calculated and the resultant gives the direction of the line of force at the

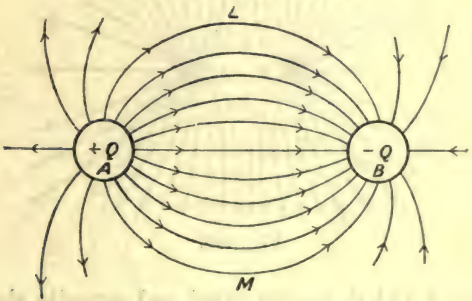

Fig. 121.

point. In general, the lines of force are not straight but curved, showing that the direction of the force varies from point to point along any given line, the direction of the line of force

being defined by the tangent to the line at that point. Hence a line of force may be defined generally as a curve, such that the

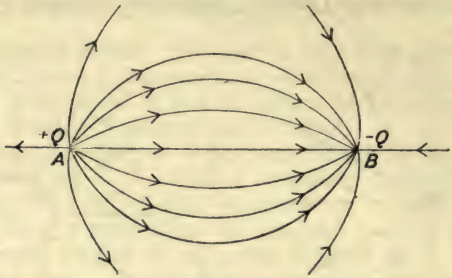

Fig. 122.

direction of the electric force at any point through which it passes is given by the tangent to the curve at that point.

Fig. 121 shows the lines of force due to two equal (conduct-

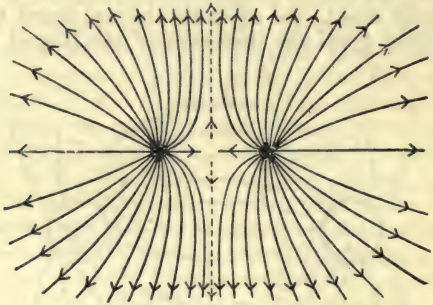

Fig. 123.

ing) spheres, A and B, having equal and opposite charges. In this *and in all cases* the following points should be noted:—

(i) There are no lines of force in the actual material of a conductor or in the interior of a hollow conductor with no electrified bodies inside; for in these regions the potential is uniform, and therefore the electric force is zero (§ 151).

(ii) In the external dielectric all the lines start from positively charged surfaces and run towards negatively charged ones. They stop short when they strike the surface of a conductor, *but nowhere else*; in the figures those broken for convenience of drawing are really continuous.

(iii) For a reason which need not be discussed here, wherever a line of force strikes the surface of a conductor *it does so at right angles* (cf. § 154). See Fig. 121.

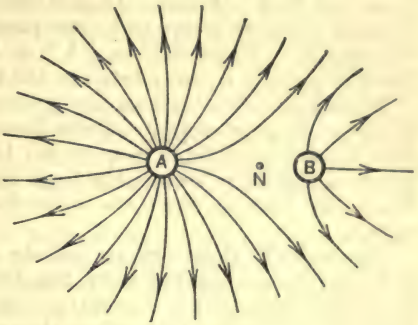

Fig. 124.

Fig. 122 shows the lines for two equal *unlike*, and Fig. 123 for two equal *like*, point-charges.

In none of these figures are the lines circular, moreover their

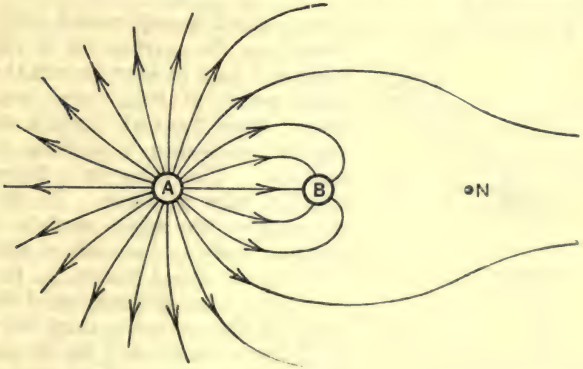

Fig. 125.

actual form depends on the distance of the charges apart, especially when, as in Fig. 121, they are not mere points, because then mutual inductive disturbance comes into play.

Fig. 124 shows the lines of force for two charges A of + 4 units and B of + 1 unit, and Fig. 125 for two charges A of + 4 units and B of − 1 unit. In each case N is a null point, *i.e.* at N the field intensity is zero and consequently there is no line of force through N. (Prove $BN = \frac{1}{2}AN$ in both cases.) In Fig. 123 the null point is halfway between the two point charges.

These figures illustrate another general theorem concerning lines of force: A *point-charge* radiates force in all directions, and the lines from it are infinite in number; but at any point of a field where no charge is located the force can have but one direction; hence *two lines of force can never intersect except at a point-charge.*

Theoretically there are an infinite number of lines of force radiating from a charged body, but by convention their number is greatly restricted; convention and the mathematical laws involved also decide that where the force is large the lines shall be drawn closely together and that where the force is small the lines shall be wider apart. Thus in Figs. 119-125 it is easily seen that in regions where the lines are closest the force is greatest.

II. Tubes of Force. Imagine a small closed curve A (Fig. 126) drawn on the surface of a charged body and lines of force drawn through every point of this curve out into space; these lines will form a tubular surface enclosing a portion of the medium. The tube thus formed is called a **tube of force**. If, further, we suppose the whole surface of the charged body to be marked out by a network of closed curves, A, B, C, D, etc., and a tube of force based on each mesh of the network, then the whole medium in which the field of force exists will be divided up into contiguous tubes of force. It is also evident that as the electric force decreases the tube widens.

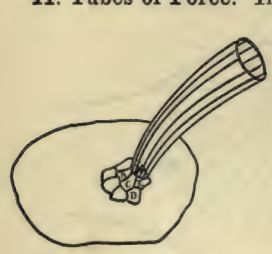

Fig. 126.

It is very important to notice that *these tubes are not endless.* They are assumed to start from the surface of the charged body, extend out into the surrounding medium, and the outer end of

each tube is found at the surface of contact of the medium, with some (conducting) substance. Thus, if a positively electrified sphere be hung up by a silk thread in an empty room, the tubes of force starting from the surface of the charged are terminated by the (conducting) walls of the room, and the medium between the body and the walls of the room may be mapped out into tubes of force. It will also be found that, where the insulating medium is in contact with the surface of a conductor, electrification exists. The charge or quantity of electricity at one end of a tube is *invariably equal and opposite to the charge at the other end.*

It is shown in the higher part of the subject that the medium is strained, and Faraday, to whom this conception of lines and tubes of force is due, showed that this strain is the same as would be produced by a tension (*i.e.* a pull) along the length of the tube and a pressure at right angles to its length. Faraday expressed this by saying that the tubes of force tended to contract, and also to repel each other—that is, the strain in the medium is such as would be produced by each tube tending to shorten and at the same time to get thicker. *The force of attraction or repulsion between charged bodies is due to this state of stress in the surrounding medium.* If it be remembered, in studying Figs. 121-125, that the tubes of force—that is, the portions of the medium between the lines of force—tend to contract and repel each other, it will be evident why attraction obtains in the case of Figs. 121, 122, and 125 and repulsion in the case of Figs. 123 and 124.

This conception of lines and tubes of force is very helpful in the study of electrification. It gives a fuller insight into the distribution of positive and negative charges in any particular case of electrification, and emphasises the fact that *the existence of a charge on any conductor implies the existence of a complementary equal and opposite charge on some other conductor or conductors* separated from the first by the insulating medium in which the field of force, due to the separation, exists. Thus, for example, when a charged body is placed in a closed pot it is obvious at once if we accept tubes of force that the charge on the interior of the pot is equal in magnitude to the charge on the enclosed body (cf. § 111); for each tube runs from the charged body to the pot, and the quantities of electricity at the two ends of each tube are equal and opposite.

148. Lines of Force in a Uniform Field. DEFINITION. A field occupied by a single dielectric, at every point of which the intensity is the same both in magnitude and direction, is said to be *uniform*. In such a field the lines of force are obviously all straight and parallel (Fig. 127).

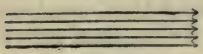

Fig. 127.

The most important uniform field is afforded by two equal and similar parallel metal plates, A B, A′ B′ (Fig. 128), directly facing one another and separated by a layer of air or other single dielectric; their distance apart must be small compared with their linear dimensions, and *they must have equal and opposite charges*. An Aepinus or parallel plate condenser affords an example of this. Except near the edges A A′, B B′, the field between them is then perfectly uniform, the lines of force being perpendicular to the faces. The widely sweeping lines which strike the outer faces belong to the comparatively weak external field where there is no pretence of uniformity.

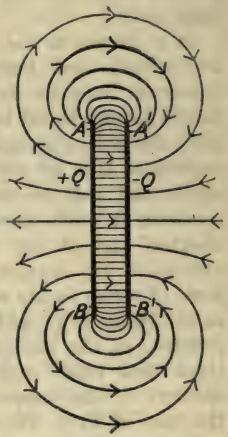

Fig. 128.

A student regarding the matter from the standpoint of the law of inverse squares (§ 142) is usually astonished to find that the intensity between the plates is the same at all points. But it must be remembered that the law of the inverse square applies only to point-charges and uniformly charged spheres *where the lines of force radiate from a centre*; in the present case where they are parallel it does not apply at all. It is just the same with light; the rays from an ordinary lamp diverge from it in all directions, and the intensity of illumination falls off rapidly as the distance increases, but when the rays are rendered parallel by means of a suitable lens or reflector (as in a search-light), the intensity is the same at all distances (except in so far as it is weakened by atmospheric absorption).

The term "uniform" is sometimes applied to a field in which the intensity at all points is the same in *magnitude only*; when emphasis is needed the kind of field represented in Fig. 127 may be termed a uniform straight field.

149. Simple instances of lines of force occurring in electrical experiments. Fig. 129 shows, diagrammatically, the relative position of the lines of force in the interior of a gold-leaf electroscope, where $a\,b$ is the metal rod supporting the leaves $s\,s$, and C is the earthed surrounding metal case. Note that the divergence of the leaves may be accounted for by the pulls in the tubes of force.

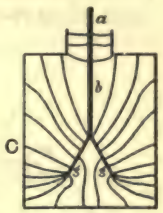

Fig. 129.

An interesting series of lines of force is obtained during the process of charging an electroscope by influence from a charged rod. Fig. 130 (*a*) shows a positive rod in position. There is a fall of potential from the rod to the gold-leaf system and from this system to the walls (earthed) of the electroscope. Lines of force therefore run from the rod to the cap and from the gold-leaf system to the inside of the walls of the electroscope. In (*b*) the cap is earthed. This reduces the potential of the gold leaves to that

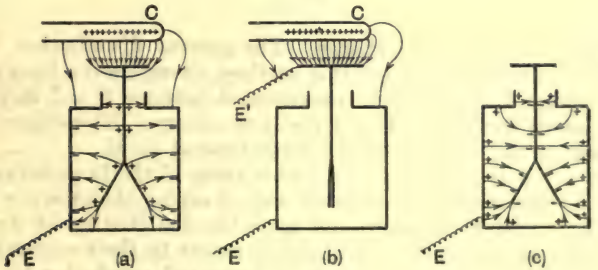

Fig. 130.

of the vessel, and there is no force within the electroscope and no lines of force. The wire to the cap is now removed, and this is followed by the removal of the charged rod. The negative charge on the cap flows to the leaves, and lines of force now run to the leaves from the inside of the electroscope (*c*).

Another interesting series is that obtained in the electrophorus. These are shown diagrammatically in Fig. 131. In (*a*) the upper surface of the ebonite B is —vely charged; lines of force

pass to this surface from the induced +ve on the sole S (which is earthed) and on other conductors. In (*b*) the brass plate B is brought up. B intercepts the lines above P but gives them off again from its under surface, *i.e.* the top of B has a −ve charge

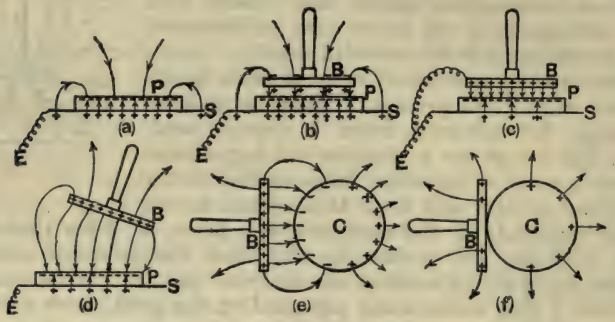

Fig. 131.

and the bottom a +ve charge. The general distribution of the lines is unaltered. In (*c*) B is earthed, therefore the lines above it vanish. Lines are now concentrated between P and B, giving the under surface of B a large +ve charge. These lines have grown at the expense of the lines from S to B.

In (*d*) B is gradually withdrawn from P, the lines between P and B being stretched—another way of saying that work is done. As B is withdrawn more and more the distribution of its lines becomes more regular and P and S revert to their original condition. In (*e*) B is held near an insulated conductor C. Lines fall from B on the near side of C and leave the far side. The lines between B and C tend to contract, *i.e.* B and C attract each other. In (*f*) B and C are in contact, so that no lines go from one to the other, and the charge originally on B is now distributed over B and C.

150. Mathematical Conception and Definition of Potential. Electrostatic Unit of Potential. In previous chapters we have seen that, regarded from the experimental side, potential presents itself as *electrical pressure*. But, from the mathematical

standpoint, another idea attaches to it *apparently* quite different, though really amounting to the same thing; thus—

Consider any system of electrified bodies, the field around them being occupied either by air, or partly or entirely by some other dielectric. Let P (Fig. 132) be any point in the field,

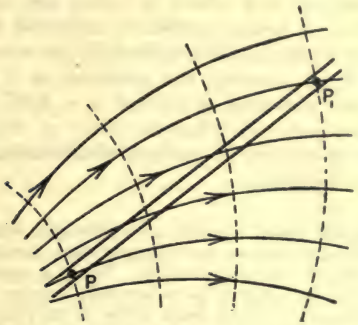

Fig. 132.

and let a positive electrostatic unit charge be placed at P. Then, neglecting the mechanical constraint of the dielectric, the field will urge it along the particular line of force that passes through P, but, as it is inexpedient to suppose its motion thus restricted, let us select *any* second point, P_1, not necessarily on that line of force, and imagine the unit charge constrained to move to P_1, say by being enclosed in a smooth narrow tube. In driving it from P to P_1 the force of the field will do work on it.

[If the field be such as to drive it the opposite way, *i.e. from* P_1 *to* P, this work must be reckoned as *negative*.]

Now, it can be proved on mechanical principles that the work so done depends only on the *positions* of P and P_1, and not in any way on the *path* by which the journey is made, *i.e.* upon the form of our supposed tube.

Now let P_1 be at an infinite distance from the electrical system. It is still true that *the work done by an electric field in driving an electrostatic unit from any assigned point P to infinity depends only on the position of P*, and not on the path travelled.

The mathematician now defines the potential as *equal* to this work; or, in formal terms, **the potential at any point of an electric field is the work which the force of the field would do in driving a positive electrostatic unit from that point to infinity.**

This definition tacitly involves a unit in terms of which potential is estimated, and which is called the electrostatic unit of potential. For, it being agreed to estimate work in ergs, if we choose a point P in the field such that the work mentioned in the definition is 1, *i.e.* one erg, the potential at P must also be 1. In other words, *the potential of a point in a field is said to be one electrostatic unit of potential when the force of the field would do one erg of work in driving a positive electrostatic unit of charge from that point to infinity.*

The student should guard against an error into which the foregoing definition is apt to lead. It seems to imply that electric potential is a form of energy. But this is not intended. What is meant is not that the potential *is* the work, but that it is *numerically represented by* the work. To put it more precisely, if the work done in moving one electrostatic unit of charge from the point P to infinity were *n ergs*, then we should say that the potential at the point P was *n electrostatic units of potential*, not *n ergs*.

For convenience of diction, the potential at a point, P, of a field due to an electrified body or system is usually called the potential "of the body" (or system) *at* P; where the body is a single conductor or a number of conductors in electrical communication, and P is in the actual material, this becomes the potential of the body in the ordinary sense.

151. Equality of Field-Intensity and Potential Gradient. If V_1, V_2 are the potentials of points P_1, P_2 in the electrical field the work required to bring a charge Q from infinity up to P_2 is QV_2 ergs, and the work required to bring a charge Q from infinity up to P_1 is QV_1 ergs. Hence the work required to bring a charge Q from P_2 to P_1 is

$$Q(V_1 - V_2) \text{ ergs.}^* \quad \ldots \ldots \quad (1)$$

* Note that this work is independent of the path taken in travelling from one point to the other.

Suppose now that P_1, P_2 are very near together and that I is the intensity of the electric field between P_1 and P_2 resolved along the direction $P_1 P_2$. Then by § 146 the force on the charge Q is I.Q dynes, and therefore the work required to bring the charge Q from P_2 to P_1 is also equal to

$$\text{I.Q. } P_1P_2 \text{ . ergs} \quad \ldots\ldots\ldots \quad (2)$$

Equating (1) and (2) we get

$$Q(V_1 - V_2) = \text{I.Q. } P_1P_2.$$

$$\therefore \text{I} = \frac{V_1 - V_2}{P_1P_2} \quad \ldots\ldots\ldots \quad (3)$$

If P_1, P_2 are on the same line of force, I is equal to the total intensity of the field at P_1 or P_2 and $\frac{V_1 - V_2}{P_1P_2}$ to the potential gradient between P_1 and P_2. Therefore (3) tells us that *the intensity of an electrical field at a given point is equal to the potential gradient at that point.*

Noteworthy instances are: (1) Within a charged conductor the potential gradient is zero, therefore the intensity there is also zero (cf. §§ 146, 147). (2) Between two equally and opposite charged parallel plates the intensity is uniform, therefore the potential gradient is also uniform.

EXAMPLE:—A small sphere containing 5 electrostatic units of positive electricity moves along a line of force in a uniform electric field for a distance of 5 cms. If the work done is 1,000 ergs, find the difference of potential between the points originally and finally occupied by the sphere, and also the intensity of the field.

The small sphere is to be regarded as a point. The work done is given directly by (1) of § 151 without reference to the distance traversed; we merely have to put work done = 1000 and Q = 5, whence we obtain

$$V_1 - V_2 = 200 \text{ electrostatic units.}$$

To find the intensity: Let F be the force on the sphere in dynes, then the work done in acting through 5 cms. is 5 F ergs; hence

$$1000 = 5\,\text{F}, \quad \therefore \text{F} = 200 \text{ dynes.}$$

Now let I be the required intensity, then, since the charge on the sphere is 5 units, we have by (3), § 146,

$$\text{F} = 5\,\text{I}, \quad i.e.\ 200 = 5\,\text{I};$$

whence I = 40 *dynes per electrostatic unit charge.*

152. General Expression for the Potential at any point of an Electric Field. Let us suppose the field occupied by air, and first let it be due to a single point-charge of q electrostatic units. If P is a point distant r centimetres from q, then the intensity of the electric field at P is $\frac{q}{r^2}$. Hence, by § 151, the potential-*gradient* at P is $\frac{q}{r^2}$. From this it follows by the Integral Calculus that the *potential* at P is

$$\frac{q}{r} \text{ electrostatic units of potential} \quad . \quad . \quad . \quad . \quad (1).$$

Next suppose the field due to any system of electrified bodies. Their total charge may be regarded as made up of a number of point-charges, and each will produce at P a potential of which (1) is the type. The potential of the whole system at P will be the sum of all these separate potentials, that is, employing the notation $\Sigma \frac{q}{r}$ to denote the sum of all the terms of which $\frac{q}{r}$ is the type*:—

$$\left.\begin{array}{l}\textbf{Potential of any system at any point of its}\\ \textbf{field, } \textit{the dielectric being air}\end{array}\right\} = \Sigma \frac{q}{r} \quad . \quad . \quad . \quad (2).$$

If the dielectric be not air, but a medium of specific inductivity, κ, the intensity at P (§ 146) will be $\frac{q}{\kappa r^2}$, the expression (1) will be replaced by $\frac{q}{\kappa r}$, and (2) by

$$\frac{1}{\kappa} \Sigma \frac{q}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

EXAMPLE :—Charges of + 10, − 20, + 30, and − 40 units of electricity are placed at the four corners of a square, the diagonal of which is 10 cms. Find the potential at the point of intersection of the diagonals.

The distance of the point of intersection from each corner is the same, viz. 5 cm. Hence by (2) the required potential is

$$\frac{10}{5} + \frac{-20}{5} + \frac{30}{5} + \frac{-40}{5},$$

i.e. − 4 electrostatic units of potential.

---

* This notation is common in Mathematics. The symbol Σ is read "sigma"; it is the Greek form of the letter S, and is intended as an abbreviation of the word "sum."

153. Potential of a Uniformly Charged Spherical Conductor, also the potential due to it at any point in its field. The determination of the potential at any point in the electric field due to a charged conductor is a mathematical process based upon the preceding article. The simplest and most important case is when the conductor is a sphere isolated in air or some other dielectric, in which case the charge on it is uniformly distributed. Let a be its radius (in centimetres) and Q its charge (in electrostatic units), then, when the dielectric is air,

$$\left.\begin{array}{c}\text{Potential at } \textit{any internal} \text{ point} \\ \text{or} \\ \text{Potential } \textit{of the sphere}\end{array}\right\} = \frac{Q}{a} \text{ electrostatic units} \quad . \quad (1),$$

$$\left.\begin{array}{c}\text{Potential at an } \textit{external} \text{ point} \\ \text{distant} \\ \text{r centimetres from the centre}\end{array}\right\} = \frac{Q}{r} \text{ electrostatic units} \quad . \quad (2),$$

while, when the dielectric is a medium of specific inductivity, K, these expressions must be respectively replaced by

$$\frac{Q}{Ka} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

and

$$\frac{Q}{Kr} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

With regard to all these formulae, it should be noted that it is only the *external* dielectric that counts; the internal, being unstrained, is quite immaterial. Similarly, it is immaterial whether the sphere be hollow or of solid metal. In all cases a is the radius of the surface on which the charge is distributed, so that for a thick hollow shell it will be the *external* radius.

EXAMPLES:—(1) A metal sphere of radius 5 cm. has a charge of 200 electrostatic units of quantity. Find its potential, and also the potential at a point, P, distant 25 cm. from its centre.

Nothing being said to the contrary, it is intended that the dielectric is air. Hence by (1) above

$$\text{Potential of sphere} = \frac{200}{5} = 40 \text{ electrostatic units of potential,}$$

while by (2) above

$$\text{Potential at P} = \frac{200}{25} = 8 \text{ electrostatic units of potential.}$$

(2) If the above sphere had been in an unlimited sea of petroleum (specific inductivity = 2) find the potential of the sphere and the potential at the point P.

By (3) the potential of sphere $= \frac{200}{2 \times 5} = 20$, and by (4) the potential at P $= \frac{200}{2 \times 25} = 4$ electrostatic units of potential.

154. Equipotential Surfaces. DEFINITION. An equipotential surface is a surface which passes through all points having the same potential.

The surface of a conductor is (by Poisson's principle) an equipotential surface. Suppose the potential of the conductor is V, where V is integral. Then if we decide to have an equipotential surface for each unit drop of potential there will be $(V - 1)$ equipotential surfaces between the conductor and the earth or earthed conductors.* It follows that if an earthed conductor is brought near a charged conductor the equipotential surfaces are brought nearer together.

Fig. 133 A.

If a sphere is isolated in space and charged, the potential due to it will fall off equally along all radial lines drawn from the centre of the sphere. Thus the equipotential surfaces are spherical (Fig. 133 A).

Fig. 133 A shows the equipotential surfaces of an isolated sphere of unit radius charged to a positive potential of 6 units. By Formula (1), § 153, the potential at the surface (and interior) of the sphere is 6. By Formula (2), § 153, the potentials at

* Just as if we decide to put contour lines around hills for every 100 feet rise there will be 9 contour lines between the base and the top of a hill 1,000 feet high.

distances 1·5, 2, 3, 6 from the centre are respectively 4, 3, 2, 1, as marked on the diagram. The equipotential surface of zero potential is at infinity. The lines of force due to the sphere are radial; and therefore cut the equipotential surfaces at right angles.

It is a general truth that lines of force cut equipotential surfaces at right angles, so that if we know the directions of the lines of force in any field we can sketch in the positions of the equipotential surfaces and *vice versa*. In Figs. 133 A and B the lines of force are indicated by dotted lines.

An analogy to equipotential surfaces and lines of force occurs in contour lines drawn around hills and lines of greatest slope drawn down the hillsides. A contour line (or rather the horizontal plane of which the contour line is the visible edge) is an equipotential surface of gravitational force. A line of greatest slope is such that at any point of it the line itself slopes more steeply than any other line drawn on the hill side through that point.

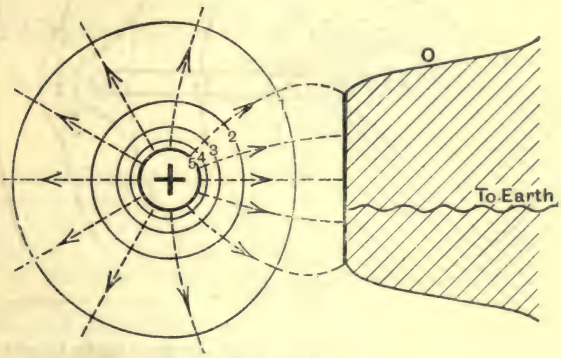

Fig 133 B.

The effect of bringing an insulated metal disc near the positively charged sphere of Fig. 133 A with the plane of the disc perpendicular to the radius of the sphere through the centre of the disc would be very small. The disc would take up the potential of the equipotential surface passing through the edge of the disc, so that this surface instead of curving over the disc would lie flat on the disc. Otherwise there is little alteration.

When the disc is earthed there is a large effect (Fig. 133 B). The potential of the sphere is lowered (§ 119) from 6 to 5, say,

and the equipotential surfaces between the sphere and the disc are flattened and the zero equipotential surface comes as a conical surface up to the edge of the disc. The shaded area in Fig. 133 B behind the disc represents a conical shaped region which is all at zero potential. This illustrates the effect of electric screening (§ 113). There are no lines of force behind the earthed disc.

Let us now draw the equipotential surfaces corresponding to the sphere and conductor of § 106 and Fig. 82. Before A B was brought up the surfaces were spherical. Let the potential

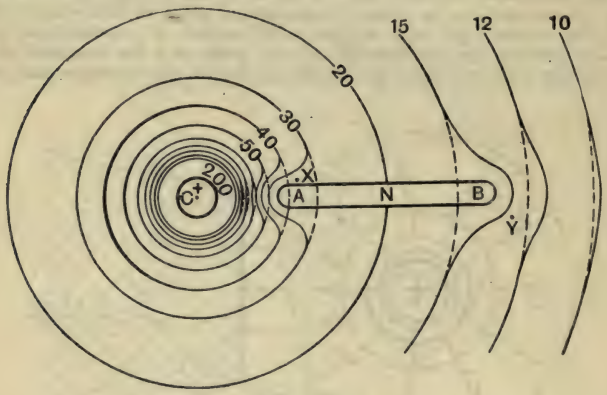

Fig. 134.

of the sphere C be 200 units and suppose we decide to place A B so that it takes up a potential of 20. Then the equipotential surfaces are bent as shown in Fig. 134 (the dotted lines near A B show the original positions of the surfaces). The 20-surface goes up to A B and expands over the whole conductor. The 30, 40, 50, etc., surfaces are pushed towards C and the 15, 12, 10 surfaces away from C. Consider a point X. Before A B was pushed into the field its potential was greater than 30. It is now less than 30. This state of affairs can only have been reached by the presence at A of a negative charge. Similarly the potential of Y, originally less than 13, is now greater than 14, thus indicating the presence of a positive charge at B.

When A B is earthed its potential is made zero and that of C is lowered. All the remaining equipotential surfaces are squeezed up between C and A. Then, arguing in the same way, it follows that the A end must have a much larger negative charge than before and that the positive charge has disappeared from the B end and left a negative one in its place.

155. Mathematical View of Capacity. Electrostatic Unit of Capacity. Capacity of Isolated Sphere. In § 120 we have seen that the capacity of a conductor means the ratio of its charge to its potential when there are no *independent* electrifications in its vicinity, and all neighbouring conductors are at zero potential; in the case of a condenser (§ 137) the same conditions are to be understood, the capacity "of the condenser" signifying the ratio of the charge on its "inner" coat to the potential of the same when its "outer" coat is earthed.

This definition leads to the algebraic relation

$$C = \frac{Q}{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

or

$$Q = CV \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

and tacitly implies a unit called the *electrostatic unit of capacity* in terms of which C is estimated. For, suppose the conductor (or condenser) to be such that when $Q = 1$ electrostatic unit of quantity, $V = 1$ electrostatic unit of potential, then (1) gives $C = 1$. The electrostatic unit of capacity thus defines itself as *the capacity of a conductor in which one electrostatic unit of charge produces one electrostatic unit of potential.*

If a sphere of radius a cm. isolated in air have a charge Q electrostatic unit of quantity, we have seen in § 153 that its potential V is given in electrostatic units of potential by

$$V = \frac{Q}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

If then C denote the capacity of the sphere in electrostatic units of capacity, we have by (1) and (3)

$$C = a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4);$$

i.e. *the capacity of a sphere (isolated in air) is, when expressed in electrostatic units, numerically equal to its radius expressed in*

centimetres. If in (4) we put $a = 1$, we get $C = 1$, i.e. *the electrostatic unit of capacity is the capacity of a sphere of one cm. radius*.

For brevity the first of these statements is often quoted as "the capacity of an isolated sphere is equal to its radius," but the phrase is open to objection as suggesting that a capacity and a length are identical, which is untrue; the two are of distinct nature, and we ought no more to speak of a capacity as so many *centimetres* than we can speak of a potential as so many *ergs*.

If the sphere be isolated, not in air, but in a medium of specific inductivity κ, its potential is (§ 152) $\frac{Q}{\kappa a}$, and its capacity is given by

$$C = \kappa a \quad . \quad . \quad .

9. General expresssion for potential, *viz.*—

$$\Sigma \frac{q}{r} \quad \text{or} \quad \Sigma \frac{q}{\kappa r} \text{ (§ 152).}$$

10. The several expressions relating to the potential of a uniformly charged spherical conductor (§ 153).

11. Equipotential surfaces and their form in certain cases (§ 154).

12. Electrostatic Unit of Capacity; Capacity of Isolated Sphere (§ 155).

EXERCISES X.

1. Two small charged spheres are placed two inches apart, and exert a certain mutual force. The charge on one of them is then doubled, and that on the other trebled. To what distance must they now be moved so that the force may be unaltered?

2. Two small spheres have charges of 25 and 64 electrostatic units respectively; at what distance must they be placed so as to exert upon one another a force of 100 dynes?

3. A B C D is a square whose side is 10 cm. Charges of 4, 6, 8, and 12 electrostatic units, all +ve, are placed at A, B, C, and D respectively. Find the resultant force on a charge of 20 units situate at the centre of the square.

4. Two small charged spheres are held a certain distance apart in air. How will the force between them be altered by immersing them in oil? If the ratio of the forces in the two cases be supposed known, show how to calculate the specific inductive capacity of the oil.

5. A sphere of radius 2 cm. has a charge of 100 electrostatic units, and a point-charge of 20 electrostatic units is placed 10 cm. from its centre. Calculate the nominal force in dynes between them.

6. A charged sphere acts on an external point-charge. Show that if the sign of the charge on either the sphere or the point (but not on both) be reversed, the nominal force is reversed in direction but unaltered in magnitude.

7. In the example (p. 203) find the intensity at the centre of the square, and also as A, (i) when the whole is in air, (ii) when the whole is in petroleum of specific inductivity 2.

8. Two spheres of radii 4 and 6 cm., and having charges of 200 and 1000 electrostatic units respectively, are placed with their centres 40 cm. apart. Find the intensity at a point midway between the centres (i) when both charges are positive, (ii) when both are negative, (iii) when the smaller is positive and the larger negative, (iv) when the smaller is negative and the larger positive, and distinguish carefully between the *directions* of the intensities in the several cases.

9. Point-charges of +ve 20 and −ve 20 electrostatic units are placed 5 cm. apart; find the force in dynes which each exerts on the other and indicate the direction in which it acts. [Remember that a point-charge exerts no force *on itself*.]

10. In the preceding question find the force which would be experienced by a point-charge of +ve 36 electrostatic units placed 3 cm. from the charge +ve 20 and 4 cm. from the charge −ve 20, and indicate in a diagram the direction in which it acts.

11. Sketch the lines of force for

(1) 3 equal charges at the corners of an equilateral triangle;
(2) 2 equal charges at the opposite corners of a square and 2 equal charges of the opposite sign at the other corners;
(3) a charged elongated conductor;
(4) a charged pointed conductor (Fig. 78);
(5) the arrangement of Fig. 134.

12. What is meant by saying that two points have unit (electrostatic) *difference* of potential?

13. The intensity of a certain uniform field is 50 dynes per electrostatic unit of charge, and a small sphere having a +ve charge of 3 electrostatic units of quantity starts from a point in it at which the potential is 1,400 electrostatic units of potential. How much work will the field do in driving it over 16 cm., and what will be the potential of the point then reached?

14. A uniform field has a gradient of 1 electrostatic unit of potential in 20 cm. Find how many ergs of work must be done against the field, in order to move a small sphere having a charge of 100 electrostatic units of quantity one metre up a line of force. Also distinguish between the cases where the charge on the sphere is (i) positive, (2) negative.

15. A B C D is a rectangle, A C and B D being the diagonals. A B is 6 and B C 8 cm. long. At A B C D are placed +ve charges of 64, 100, and 36 units respectively. Find the potential at D.

16. A and B are two point-charges of +16 and −9 units respectively 14 cm. apart. Find the position of that point in A B, or A B produced, at which (i) the intensity, (ii) the potential, is zero.

17. Find the radius of a sphere which, when isolated in oil of specific inductivity 2·5 and charged with 100 electrostatic units of charge, has a potential of 8 electrostatic units of potential.

18. A sphere of radius 4 cm. has a potential of 20 electrostatic units of potential. Find the potential at a point 12 cm. from its centre.

19. Sketch the equipotential surfaces for

(1) the arrangement of Figs. 120-125, 128;
(2) a charged gold-leaf electroscope;
(3) an electrophorus during the process of charging;

(4) a charged insulated hollow conductor;
(5) an uncharged metal tin when a small charged sphere is placed within it, also after the tin is earthed.

20. Two Leyden jars of capacities C_1, C_2 have respective charges Q_1, Q_2. They are placed a long way apart, their outer coatings earthed, and their knobs connected (by a fine insulated wire). Find (i) their common potential; (ii) the charge on each; (iii) the condition that the connection shall make no alteration in the charges.

21. A small metal sphere is hung by a silk thread from one pan of a balance and is counterpoised by weights placed in the other. A second insulated and electrified metal sphere of equal radius is made to touch it for a moment, and is then fixed at a distance of 10 cm. below the first. It is found as the result that the counterpoise has to be reduced by ·001 gramme to restore equilibrium. Find the charge on each sphere.

What precautions would it be necessary to take to perform this experiment successfully?

22. Two insulated spheres of radii 3 and 1 cm. respectively are placed a long way apart. A charge of 15 units is given to the larger sphere; what charge must be given to the smaller in order that neither may gain nor lose charge when the two are connected by an insulated wire?

23. Two circular metal discs of 50 cm. radius are placed parallel to one another 1 cm. apart, and are equally and oppositely charged. Draw a diagram showing fully the directions of the lines of force between them. If the difference of potential between the discs is 10 C.G.S. units, find the force that would be experienced by a small particle containing ·005 unit of electricity when placed midway between them.

24. An insulated and electrified metal ball is let down into the interior of an insulated metal can, but without touching it. The can is then momentarily earth-connected, after which the ball is removed. Describe the electric changes that accompany these processes, and illustrate your answer by diagrams showing the distribution of lines of force and potential at each stage.

PART III.

VOLTAIC ELECTRICITY OR ELECTRODYNAMICS.

CHAPTER XI.

GENERAL FACTS AND PRINCIPLES.

156. Chemical Preliminaries.

Exp. 105. Place a common piece of zinc in dilute sulphuric acid. Observe that the zinc dissolves and that bubbles of gas are evolved. Test this gas. It is hydrogen. Note also that the mixture becomes hot, showing that energy is being liberated by the chemical action.

Let us regard this reaction from the standpoint of energy. The equation of the reaction is

$$\underset{\text{Zinc}}{Zn} + \underset{\text{Sulphuric Acid}}{H_2SO_4} = \underset{\text{Zinc Sulphate}}{ZnSO_4} + \underset{\text{Hydrogen.}}{H_2}$$

The group of atoms (SO_4), called *Sulphion*, which exists in the sulphuric acid has a certain chemical affinity for the zinc, more affinity for the zinc than it has for the hydrogen, so that when the acid is poured upon the zinc an interchange is effected. There is a certain amount of potential energy locked up in the molecule H_2SO_4 and a certain, but less, amount locked up in the molecule $ZnSO_4$. When the interchange described above is effected there is on the whole a conversion of chemical potential energy into heat.

The same thing occurs in a great many other chemical actions.

When any action (chemical or otherwise) results in the conversion of potential into some other form of energy, the action is often spoken of as a *source* of energy though of course (see Appendix, § 242) no energy is really created; thus the combustion of coal and the action of sulphuric acid on zinc are both

"sources" of energy. In the course of this chapter we shall see how the latter and several other chemical actions may be made sources of *electrical* energy—*i.e.* of energy manifesting itself in association with electricity.

Exp. 106. While Exp. 105 is in progress pour some mercury upon the zinc. The zinc at once becomes coated with mercury and the chemical action ceases.

In like manner if the zinc is first well cleaned and then rubbed with mercury (in which case it is said to be *amalgamated*) the acid under ordinary circumstances has no action upon it. The same is the case if the zinc be perfectly pure; in fact, the commonly received explanation of the action of the mercury is that it dissolves the outer layers of the zinc, leaving the impurities behind, so that the portion in contact with the acid virtually *is* pure. *Why* there should be this difference in the behaviour of pure and impure zinc will be explained in § 188.

157. Simple Voltaic Cell.

Exp. 107. Take a vessel containing dilute sulphuric acid (Fig. 135), and in it put a plate of amalgamated zinc, Z. No action whatever occurs. Next put into the acid a plate of copper, C (not touching the zinc). Again no action occurs. Now connect the zinc and copper plates by a copper wire, M, which is most conveniently done by aid of binding screws.* Bubbles of hydrogen are then given off from the *copper* plate; and if the action is allowed to go on for some time, and the plates are weighed before and after, it is found that the zinc loses weight while the copper does not, and that the solution contains zinc sulphate.

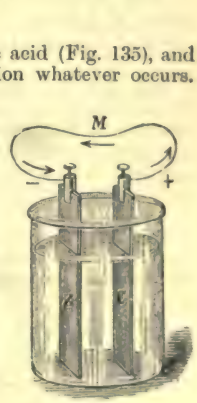

Fig. 135.

You will have observed that in Exp. 107, as soon as metallic connection is established between the two plates, the same chemical action sets in as in § 156: the zinc dissolves, and hydrogen is liberated. *Why* the hydrogen makes its appearance on the copper instead of the zinc is a secondary point, which we shall discuss in §§ 177, 178.

* See § 172.

The contrivance in Fig. 135 is called a **simple voltaic** or **galvanic*** **cell.** It serves as a convenient introduction to the study of electrodynamics, although in practice it is seldom used, there being other kinds of cells which answer much better. It frequently happens too that a single cell of any kind is insufficient to do what we want, and we then employ a combination of several cells, such combination being termed a **battery.** The most important cells and batteries we shall study in due course, but there are certain general points connected with the whole subject which it is desirable to understand at the outset, and these we shall consider in the remainder of the present chapter. It must be understood that though our remarks may be illustrated by reference to the simple cell, they apply with mere changes of detail to cells and batteries in general and also to dynamos.

In the simple cell we may, in place of a copper plate, employ one of platinum or gas-coke (carbon) or any substance which is a good conductor of electricity and is not acted on chemically by the acid.

158. Introductory Considerations and Nomenclature. The wire, M, joining the plates of a cell (Fig. 135) is found to possess several remarkable properties.

Exp. 108. Join the two plates of a voltaic cell by a long stout copper wire (1 mm. diam. say) and hold it lengthwise just over a compass-needle. The needle is deflected from the meridian (see Fig. 186). Detach the wire from one of its binding screws or cut the wire at any point so as to interpose an air gap. The needle at once returns to the meridian ; the chemical action in the cell ceases at the same time.

Exp. 109. Cut the wire at one point and dip the ends in a solution of lead acetate in a small beaker. Observe that bubbles of gas arise from one of the immersed ends and that crystals of lead grow on the other.

Exp. 110. Mix a little potassium iodide solution with some starch solution and pour it on some white blotting-paper. Take the free ends of the two wires joined to the poles of the cell and hold them near together and resting on the damp blotting-paper. Note the blue colouration (cf. Exp. 99).

Exp. 111. Join the free ends of the copper wires by a short piece of thin platinum wire. Note that the wire becomes warm, possibly hot.

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* From Volta and Galvani, two Italian physicists of the eighteenth century.

It is therefore natural to attribute the properties in question to some influence flowing along the wire, which can travel through a solution of a metallic salt but which cannot cross an air-gap, and this influence is called an **electric current.**

Now let us look more fully into the matter:—The first point to notice is that (as we shall show experimentally in Exp. 112) before the plates are connected by the wire they are at different potentials, that of the copper being the higher. This being so, then, by Poisson's Principle (§ 92), as soon as connection is made, electricity flows along the wire *from the copper to the zinc plate, as indicated by the arrows* (Fig. 135). According therefore to this view, the so-called electric current is a real flow of electricity, just as truly as a water current in a pipe is a real flow of water, and cutting the wire at any point, or detaching from one of the plates so as to interpose an air gap, corresponds to turning off a tap in the pipe.

This, however, is not all that takes place in a voltaic cell. If we take two electrostatically charged conductors at different potentials and connect them by a wire, a momentary current flows and the potentials become equalised. But if the conductors are respectively attached to the prime conductors of two machines working to different potentials then the potentials cannot become equalised because the machines are always unequalising them; we have in fact in the machines a source of energy, and this energy maintains a continual potential difference in the conductors, giving a continual current. It is the same with the voltaic cell. The chemical action of the sulphuric acid upon the zinc is a source of energy, it maintains a continual potential difference between the plates, and we have a continuous current so long as the cell is in action.

The current in a voltaic cell does not exist in the wire only, but also in the liquid of the cell. The electricity having flowed from the copper to the zinc by virtue of the potential difference of the plates, is driven *from the zinc across the liquid to the copper by virtue of the energy of the chemical action.* That is, it flows *down the potential-gradient* through the wire, so to speak "of its own accord," and is then forced *up the gradient* by the energy liberated by the chemical action.* Having got up, it is ready to

* In § 166 we shall see that there is not an up-gradient *all* the way from the zinc to the copper through the liquid, but at present this is a detail which does not concern us.

flow down again and so on, so that it keeps on flowing round. The whole cell may be likened to a couple of water tanks at different levels connected by a flow-pipe and a pump. Water flows from the upper tank to the lower "of its own accord," it is then forced up again by the energy of the pump, flows down again, and so on, so that there is a continuous current as long as the pump works.

The entire path of the current through the wire and cell is called the **circuit**, the portion through the wire being the *external* circuit and that through the liquid (including the plates, screws, etc.) the *internal*. Now in the water illustration just given there is a gradual fall in *level* from the upper to the lower tank, but the water *current* is the same at all parts of the circuit, and this is true whether the pipes be of uniform bore or not.* In like manner in the electric circuit there is a gradual fall of *potential* along the wire from the copper to the zinc, but the electric *current* is the same at all parts of the circuit both external and internal; the student should note this very carefully—a common error, for example, is to imagine that if the wire is thicker at one end than at the other the current will be stronger at the thin end, but it is *not*, it is just the same all along the wire.

Another common though less serious misconception is to look upon a cell as a *generator* of electricity, but it no more makes the electricity than a pump creates water; in fact, voltaic cells, dynamos, and all other so-called "electric generators" are nothing more than *electric pumps*: the electricity is in the circuit already, and all the "generator" does is to pump it round and so make it produce numerous effects which it cannot when at rest.

Frequently the external circuit consists not of a mere wire, but includes some other piece or pieces of apparatus placed in the course of the wire and through which the current is conducted. A mere wire, or any instrument in which no mechanical or

* By the water current at any point of a pipe is strictly meant the quantity of water which in a given time, say one second, crosses a section of the pipe through that point and perpendicular to its axis. Whatever quantity of water passes through a section at the beginning of a given portion of a pipe, an *equal* quantity must in the same time pass out through a section at the end of the portion; that is, *the currents across the two sections are the same.* Precisely the same reasoning applies to the electric current.

chemical action occurs, is called a **dead connection**, and unless the contrary is stated we shall always suppose our connections to be dead. The characteristic of a dead connection is that it *contributes* no energy and that *all the energy it absorbs from the current is converted directly into heat*; in other words it is neither a "source" nor a "sink" of energy. When on the other hand any part of a circuit is a source or a sink it may be termed "**alive**"; an electromotor such as is used to drive tramcars is "alive"—it absorbs energy from the current supplied to it which is converted into mechanical work.

The binding screws attached to the plates of any cell are called its **terminals or poles**; the plate or pole of higher potential is termed the *positive* or *high-potential* (*H. P.*) plate or pole, and the one of lower the *negative* or *low-potential* (*L. P.*) plate or pole; thus in the simple cell the copper is the H. P. or + ve plate and the terminal thereto attached the H. P. or + ve pole.*

A voltaic cell may be regarded as a device for *redirecting the energy of chemical action*. If we merely dissolve zinc in sulphuric acid, as in Exp. 105, all the surplus energy of the action goes direct to heat; but if we dissolve it in a voltaic cell this energy, instead of going *direct* to heat, goes *first* to pump electricity from a lower to a higher potential. Mark the words italicised. We do not say that in a voltaic cell the surplus energy *does not* go to heat, but that it does not do so *directly*. If the plates are merely connected by a wire all the surplus energy ultimately goes to heat, part in the liquid and plates, and part in the wire; but it does not do so straight away,—it first pumps up the electricity, conferring energy upon it, and this energy afterwards becomes heat. But we can prevent some of it from becoming heat if instead of joining the plates by a mere wire we connect one of them to one terminal of a small electro-motor and the other to the other; the current then drives the machine, which might in turn be used to drive a small model tramcar up an incline; in this way some of the energy liberated in the chemical action is converted, not into heat *at all*, but into the potential and kinetic energy of the uplifted car.

159. Electrodynamic Measurements. In electrostatics there are two things we are more particularly called upon to measure or compare—viz. *potentials* and *charges*; these measurements we effect by the methods of § 99 and § 116 respectively.

* The older writers, for reasons into which we need not enter, were in the habit of calling the copper itself the *negative plate* and the terminal attached to it the *positive pole*, but this confusing nomenclature is now abandoned.

In electrodynamics there are two things we are more particularly called upon to measure—viz. *potential-differences* and *currents*. For the present the most important thing to measure is the difference between the potentials of the two poles of a voltaic cell.

If the two poles are not connected by a wire or other conductor, no current will be running and the cell is then said to be *on open circuit*. In this case it will be found that the two poles of a given cell always have the same *difference* of potential, though it is a matter of accident what their *actual* potentials may be (see § 165).

If the two poles are connected by a wire, there will be a current. The circuit is then said to be *closed*. There will again be a difference of potential between the two poles, but this is found to be always *less* than when the cell is on open circuit

The difference in potential between the two poles of a cell when on open circuit is called its **Electromotive Force**—for which term the letters E.M.F. are the recognised abbreviation.

160. Measurement of Potential-differences. The Condensing Electroscope. It is most important at this stage for the student to understand that the potential differences which occur in voltaic cells are very much smaller than those which occur in electrostatic experiments. In fact the ordinary form of electroscope is not sufficiently delicate to respond at all to the potential differences with which we are dealing at present. We therefore need something further, and the requirement is met by the **Condensing Electroscope**; this was first invented by Volta, but in its original form it is very difficult to work; the arrangement here described will be found to yield excellent results without trouble.

The arrangement is shown in Fig. 136. The electroscope V is the identical instrument depicted in Fig. 75, with its cap removed. F and G are two brass plates, which should be fairly thick, so that they be made and remain perfectly flat; the lower one is mounted on an ebonite or other insulating stem and the upper is furnished with a handle whose material is of no consequence; each plate also has a pair of binding screws, E E′, I I′. The plate F is placed on G, *with a sheet of foolscap paper*, M M′, *between*.* One of the binding screws, E′, of the upper plate is

* A thin sheet of mica is ideal for this purpose.

connected by a copper wire Y to the terminal t' of the electroscope; t' being connected to a gas pipe, so that the walls of the electroscope are well earthed. One of the binding screws I′ of the lower

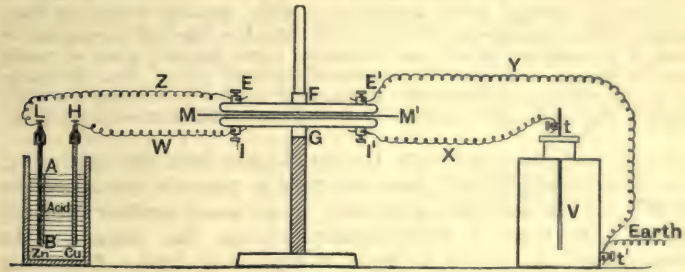

Fig. 136.

plate G is connected by another copper wire X to the terminal t of the electroscope; in this way any potential which in any experiment may be conferred upon the plate G will if sufficiently strong cause the leaves to diverge.

Now the two plates F and G clearly constitute a condenser (§ 135) of which the lower, G, is the insulated plate, the upper, F, the earth plate, and the paper the dielectric;* also since the plates are very close together the capacity of the condenser is considerable.

Exp. 112. *Use the condensing electroscope to show that there is a difference of potential between the poles of a voltaic cell.* Arrange the apparatus as in Fig. 136 where H and L are the terminals of the cell. Connect L by a wire Z to the screw E on the upper plate and connect H loosely by a wire W to the screw I on the lower plate. Then remove W by insulating tongs, and lift F. The leaves of the electroscope diverge.†

These results are explained as follows:—When L is joined to F the potential of L will be zero, for E′ is connected to t', which is earthed.

* The paper is of course but an indifferent insulator, but as the condenser is only going to be charged to a weak potential this does not matter.

† In practice, unless the electroscope be very delicate and great care be taken, experiments with a *single* cell are sometimes disappointing, but by employing three or four connected together so as to form a series-battery (§ 186) excellent results may be ensured. For very accurate work with a single cell Lord Kelvin's Quadrant Electrometer should be employed instead of the Condensing Electroscope.

When H is joined to I by the wire W this plate acquires a +ve potential equal to the E.M.F. of the cell, *which is insufficient to affect the electroscope*: so far, therefore, the manipulation was incomplete. Now, not only has the plate G acquired a +ve potential, but also a +ve electrostatic charge, and our next step is to *disconnect the wire W from the terminal I*, so as to prevent this charge being subsequently lost. Finally comes the main feature of the experiment; *when the earth-plate F is lifted off* G ceases to be part of a condenser at all, and becomes a mere isolated plate of vastly smaller capacity than when in presence of the earth-plate. But the charge on G still remains, and therefore *its potential is greatly increased*: accordingly *as soon as the earth-plate is lifted the leaves of the electroscope diverge.*

If we had dispensed with the earth-plate from the beginning, the plate G would still have acquired a potential equal to the E.M.F. of the cell, but on account of its much smaller capacity would have taken a much smaller charge; the object of the arrangement is, in fact, to make the plate G draw a fairly big charge at the weak potential furnished by the cell, and then by lifting the earth-plate cause that charge to develop a much stronger potential.

Exp. 112 *a*. Beat an insulated metal ball with india-rubber, and hold it over the electroscope when the leaves are diverging from the results of Exp. 112. Increased divergence results, thus *proving* (cf. § 85) that the charge on G is +ve and therefore by Poisson's Principle that the potential of H is +ve with respect to L.

Exp. 113. Repeat Exp. 112, but now connect H to E and L to I. A divergence is again obtained. Show that the divergence is increased by holding over the electroscope a metal ball beaten *with fur*, thus showing the charge on G and the potential of L to be negative with respect to H.

In the original form of condensing electroscope as used by Volta the plate G (Fig. 136) constituted the cap of the gold-leaf electroscope so that the wire X was not required. Also the wire Y was not used, F being earth-connected by contact with the finger.

161. Measurement of Currents: the Tangent Galvanometer. To measure currents we may employ an instrument called a *galvanometer*. There are many forms of galvanometer, but they all depend upon the same principle—*viz.* that when a current flows along a wire near to or encircling a magnetic needle, it in general *deflects the needle*. This fact, which is of very great importance in electrodynamics, may be shown by the apparatus shown in

Fig. 137. This is a simple form of the particular kind of galvanometer known as a *tangent* galvanometer. A B A′ B′ is a horizontal graduated circle of brass or stiff cardboard; and on a pivot, P, at its centre, is balanced a compass-needle. C D E D′ C′ is a vertical hoop of stout copper wire, and the zeros of the graduated circle are in the plane of this hoop. The whole is fixed on wooden supports, as shown. The ends C, C′ of the hoop enter the supports, and are prolonged beneath the woodwork to binding-screws T, T′, into which they are permanently soldered, while the upper parts of T, T′ stand above the wooden base in which they are firmly screwed. The instrument is set on the table, and turned until the needle *n s* points to the zero; we then know that the hoop C D E D′ C′ is in the magnetic meridian.

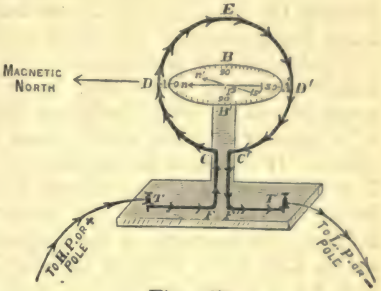

Fig. 137.

Exp. 114. Set up and adjust a simple form of tangent galvanometer. (The essentials of the apparatus shown in Fig. 137 may be set up in five minutes.) Connect the terminals T, T′ to the H.P. and L.P. poles of a voltaic cell. This closes the circuit; a current flows round the hoop in the direction of the arrows, the needle becomes deflected, and (after a few swings) comes to rest in a new position, *n′ s′*. The angle *n P n′* between its old and new position is called its *deflection*, and can be read off on the graduated circle.

The needle in Exp. 114 is under influences analogous to those shown in Fig 41. As we shall see in Chapter XIV., the current produces a magnetic field which tends to make the N. pole of the needle point (magnetic) east* and the S. pole (magnetic) west, while the earth's field tends to make the magnet point magnetic north and south. The needle therefore takes up a position in which the forces on it due to these two fields balance one another. Moreover, the stronger the current the greater will be the force due to its field, and the greater will be the deflection, so that the magnitude of the latter gives us an estimate of the strength of the former. If

*It will be east as the figure is drawn; if the current went the opposite way round the hoop it would be west. This will receive detailed study in Chapter XIV., but meanwhile it should be noted that with a given instrument reversing the current reverses the deflection of the needle.

the current were enormously strong the deflection would be very nearly 90°, but it is never possible to make it *exactly* 90°.

The instrument in Fig. 137 is called a *single-coil galvanometer*, because the hoop consists of one turn only. Such an instrument is not much good for very weak currents, and for these a *multiple-coil galvanometer* is employed. In the latter instrument we use *insulated* copper wire—that is, wire covered with silk, cotton, gutta-percha, or some insulating material—and the hoop consists of a number of turns wound close together in the form of a flat spiral. If n denote the number of turns, the current entering at T will pass up to C and thence travel n times round the hoop before emerging along the route C′ F′ T′. The effect is to multiply the strength of the magnetic field due to the current by n (see Exp. 141), without of course altering the earth's field, so that for a given current the deflection is greater than with a single coil, and the greater n is the greater is the deflection. In this way, by using a few dozen coils, weak currents can be readily recognised.

In the instrument shown in Fig. 137 the needle is long: for elementary purposes, where we merely wish to see which of the two currents is the stronger, this does not matter, but when the galvanometer is required for the *accurate measurement* of a current the needle must be very short. The best instruments are accordingly made in this way, and the needle has fixed to it a light aluminium pointer whose extremities move over the circular scale by means of which the deflections are read (Fig. 202).

Any simple galvanometer not intended for accurate measurements is sometimes called a *galvanoscope*.

162. Resistance; External and Internal. The substances which permit an electric current to flow along them are, in general, the same as those hitherto termed conductors (§ 72): the terms "conductor" and "insulator" thus apply broadly to the same respective substances in electrodynamics as in electrostatics. But there are two important differences. The first arises from the fact that the electrical pressures with which we have to deal are very much smaller in Electrodynamics than in Electrostatics, so that a substance may be a good electrodynamic insulator which is an electrostatic conductor: this is so with wood, and

generally with all substances termed in electrostatics "partial insulators (§ 72). Pure water, though electrostatically a good conductor, is electrodynamically a very bad one.

The second difference arises from the fact that we are dealing now with electricity *in motion*. Now, in electrostatics we have regarded all good conductors as practically alike—for example, an iron ball behaves precisely like a copper one of the same size and similarly situated: this, of course, is because the charge is on the external surface, it being *the surrounding dielectric* which regulates the behaviour of the ball and not *its own material*. But electric currents flow *through the actual substance of wires and other conductors*, and in so doing encounter opposition, the amount of which depends upon the material of the conductor as well as its size, etc.; this opposition is called the *resistance* of the conductor. As the current flows round the circuit, it has to encounter the resistance of the wire and of any instruments in the external circuit, and also that of the liquid in the cell. The former constitutes the *external resistance*, the latter the *internal*, the two together making the *total resistance*.

Exp. 115. *Show that conductors offer resistance to the passage of an electric current.* Join up in series a Daniell's cell,* a multiple coil galvanoscope made of thick wire, and a length—half a metre say—of thin german-silver wire. Note the deflection of the needle of the galvanoscope. Now replace the wire by a longer piece of the same diameter. Note that the deflection has decreased. This proves that the long wire offers more resistance to the passage of the current than a short wire.

Repeat the experiment with two wires of the same material of the same length, but of different cross-sections. You will find that the one of greater cross-section allows a greater current to flow.

Repeat the experiment with wires of the same length and diameter of cross-section, but of different materials. In general the currents will be different, showing that the resistance of a wire depends upon the material of which it is made.

The resistance of a wire thus depends upon three things—*viz.* its *length*, its *thickness*, and its *material*. For wires of the same material the resistance is greater the *longer* and *thinner* the wire. The reason for this is fairly obvious, for the longer the wire the

* For this experiment a steady generator of electricity is required. The output of a simple voltaic cell falls off as time goes on, but a Daniell cell (§ 180) is very steady.

greater the distance through which the electricity has, so to speak, to "grind" its way; while the thicker it is, the broader is the path it affords to the electricity and therefore the less the resistance—just as a broad pipe offers less resistance to the passage of water than a narrow one. In the higher parts of the subject, methods are devised for the actual measurement of resistances, and it is then found that there is a definite law—*viz.* that for wires of the same material the resistance is *directly* proportional to their length and *inversely* proportional to the area of their cross-section; thus if we had two copper wires of the same length but of which one had four times the cross-sectional area of the other, the thick one would have only a quarter the resistance of the thin one.

If we consider wires all of *the same length and cross-sectional area* their resistances vary greatly according to their material. The numbers expressing the relative resistances of wires of different materials, but of the same length and thickness, constitute the *relative resistances of the materials*; their values depend simply upon the materials, and in no way upon the actual lengths, etc. Of all substances, silver has the lowest resistance, that of copper is a little greater, that of iron and platinum much greater, and that of German silver greater still. The resistance of insulators is, of course, enormous. Pure water has a very high resistance, but the resistance of dilute sulphuric acid, and of the other liquids employed in voltaic cells, is in general fairly small.

The following numbers represent approximately the relative resistances of the most important metals and alloys, that of silver being taken as unity; it should be observed, however, that the numbers vary considerably in different specimens of the same metal, according to purity and the way the wire has been drawn. In some specially prepared copper wires the resistance is *less* than that of silver.

Silver	1	Iron	6·4
Copper	1·1	Tin	9
Gold	1·3	Lead	13
Aluminium	2·0	German Silver	17
Brass	4·5	Platinoid	22
Platinum	6·0	Manganin	32
Phosphor-bronze	6·3	Mercury	63

Let us now compare the resistances of several cells all of the same *kind*, and differing only in respect to the distances of their plates apart, and the areas of them below the surface of the liquid. The passage of the current through the liquid from one plate to the other is analogous to its flow along a wire, the distance between the plates corresponding to the length of the wire, and their immersed area to the cross-sectional area of the wire; hence *the resistance of a cell will be greater the greater the distance between the plates, and the less their immersed area*, or, speaking broadly, *the internal resistance is less the larger the cell*. As will be more fully seen in § 163, the effect of resistance, other things being the same, is to weaken the current; it is therefore in general an advantage to employ cells as large as possible and with their plates as close together as possible.

When the poles of a cell (or battery) are not connected by a wire or some other conductor it is said to be an **open circuit**; the external resistance is then practically infinite and the current *nil*. If they are so connected the circuit is said to be **closed** or "made" or "complete." If they are connected by a thick and fairly short wire, whose resistance is practically *nil*, the cell is said to be on **short circuit**, in which case the current is as great as the cell can possibly yield.

In experimental work it is usually necessary to pass the current through some instrument, into which it is conducted by so-called *leads*. These are stout copper wires, which, having very small resistance, do not appreciably weaken the current (see § 172).

The main body of the earth is an excellent conductor, electrodynamically as well as electrostatically. If we connect one pole of a cell to the gas-pipe and the other to the water-pipe the circuit will be completed through the earth, and the current will flow just as if the two poles were joined by a single wire. In telegraphic work the circuit is commonly completed "through earth," thus saving the expense of a "return wire," and also diminishing resistance, since the resistance of the earth portion of the circuit is very small.*

* One disadvantage of this is that telegraphic communication is upset by magnetic storms. Thus during the great magnetic storm of Sept. 25th, 1909, telegraphic communication in England was stopped for three hours between the towns which used a "return through earth," whereas in the case of towns which used wires each way no inconvenience was felt.

163. Ohm's Law. The Volt, Ohm, and Ampere. Potential Drop. In Fig. 138 let A and B be any two points on an electric circuit, and suppose the current flowing along the circuit in the direction of the arrow, so that by Poisson's Principle the potential at A is higher than that at B. Then confining our attention to the portion A B of the circuit, there are three things about it which we have to consider, *viz.*:—

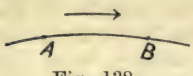

Fig. 138.

(1) The *current* in it: this we will denote by C.

(2) The *potential-difference* (*P.D.*) of its extremities A and B: this we will denote by V.

(3) Its resistance: this we will denote by R.

Now the current in the wire may be measured by its effects, either the magnetic or chemical or heating effect (Exps. 108-111). These measurements will be considered in detail later, but it is sufficient at present to know that currents can be measured. The potential difference between A and B may be indicated by a gold-leaf electroscope, and measured by another but more complicated instrument called a quadrant electrometer.

In 1827 Dr. Ohm found in all cases that as long as we restrict ourselves to one portion of the circuit and keep that portion of the circuit under the same physical conditions the current through that portion is exactly proportional to the potential difference between its extremities, *i.e.* the ratio

$$\frac{\text{P.D. between the extremities of that portion}}{\text{Current through that portion}}$$

is a constant. This constant he designated the **resistance** of that portion of the circuit, and thus Ohm's law reads

$$\frac{\textbf{P.D. between the extremities of a conductor}}{\textbf{Current through the conductor}} = \textbf{Resistance of the conductor.}$$

It is easy to see that this ratio may fairly be used as a measure of the "resistance" of any part of the circuit. For the ratio (P.D. ÷ current) is large for any part of the circuit where a large P.D. is required to produce a small current, and is small where only a small P.D. is required to produce a large current (cf. § 162).

Ohm's law is thus primarily a definition of resistance, but as resistance may now be defined in other ways independent of, but consistent with, Ohm's definition the law may be considered a

pure law. Consistent units of potential difference, current, and resistance have been devised. They are named after distinguished electricians, and are the **volt**, **ampere**, and **ohm** respectively. As long therefore as the three quantities, potential difference, current, and resistance are expressed in these units or in the same multiple of these units we have

$$\frac{\text{P.D. between the extremities of a conductor (in volts)}}{\text{current through the conductor (in amperes)}} = \text{resistance of the conductor (in ohms)}$$

or shortly $$\frac{\text{volts}}{\text{amperes}} = \text{ohms.}$$

164. Definitions of the Ohm, Ampere, and Volt. At the International Congress of Electrical Units which met in London in 1908, the following definitions of the ohm, ampere, and volt were decided on, *viz.*:—

The international ohm is the resistance offered to an unvarying current by a column of mercury at the temperature of melting ice, 14·4521 gms. in mass, of a constant cross-sectional area, and of a length of 106·300 cms.

The international ampere is the unvarying electric current which when passed through a solution of silver nitrate in water of certain strength, etc., deposits silver at the rate of 0·00111800 gm. per second.

The international volt is the electrical pressure which, when steadily applied to a conductor whose resistance is one international ohm will produce a current of one international ampere.

These definitions should be committed to memory.

You will not fully understand the definition of the ampere unless you are acquainted with some of the facts of electrolysis, so that at the present stage it is perhaps best to give an independent definition of the volt and use Ohm's Law to define the ampere. The following definitions are meant to be intelligible rather than accurate.

An **ohm** is the resistance of 74·1 yards (67·7 metres) of No. 18 S.W.G.* pure copper wire.†

* S.W.G. = Standard wire gauge. Wire of No. 18 S.W.G. is ·048 in. (= 1·21 mm.) in diameter.

† It is interesting to note that the resistance of a mile of ordinary telegraph wire is about 10 ohms.

A **volt** is $\frac{10}{11}$ of the electromotive force of a Daniell's cell (§ 180).

An **ampere** is the current which a potential difference of one volt would drive through a resistance of one ohm; that is, if we had a piece of wire of 1 ohm resistance and maintained a potential difference of 1 volt between its ends, the current in it would be 1 ampere.

Ohm's Law is the fundamental law of Electrodynamics, and should be thoroughly grasped by the student; it must, however, be observed that in the forms above given it is only true when the portion of the circuit considered is "dead" (§ 158)—if it contains any source or sink of energy the statement of the law needs modification.

Exp. 116. *To find how the potential difference between the poles of a voltaic cell depends upon the size, shape, etc., of the cell.* Take several cells, all of the same kind, but differing in other respects. Taking first one of them, connect its terminals (on open circuit) with the earth and condensing electroscope, as in Exp. 112; on lifting the earth-plate a certain divergence is obtained. Do the same in succession with the others: *in each case you will obtain the same divergence.* A gallon-size cell, for example, gives the same divergence as one holding only a quarter of a pint; therefore the potential difference depends only on the nature of the cell.

165. Electromotive Force. An extremely important quantity in connection with a cell or battery is what is called its *Electromotive Force* (*E.M.F.*). This may be roughly defined as "the influence which drives the current," but its strict definition is *the difference of potentials of its terminals when the cell is on open circuit.*

Now it is found by measurement (see Exp. 115) that the E.M.F. of a cell, *when freshly fitted up and in good condition*, depends simply upon the *kind* of cell—*i.e.* whether it be a simple cell or a Daniell's, Leclanché's, Poggendorf's, etc. (Chap. XII.); it in no way depends upon the *size* of the cell or the arrangement of its plates: the student should note this carefully and contrast it with what was said in § 162 regarding internal resistance. It is thus possible to tabulate the E.M.F.'s of the different types of cell in definite numbers. Daniell's cell (§ 180), for example, has an E.M.F. of 1·1 volt, and this furnishes the elementary definition of the volt given in the preceding article.*

* Strictly, the expression "kind" of cell must be taken to include the *quality* of the materials employed in its construction—*i.e.* the purity of

The *difference* of potentials of the terminals of a cell (on open circuit) is a perfectly definite thing depending only on the kind of cell, but the *individual* potentials of the terminals are, so to speak, more or less accidental. Thus, if we have a Daniell's cell with its terminals insulated from the earth, we know their potential difference is 1·1 volt, but we have no guarantee as to potential of either of them separately. But suppose we earth the low potential or negative terminal (§ 158), thus rendering its potential zero, then the potential of the high potential or positive terminal becomes 1·1 volt; while if we earth the high potential terminal the potential of the low potential one becomes −1·1 volt. If we earth *both* terminals the cell is no longer on open circuit, and the state of affairs is altogether different.

If we earth the low potential terminal of a cell and connect its high potential terminal to an insulated conductor—*e.g.* a brass ball—the latter receives a +ve electrostatic charge (though exceedingly feeble) in accordance with Poisson's Principle; but unlike the corresponding case in § 92, the potential does not fall during the process, because the chemical action within the cell keeps it up to its standard value; thus in the case of a Daniell's cell the ball would acquire a potential of +1·1 volt. In like manner if the high potential terminal were earthed and the low potential connected with the ball, the latter would acquire a negative charge and a potential of −1·1 volt—*i.e.* 1·1 volt below the potential of the earth.

In Exp. 116 the cell was on open circuit. Let us now see what happens to the potential difference between the poles of the cell when the circuit is closed.

Exp. 116*a*. Join the poles of the cell by a long German silver wire and repeat Exp. 112, or Exp. 116. The divergence is less than it was before. Repeat again with half the length of wire and again with the shortest piece that can be used. The divergence gets less as the wire used gets shorter.

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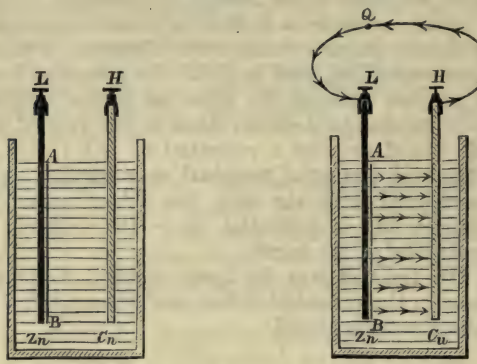
the plates, and more particularly the purity and strength of the acid or other exciting material. Thus the E.M.F. of a Daniell's cell in which the plates are *pure* zinc and copper, and the solutions are pure zinc sulphate and pure copper sulphate *of equal density*, is 1·1 volt, and this is the kind of cell referred to in defining the volt in § 164. For other forms of Daniell's cell the E.M.F. may vary from about 1·05 to 1·15 volt.

166. Distinction between Electromotive Force and Terminal Potential-difference. More Complete Theory of the Voltaic Cell. Application of Ohm's Law to the External, Internal, and Complete Circuit. The E.M.F. of a cell is defined (§ 165) as its terminal potential-difference *on open circuit*. But it was found by experiment (§ 165, Exp. 116 *a*) that the terminal potential difference on *closed* circuit is *less* than the E.M.F., and, moreover, depends on the resistance of the wire used to connect the terminals, being less the less the resistance of the wire. Now let us look into this point; in so doing we shall learn a good deal more of the action of the voltaic cell:—

In § 158 we have said that the chemical action in a cell pumps the electricity up the potential-gradient through the liquid from



No. 1. No. 2.

Fig. 139.

the zinc to the copper. But this statement is somewhat inadequate, inasmuch as it gives the impression that there is a *gradual* up-gradient, which there is reason to believe is not the case. A nearer approach to the true condition of things will be gathered from Fig. 139, where H represents the copper plate and L the zinc plate. No. 1 represents the cell on open circuit. Some action, into whose nature we need not enter, takes place between the sulphuric acid and the amalgamated zinc, this action being confined to a very narrow region lying between the plate L and a

layer A B.* The effect is to establish a difference of potential between the zinc plate and the liquid layer A B, so that, again for a reason into which we need not enter, Poisson's Principle does not apply to this narrow region.† But it applies to all the rest of the circuit, so that the potential of the liquid from A B towards and including the copper is the same throughout, and there is no gradient anywhere except from the zinc to the layer A B, where there is steep jump-up equal to the E.M.F.

We next connect the plates by a wire (Fig. 139, No. 2). Electricity flows through the wire from H to L, which lowers the potential of the copper plate and raises that of the zinc plate. This establishes a down-gradient from A B across the liquid towards the copper, and electricity flows through the liquid down this gradient. The tendency of this flow is to lower the potential of A B and raise that of the zinc plate. But now chemical action sets in between the zinc and sulphuric acid, the effect of which is to maintain a P.D. between the zinc and A B equal to the original E.M.F. Thus *throughout the whole action the P.D. between the zinc and the layer A B remains equal to the E.M.F.* But since there is now a down-gradient from A B through the liquid to the copper, it is obvious that the potential of the copper cannot be so high as that of the layer A B; in other words, *the terminal potential difference of the cell on closed circuit must be less than its E.M.F.*

The above account will perhaps be clearer if we draw a potential diagram. In Fig. 140 distances above the base line represent potentials. No. 1 shows the state of affairs when the cell is on open circuit. The potential is constant over the zinc and also constant but at a higher value over the acid to the right of P Q and the copper. The state of affairs when the copper and zinc are joined by a wire is shown in No. 2. There is the same rise of potential from the zinc to the layer P Q, and a gradual fall over the rest of the liquid, and also along the wire shown by the sloping lines P T, H L, the total fall being equal to the rise from the zinc to P Q.

* In reality the action occurs on both sides of the plate, but we have taken only one for simplicity.

† Lest this should look like a violation of the generality of Poisson's Principle, it may be pointed out that this region appears to be in a state of electric strain, and in certain respects to resemble an insulator rather than a conductor.

Following the course of the current in Figs. 139, 140, No. 2, the electricity starting from A B, the top of the gradient, flows across the liquid to the copper plate where it is part of the way down, and thence continues its course through the wire to the zinc plate which is at the bottom. All this part of the circuit is "dead" (§ 158); the "live" part is the narrow region between

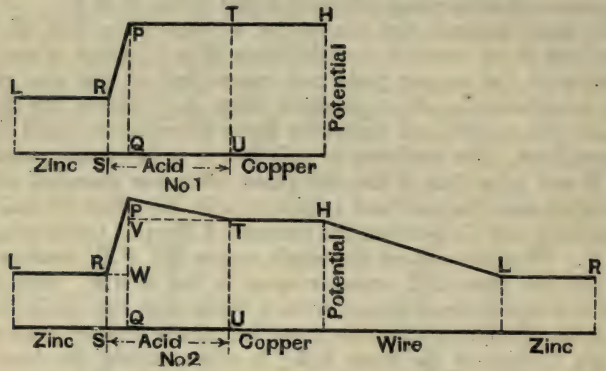


Fig. 140.

the zinc and the layer A B, where the chemical action is going on. As soon as the electricity arrives at the bottom of the gradient (*i.e.* at the zinc) the energy of this live portion pumps it up to the top (*i.e.* to A B), whence it runs down again, and so on. If the circuit be broken the chemical action and the current simultaneously stop, and everything returns to the condition of Figs. 139, 140, No. 1.

The internal resistance of the cell is that of the liquid between A B and the copper plate, and the external resistance is that of the wire: these together make up the total resistance, and it is this which the current has to overcome in the course of its flow from the top to the bottom of the gradient.*

* The resistance of the narrow region between the zinc and A B appears to be *nil*, although in the footnote on p. 245 we said this region might be considered to act as an insulator. Anyway, the resistance is overcome by the chemical action and lies altogether beyond the range of our enquiries.

What may be called the "driving influence" for the *complete* circuit is the potential difference between A B and the zinc—*i.e.* the E.M.F.—and this has to overcome the *total* resistance. It is represented by P Q − R S or P W in Fig. 140. The driving influence for the *external circuit alone* is the terminal potential difference, which has to overcome the external resistance. It is represented by T U − R S or V W in Fig. 140. The driving influence for the *internal* circuit alone is the potential difference between A B and the copper plate—*i.e.* the fall of *potential across the cell*; this is usually termed the *internal fall of potential*, and is obviously the excess of the E.M.F. over the terminal potential difference. In Fig. 140 it is represented by P Q − T U or P V. Electrical engineers frequently speak of the internal fall of potential as the "lost volts," from the fact that they are spent in driving the current against the internal resistance and are not available for external work.

Let us now, bearing in mind (§ 158) that the current is the same throughout the entire circuit, apply Ohm's Law to its different parts: The potential difference of the extremities of the *complete* circuit is the E.M.F., hence by Ohm's Law (§ 163) we have—

$$\text{Current} = \frac{\text{E.M.F.}}{\text{Total Resistance}} \quad . \quad . \quad . \quad . \quad (1).$$

Applying Ohm's Law to the external circuit we get

$$\text{current} = \frac{\text{terminal potential difference}}{\text{external resistance}}.$$

Lastly for the internal circuit we get

$$\text{current} = \frac{\text{internal fall of potential}}{\text{internal resistance}}.$$

If E denote the E.M.F., V the terminal potential difference, C the current, B the internal resistance, and R the external resistance, all in appropriate units, the above equations may be written in algebraic forms, respectively as follows:—

$$C = \frac{E}{B + R} \quad \text{or} \quad E = C(B + R) \quad . \quad . \quad . \quad . \quad (2).$$

$$C = \frac{V}{R} \quad \text{or} \quad V = CR \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

$$C = \frac{E - V}{B} \quad \text{or} \quad E - V = CB \quad . \quad . \quad . \quad . \quad (4).$$

It is now easy to see why the terminal potential difference of a given cell becomes less as the external resistance is diminished. For as the external resistance decreases so of course does the total, and since the E.M.F. remains unchanged, equation (2) tells us that the current increases; then, since the internal resistance remains unaltered, equation (4) tells us that the internal drop increases, or in other words the terminal potential difference decreases. In the extreme case where the current is on short circuit the terminal potential difference is practically *nil*.

The results of this article are of great service in Electrodynamics.

EXAMPLE.—A battery whose E.M.F. is 12 volts and resistance 6 ohms, has its terminals joined by a wire of resistance 9 ohms. Find the current, and also the potential difference of the terminals.

To find the current we apply equation (2). We have $E = 12$, $B = 6$, $R = 9$; hence

$$C = \frac{12}{6+9} = \frac{4}{5} \text{ ampere.}$$

Next, to find the terminal potential difference we apply equation (3). R denoting the resistance of the wire and V the required terminal potential difference, we have

$$C = \frac{V}{R} = \frac{V}{9}.$$

But we have just found $C = \frac{4}{5}$ ampere,

$$\therefore \frac{4}{5} = \frac{V}{9}$$

whence $V = 7{\cdot}2$ volts.

After having found the current, we may also find the terminal potential difference from equation (4) thus:—

Internal fall of potential = current × internal resistance

$$= \frac{4}{5} \times 6 = 4{\cdot}8 \text{ volts,}$$

and if we now subtract this drop from the E.M.F. we get the terminal potential difference thus:—

Terminal potential difference = $12 - 4{\cdot}8 = 7{\cdot}2$ volts, as before.

167. High and Low Resistance Galvanometer. In the tangent galvanometer the needle turns horizontally, but for some purposes one in which it turns vertically is to be preferred. A very convenient form of vertical galvanometer (or rather galvano-

scope) is shown diagrammatically in Fig. 141, Nos. 1 and 2. No. 1 represents a vertical section, and No. 2 a plan. Some of the details are not shown in both views. E F G H is the top of a flat bobbin. In it is cut a slot, C D, about $\frac{1}{4}$ in. wide, and in this slot swings a rather heavy magnetic needle, N S, about a horizontal axis, K K. The plane of swing of the needle is therefore vertical. Attached to the needle is an aluminium pointer, P, whose extremity moves along a circular graduated scale. The axis K is a trifle above the centre of gravity of the needle, so that the weight of the latter practically counteracts the tilting force of the earth's magnetic field. When no other magnetic influence is present the needle sets horizontally with the pointer P at the zero of the scale. Around the groove, E E′ G G′, of the bobbin are wound two separate coils of insulated copper wire; one of these is fairly stout and of only a few turns, so that its resistance is low; the other is fine and of many turns, its resistance is therefore high.

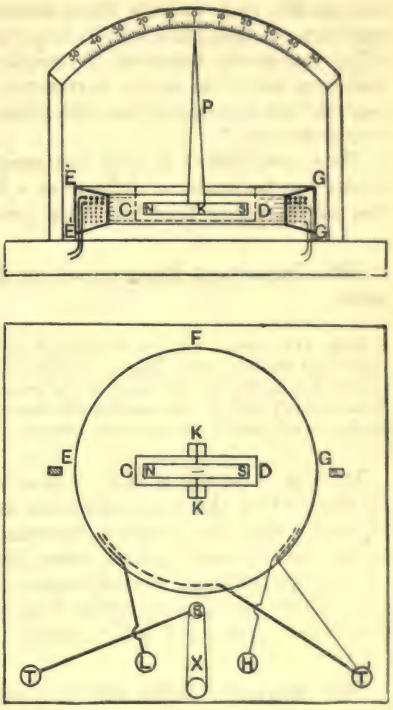


Fig. 141.

The instrument is furnished with a switch X working on a pin B, which can be turned to make contact with either of the buttons, H or L, as required, and the connections are so arranged that when the switch is on L, the low resistance coil only is in series, while when it is on H the high resistance coil only is in circuit; we thus have a high and low resistance

galvanometer in one instrument, which for many purposes is a great advantage.

Suppose now, the switch being on either of the buttons H or L, the terminals T, T′ are connected with the poles of a cell or battery: the current will flow round one of the coils and deflect the needle, thus causing the pointer to travel over the scale. The amount of deflection will in any case be such that the turning effect due to the weight of the needle in its displaced position just balances that due to the currency. For *a given one of the coils*, say the low-resistance one, the stronger the current the greater is the deflection.*

This instrument is very convenient, but no vertical galvanometer can be made so *delicate* as a horizontal one on account of the much greater friction on the pivot.

168. Important Experiment on E.M.F., Current, and Resistance.

Exp. 117. Take any two cells of the same kind, say two Daniell's, one small and one very large. Connect the terminals of the small one to those of a multiple coil tangent galvanometer, and notice the deflection. Then detach and do the same with the big one. Note that the deflection in the latter case is the greater, showing that we have a greater *current*.

By Exp. 116, the E.M.F. of the two cells is the same. Hence, by Ohm's Law the total resistance must be less when the big cell is used. But the external resistance—*viz.* that of the galvanometer—is the same in both cases, because it is the same identical instrument. Hence, *the resistance of the big cell must be less than that of the small one.* Thus Exp. 117 confirms the theoretical deductions given in § 162 (p. 239).

169. Magnetic Field due to a circular current: Law of the Tangent Galvanometer. Consider a circle of wire of radius r centimetres, and let a current of C amperes flow in it. Let I be the intensity of the magnetic field produced by the current at the centre of the circle, then the *direction* of I is perpendicular to the

* We cannot immediately judge of the relative strengths of two currents one of which passes through one coil and one through the other, nor indeed do we require to.

plane of the circle, and in the higher parts of the subject it is shown that its *magnitude* is given by

$$I = \frac{\pi C}{5r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).*$$

If, instead of a single circle of wire, we have a squat spiral of n turns, the current, C, will flow through all the turns one after another, and each turn will produce a field whose intensity (at the centre) is given by (1); these fields will add together, and the actual intensity at the centre will be

$$I = \frac{n\pi C}{5r} \quad . \quad . \quad (2).\dagger$$

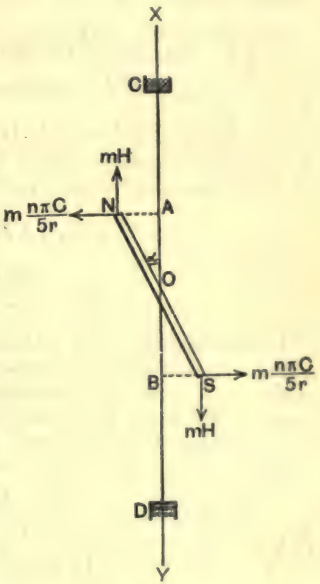


Fig. 142.

From these we can determine the exact law connecting the current in a tangent-galvanometer with the deflection of the needle.

For in Fig. 142 let X Y be the magnetic meridian, C, D sections of the coil made by a horizontal plane through the needle, N S the needle in the deflected position, and α the deflection. Let the coil consist of n turns, and let m be the pole strength of the needle, and H the horizontal intensity of the earth's field. Then the earth exerts on the north pole N of the needle a northward force m H parallel to the magnetic meridian, and on the south pole an equal southward force also parallel to the meridian. Also, *since the needle is very short*, the part of the

* The formula is sometimes given as $I = \frac{2\pi C}{r}$, but here C is estimated not in *amperes*, but in *electromagnetic units of current*; an ampere is $\frac{1}{10}$th of an electromagnetic unit.

† An experimental proof of this formula is given in Exp 141.

current's field over which it can range is practically uniform, and therefore the intensity of this field at N and S is practically equal to that at the centre of the coil, *i.e.* to $\frac{n\pi C}{5r}$. Hence the force exerted by the current on N is $m \cdot \frac{n\pi C}{5r}$, say, westwards (as in the figure), and on S a force $\frac{m \cdot n\pi C}{5r}$ eastwards.

The two forces mH form a couple* whose arm is NA + BS or 2AN. This couple tends to rotate the needle anti-clockwise. The forces $m \cdot \frac{n\pi C}{5r}$ constitute a couple of arm AB or 2OA and the tendency of this couple is to rotate the needle anti-clockwise. The needle therefore takes up a position in which the moments of these two couples are equal and opposite. We have therefore

$$m\mathrm{H} \cdot 2\mathrm{AN} = m\frac{n\pi C}{5r} \cdot 2\mathrm{OA},$$

$$\therefore C = \frac{5\mathrm{H}r}{n\pi} \cdot \frac{\mathrm{AN}}{\mathrm{OA}} = \frac{5\mathrm{H}r}{n\pi} \tan\alpha. \quad . \quad . \quad . \quad (3)$$

It thus appears that with a given instrument used at a given place the *current is proportional to the tangent of the angle of deflection*: from this circumstance the instrument derives its name. Since for small angles, say up to about 10°, the tangent is very nearly proportional to the angle, we may for small deflections take the current as proportional to the deflection.

It appears from (3) that the deflection for a given current is quite independent of the pole-strength of the needle. But *prior to the ultimate deflection being attained* the influence tending to move the needle can be shown to be greater the greater the pole-strength; hence a strong needle responds to the current more readily than a weak one, and thus renders the instrument *more sensitive.*

The coefficient $\frac{5\mathrm{H}r}{n\pi}$ in (3) is called the *reduction factor* of the galvanometer; it can be calculated once for all for the same place, and marked on the instrument. Denoting it by K, (4) becomes

$$C = K \tan\alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

* See Appendix, § 248.

The following is another proof of the formula of a tangent galvanometer:—

Let R be the resultant of the forces with $m \cdot \frac{n\pi C}{5r}$ on the pole N (Fig. 143). The needle sets itself so that the direction of R is along O N produced, hence if we draw a parallelogram of forces* N E G F we get a $\triangle$ G E N similar to $\triangle$ N A O of Fig. 142.

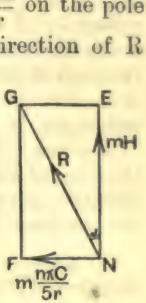


Fig. 143.

By the parallelogram of forces we get

$$\frac{mH}{m \cdot \frac{n\pi C}{5r}} = \frac{NE}{GE} = \frac{OA}{AN} \text{ of Fig. 142,}$$

or
$$C = \frac{5Hr}{n\pi} \frac{AN}{OA},$$

i.e.
$$C = \frac{5Hr}{n\pi} \tan \alpha.$$

Examples:—1. A tangent galvanometer is wound with 30 turns of wire. The internal diameter of the coils is 19·1 cm. and its external diameter 19·9 cm. Find the reduction factor of the instrument when used in London, where H = ·18 dyne per unit magnetic pole.

The *mean* diameter of the coils is $\frac{1}{2}(19{\cdot}1 + 19{\cdot}9)$, *i.e.* 19·5 cm., and ∴ the mean radius 9·75 cm.; this is the value we must assign to r. Now $K = \frac{5Hr}{n\pi}$, whence, making the substitutions, we have

$$K = \frac{5 \times {\cdot}18 \times 9{\cdot}75}{30 \times 3{\cdot}1416} = {\cdot}093.$$

2. With the instrument in the previous example, if the deflection is 30°, what is the current?

We have $C = K \tan \alpha$. Here K = ·093 and $\tan \alpha = \frac{1}{\sqrt{3}}$. Hence

$$C = \frac{{\cdot}093}{\sqrt{3}} = {\cdot}031\sqrt{3} = {\cdot}054 \text{ ampere.}$$

170. Specific Resistance and Law of Resistance of a Wire.—In § 162 we saw that the resistance of a wire of given material is proportional directly to its length and inversely to the area of its cross-section. If R denote the resistance of a wire of length l and cross-sectional area a we have

$$R = k\frac{l}{a}, \quad \ldots \ldots \quad (1)$$

* See Appendix, § 245.

where k is a "constant" depending only on the material of the wire and the units of measurement, not in any way upon its length or thickness. This quantity, k, is called the *specific resistance of the material.* Consider now a cubical block of the material whose edge is of unit length, and let the current flow through it parallel to an edge; from the electrical standpoint it is a wire in which $l = 1$ and $a = 1$, and therefore by (1) its resistance is k. We may therefore define the specific resistance of a given material as *the resistance of a unit cube of that material, the current being supposed to flow parallel to one of its edges.* For scientific purposes it is usual to take the cube as having its edge one *centimetre* in length.

The annexed table gives the specific resistances of a few of the most important substances in *microhms* per centimetre cube* at 0° C. (cf. § 224) :—

Silver	1·49
Copper (pure)	1·57
Copper (hard-drawn)	1·70
Aluminium	2·90
Platinum	9·00
Steel (rails)	12·00
Iron (telegraph wire)	15·00
German silver†	20·76
Platinoid†	32·50
Manganin†	47·50
Mercury	94·07
Carbon (lamp filaments) about	4000
Gutta-percha about	350 million MEGohms.

EXAMPLES :—**1.** The specific resistance of a substance per centimetre cube being r, find its specific resistance per inch cube, it being given that 1 inch = 2·54 cm.

An inch cube has a length 1 inch, *i.e.* 2·54 cm., and a sectional area 1 square inch, *i.e.* $(2{\cdot}54)^2$ square cm. Hence by (1) its resistance is $\frac{r \times 2{\cdot}54}{(2{\cdot}54)^2}$; *i.e.* $\frac{r}{2{\cdot}54}$.

2. The specific resistance of hard-drawn copper such as used for overhead tramway wires is $\frac{2}{3}$ microhm per inch cube. Find the resistance of 4 miles of such wire ·325 inch diameter.

Our formula is $$R = \frac{kl}{a} \quad \ldots\ldots\ldots\ldots \quad (1)$$

* A microhm is one-millionth of an ohm.

† These are alloys : their resistance is practically unaffected by changes of temperature (cf. § 224).

Since k is referred to an inch as unit, we must express l in inches and a in square inches.

Now $l = 4$ miles $= 4 \times 5280 \times 12$ inches.

Also $a = \cdot7854 \times (\cdot325)^2 = \cdot083$ square inch, very nearly.

Hence by (1)

$$R = \frac{\frac{3}{8} \times 4 \times 5280 \times 12}{\cdot083} \text{ microhms} = 2 \text{ ohms very nearly.}$$

171. Resistance of a Combination of Wires.—Suppose we have a number of wires of resistances R_1, R_2, R_3, ...

We may connect them **in series**, *i.e.* end to end, as in Fig. 144, and then join the ends of the combination to the battery or the

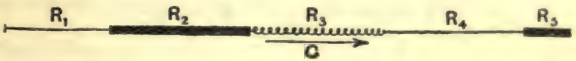


Fig. 144.

ends of any uncompleted circuit, so that the current goes through one wire after the other. Obviously since there is no accumulation of electricity at any point, the current through each wire is the same and the resistance of the combination is the sum of the separate resistances. That is, if R denote the resistance of the combination,

$$R = R_1 + R_2 + R_3 + \ldots\ldots$$

Another mode of connection is shown in Fig. 145. A D B, A E B, ... are the wires; one set of ends is joined together at

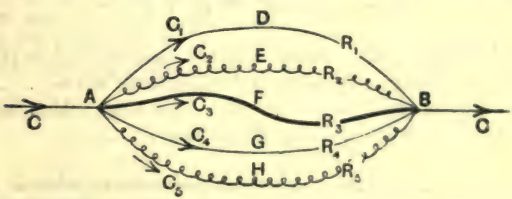


Fig. 145.

A and the other at B, and the junctions A and B are connected with the terminals of a battery or the ends of an uncompleted circuit, as shown. This arrangement is known as connecting in **parallel**, or **multiple arc**. The current C, arriving at A, *divides into*

parts, $C_1, C_2, \ldots$, travelling round the wires ADB, AEB, ... respectively, these parts meeting at B and reconstituting the total current C: we thus have

$$C = C_1 + C_2 + C_3 + \ldots \quad \ldots \quad (1)$$

To find the resistance of the combination we proceed in the following way:—Let V_A and V_B be the potentials of A and B. Then, applying Ohm's Law to the wire ADB, we get

$$C_1 = \frac{V_A - V_B}{R_1}.$$

Similarly applying it to the other wires in turn we get

$$C_2 = \frac{V_A - V_B}{R_2}, \quad C_3 = \frac{V_A - V_B}{R_3}, \ldots.$$

whence, noting the relation (1), we add these expressions and get

$$C = (V_A - V_B)\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots.\right). \quad \ldots \quad (2)$$

Now let R denote the resistance *of the combination*, then, applying Ohm's law to the whole group between A and B, we have

$$C = \frac{V_A - V_B}{R} \quad \ldots \quad (3)$$

whence equating (2) and (3) and dividing by $V_A - V_B$ we get

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots., \quad \ldots \quad (4)$$

i.e. *the reciprocal of the resistance of the combination is the sum of the reciprocals of the resistances of the separate wires* . . . (5)

From this it is easily seen that R is less than either R_1, R_2, R_3. This can readily be proved experimentally.

Exp. 118. *Show that the resistance of a combination of wires in parallel is less than that of a single wire.* Join up in series a Daniell's cell,* low resistance galvanoscope and a piece of german-silver wire, the ends of this wire being attached to two binding screws A and B. Note the deflection

* A simple cell will not do for this experiment. It is necessary that the E.M.F. and internal resistance of the cell used remain constant and a Daniell's cell (§ 180) fulfils these requirements fairly well.

of the needle of the galvanoscope. (If the deflection is more than half the maximum deflection which the instrument can register, add resistance to the circuit in the shape of some fine wire of high resistance in series with the rest of the circuit.) Now connect the ends of another length of wire to A and B so that the second wire is in parallel with the first piece. Note that the deflection of the galvanoscope is increased, showing the current in the circuit is increased. Since the E.M.F. has remained constant it follows that the resistance of the combination of wires in parallel is less than that of a single wire. Now put a third wire in parallel with the other two. The current is still further increased, showing that the resistance of the combination is still decreased.

In the particular case of n wires of *equal* resistance it easily follows that *the resistance of the combination is* $\frac{1}{n}$*th part of that of a single wire* . . . . . (6)

In the case of two wires,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

or
$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{\text{product of the resistances}}{\text{sum of the resistances}} \quad . \quad . \quad (7)$$

Exp. 118a. Repeat the first part of Exp. 118 and note the deflection of the needle of the galvanoscope. Now remove the piece of german-silver wire from the circuit. Cut off from the same reel of wire two pieces of wire each twice the length of the first piece and join them in parallel to the binding screws A and B, taking care that they touch nowhere else. Again note the deflection of the galvanoscope. If the cell has remained constant the reading will be found to be exactly the same as before. This proves that the resistance of the combination is equal to that of the first wire. Show that this verifies the formula $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.

Since also

$$V_A - V_B = C_1 R_1 = C_2 R_2 = C_3 R_3,$$

it follows that in all cases the current in any one wire is inversely proportional to the resistance of the wire.

In the case of two wires,

$$\frac{C_1}{C_2} = \frac{\frac{1}{R_1}}{\frac{1}{R_2}} = \frac{R_2}{R_1},$$

i.e. $$C_1 = \frac{R_2}{R_1 + R_2} C \quad \text{and} \quad C_2 = \frac{R_1}{R_1 + R_2} C. \quad . \quad . \quad (8)$$

EXAMPLES :—1. Four wires of resistance 2, 3, 4, 12 ohms are connected in parallel ; find the resistance of the combination.

Let R denote the required resistance, then by (5)

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{12} = \frac{7}{6},$$

$$\therefore R = \frac{6}{7} \text{ ohm.}$$

2. Two wires of resistances 5 and 20 ohms are joined together in parallel and their extremities joined by wires each of 1 ohm resistance to the terminals of a voltaic cell of resistance 6 ohms and E.M.F. 2 volts. Find the current through the battery and through each of the two wires.

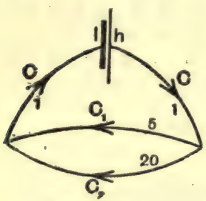


Fig. 146.

Draw a diagram first (Fig. 146). The attention of the student is drawn to the conventional way of drawing a voltaic cell. The long thin line h (Fig. 146) represents the high potential plate and the short and thick line l the low potential plate ; the exciting liquid is imagined between them. The joint resistance of the 5 and 20 ohms is

$$\frac{5 \times 20}{25} = 4 \text{ ohms.}$$

The total resistance in the circuit is therefore

$$6 + 1 + 4 + 1 = 12 \text{ ohms.}$$

The current C, sometimes called the total current,

$$= \frac{\text{E.M.F.}}{\text{Total resistance}} = \frac{2}{12} = \frac{1}{6} \text{ ampere.}$$

The current through the 5 ohm resistance

$$= \frac{20}{25} \text{ of } \frac{1}{6} = \frac{2}{15} \text{ ampere,}$$

and the current through the 20 ohm resistance

$$= \frac{5}{25} \times \frac{1}{6} = \frac{1}{30} \text{ ampere.}$$

172. Accessories required for Electrical Experiments. There are certain small but useful pieces of apparatus which are convenient and sometimes necessary in electrical work. A few details of these are given below.

(1) **Connecting wires or leads.** These are stout copper wires, which, having very small resistance, do not appreciably weaken the current. A convenient size is No. 18 Standard Wire Gauge (S.W.G.), which is about $\frac{1}{20}$ in. or $1\frac{1}{3}$ mm. in diameter. This will carry five amperes without undue heating. Its resistance is about 1 ohm for 60 metres. When larger currents have to be used twist two or three wires of equal length together or merely put them in parallel (§ 171). The insulating material round the wire is removed (by scraping with a blunt knife) from the ends of the wires over a length of about $\frac{1}{2}$ in. The bare metal ends are clamped by binding screws.

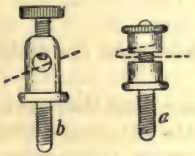


Fig. 147.

Fig. 148.

(2) **Binding and connecting screws** (Fig. 147). These are of various shapes and sizes; *a* is more suitable for joining thin wires to instruments, *b* for thick wires, the hole prevents slipping. Fig. 148 shows a connector for two wires.

(3) **Tapping or press key.** It is convenient to have an instrument by which an electric circuit may be opened or closed at will. Such an instrument is called a *key*. Fig. 149 shows a useful form of tapping key. The terminals T, T′ are screwed to metal strips B, B′, of which B is "springy." The base C is of wood or ebonite. (Ebonite is used when high insulation is required.) The ends of the connecting wires are joined to the terminals. On pressing H with the finger, the break at D is closed and current flows through the key. When the finger is removed the piece B springs back and breaks the circuit. Thus the tapping key is useful when current is required for a very short time.

Fig. 149.

(4) **Plug key** (Fig. 150). The terminals are screwed to metal plates mounted separate from one another on a wood or ebonite base. A plug can be pushed into the gap between the plates. When the plug is out the circuit is broken; when pushed in, it is made and current flows until the plug is removed. Thus

Fig. 150.

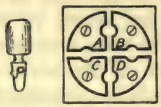


Fig. 151.

the plug key is useful when a current has to be maintained for some time.

(5) **Commutator**. This is any arrangement that permits the direction of the current in a section of the external circuit to be reversed when required. Fig. 151 illustrates a convenient form. Four metal plates, A, B, C, D, separated from one another, are

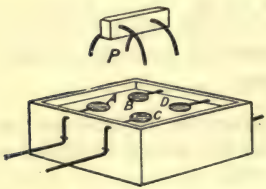


Fig. 152.

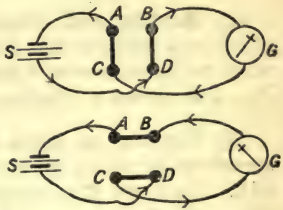


Fig. 153.

fixed to an ebonite or wooden base. Each plate carries a terminal. There are two plugs like P, that can be pushed into the gaps between the plates.

Fig. 152 illustrates a second form of commutator. A thick piece of wood has four holes, A, B, C, D, bored half-way through it. Thin sides are fixed to it, making a shallow box. From each hole a stout copper wire passes through the sides. These are joined into the circuit by connecting screws. Mercury

is put in the holes. The piece P is a strip of wood carrying two stout bent copper wires, the ends of which are far enough apart to reach between adjacent holes. Then the piece may be placed so that (i) hole A is connected with B, and C with D, or (ii) A with C, and B with D. The circuit is broken by lifting the piece out of the holes.

The method of joining either commutator with the circuit is shown in Fig. 153, where S represents a battery, G a galvanometer. It is required to pass the current through the galvanometer, first in one direction and then in the other. Join the battery wires to one pair of opposite (not adjacent) terminals of the commutator, the galvanometer wires to the remaining pair. The arrows in the diagrams indicate the direction of the current. In the upper figure the adjacent plates or holes A, C and B, D are joined, in the lower figure A, B and C, D. The directions of the current in the battery section are the same in both cases, in the galvanometer section they are opposite.

(6) **Wire resistances, Rheostats.** Wires differing in length, cross-section, and material are introduced into electric circuits to control the strength of current. Such wires are, when fine, insulated and wound on reels; their ends are joined to terminals; when coarse, the bare wire is laid over a board or wound in a spiral and stretched across a frame. A *rheostat* or *regulating resistance* is a conductor so arranged that the amount of it through which the current flows can be readily altered. **Resistance boxes** (§ 173) are rheostats in which definite amounts of resistance may be introduced or withdrawn as desired.

Fig. 154.

(7) **Varley's Carbon Rheostat** (Fig. 154). This consists of a number of discs of carbonised cloth, strung on a vertical rod. Three brass plates with terminals attached are introduced, one at the top, one at the bottom, and one intermediate. The connecting wires of the circuit are joined to two of these, current may then be passed through the cloth. A thumb-

screw working at the upper end of the vertical rod presses the cloth discs together. By tightening, the resistance is diminished; by relaxing, it is increased.

When the current is likely to be considerable carbon (gas-coke) plates in a box or frame are used. These are pressed together, more or less, by a screw that can be turned by hand.

173. Resistance Boxes. The standard ohm is defined in § 164 as the resistance of a column of mercury of certain dimensions at 0° C. It would be extremely difficult to use standards of resistance of this nature in practice, hence the resistances used in practical measurements are generally lengths of insulated wire wound on bobbins and having their ends joined to terminals. The lengths and diameter of wire, etc., are such that the resistance of the wire is some multiple of the ohm. In order that there may be no electromagnetic induction effect (cf. Ch. XVI.) when the current is sent through the wire, the length of wire is first doubled back and then wound on the bobbin (Fig. 155). Resistances of various values are joined up to form a *resistance box* or *set*.

Fig. 155 illustrates one method: *a*, *b*, *c*, *d*, *e* are brass pieces, usually fixed to an ebonite slab. Brass plugs, *k*, *l*, *m*, *n*, slightly conical in shape, and with ebonite or wooden heads, can be pushed between the pieces, or withdrawn as desired. The wire resistance has an end fixed to each of two adjacent pieces. Hence if a plug, *n*, is out the resistance wire is the only conducting connection between the pieces; if a plug is in it forms a path of negligible resistance, it is said to "short-circuit" the resistance, or "cut it out" from the circuit. The number expressing the resistance of the wire in ohms is engraved close to the space between the brass pieces. Thus in Fig. 155 the resistance between the terminals, A, B, is 5 ohms. If *n* is put in and *k*, *m* are pulled out it will be 3 ohms.

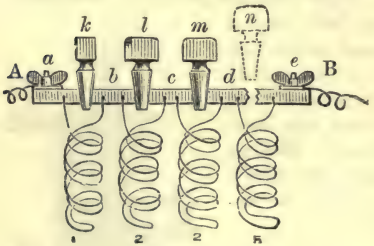


Fig. 155.

Fig. 156 illustrates a resistance box by which any integral number of ohms up to 1000 can be obtained. The resistances are in multiples 1, 1, 2, 5, like a set of weights. In the particular

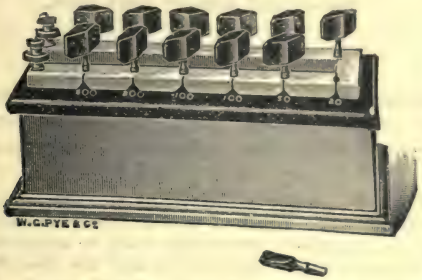


Fig. 156.

box shown the order of the resistances, starting from the left hand back, is 1, 1, 2, 5, 10, 10, 20, 50, 100, 100, 200, 500, so that any resistance from 1 ohm up to 999 ohms can readily be set up.

174. Determination of Resistance by substitution.

Exp. 119. *Find the resistance of a coil of cotton-covered wire (German silver or platinoid).* Join in series a constant cell C, sensitive galvanoscope G, the unknown resistance X, a resistance box B, and a plug key K (plug out) (Fig. 157). (1) Make the box resistance of such a value (R_1 say) that when the circuit is completed through the plug key an adequate deflection (about half the scale) of the galvanoscope needle is obtained. (2) Remove X: alter the box resistance to a value (R_2 say), such that the deflection is the *same* as before. Then

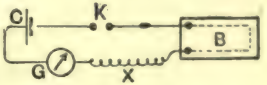


Fig. 157.

$$X = R_2 - R_1.$$

(3) Repeat (1). If the values are unaltered it may be assumed that the cell is constant. If the values are different repeat the experiment with another cell.

EXPLANATION.—The deflections being the same in both cases, the currents through the circuit are equal. Also the E.M.F. of

the battery does not change. Hence the total resistances of the circuit in the two cases are equal; that is,

X + R_1 + galvanometer and battery resistance = R_2 + galvanometer and battery resistance.

Then

$$X = R_2 - R_1.$$

By this method the resistance can only be obtained to the nearest ohm (if the smallest resistance in the box is an ohm). For more accurate work other methods are employed.

Exp. 119a. *Find the specific resistance of German silver or platinoid.* Measure the resistance of a length of wire as described above. The result to be accurate to a reasonable percentage must be many ohms. Measure the length of the wire with a metre scale and the diameter of the bare wire at many places with a micrometer screw gauge. Substitute the results in the formula $R = k\frac{l}{a}$ (§ 170) and so find k. Verify by the table, p. 254.

SUMMARY.—CHAPTER XI.

1. Chemical action of dilute sulphuric acid on metallic zinc (§ 156).

2. The simple voltaic cell (§ 157).

3. The current of a voltaic cell is of the same strength at all parts of the circuit, external and internal, but the potential is not. The top of the potential gradient is at a layer, A B (Fig. 139, No. 1), very near the zinc plate, and the bottom at the zinc plate itself; the electricity flows of its own accord down the gradient from A B across the liquid to the copper plate, and thence through the wire to the zinc, whence it is pumped up again to the top, A B, by the chemical action of the cell (§§ 158, 166).

4. The Condensing Electroscope and the explanation of its action. How to use it to prove (i) that the potential of the terminal commonly called positive really is higher than that of the one called negative; (ii) that the terminal potential difference of a cell on a closed circuit is less than its E.M.F.; (iii) that the E.M.F. of a cell is independent of its size, etc. (§ 160).

5. The E.M.F. of a cell or battery is defined to be the Potential Difference of its terminals *on open circuit*; it depends simply upon the *kind* of cell. But the internal resistance depends in addition upon the *size*, etc., of the cell. The terminal potential difference of a cell on *closed* circuit is *less* than its E.M.F. (§§ 162, 165, 166).

6. Ohm's Law (1) in the general form (§ 163), applicable to any dead circuit; (2) in the specialised forms, applicable to cells and batteries. Potential Drop (§ 166).

7. The Ampere, Ohm, and Volt (§ 164).

8. The resistance of a wire depends upon its material, length, and thickness. For wires of the same material the resistance is proportional *directly* to the length, and *inversely* to the area of cross-section. The exact law is $R = \frac{kl}{a}$, where k is the specific resistance of the material (§§ 162, 170).

9. Galvanometers and Galvanoscopes; their construction and principle of action. The Tangent Galvanometer (§§ 161, 167).

10. How to prove experimentally that the resistance of a cell depends on its size, etc. (§ 168).

11. Magnetic field due to a circular current, and Law of Tangent Galvanometer (§ 169).

12. Resistance of a combination of wires in series and in parallel (§ 171).

13. The Resistance Box, Measurement of resistance by substitution (§§ 173, 174).

EXERCISES XI.

1. A piece of copper and zinc are put side by side in a vessel of dilute sulphuric acid. What takes place *in the vessel* when the copper and zinc are joined by a wire?

2. In the preceding question, if, instead of employing a wire, the copper and zinc plates were made to touch below the surface of the acid, what would happen?

3. Give a drawing of a galvanic cell of copper, zinc, and dilute sulphuric acid, showing in what direction the current passes through a wire connecting the two metals, and also through the acid.

4. A piece of zinc and copper are each carefully weighed; they are then connected by a copper wire, and dipped side by side into dilute sulphuric acid contained in an earthenware jar. After, say, half an hour, the pieces of zinc and copper are taken out, washed, dried, and weighed again. Would the weights be the same as at first? if not, how and why would they differ?

5. A current flows through a copper wire which is thicker at one end than at the other. Is there any difference either (1) in the strength of the current, or (2) in the potential at the two ends of the wire? Explain your answer.

6. There are two wires, A and B, of the same material, but of which B is 12 times as long as A and double its diameter. Compare their resistance.

7. A piece of wire has a resistance of 6 ohms, and the P.D. of its ends is 18 volts; what is the current in it?

8. The terminals of an incandescent electric lamp have a P.D. of 50 volts, and it takes a current of $\frac{3}{4}$ ampere; what is its resistance?

9. An over-head trolley-wire has a resistance of half an ohm per mile, and carries a current of 20 amperes; find the potential-drop per quarter-mile along it.

10. A battery, whose E.M.F. is 12 volts and resistance 6 ohms, has its terminals connected by a wire of resistance (1) 18 ohms, (2) 2 ohms. Find the current and terminal potential difference.

11. The E.M.F. of a cell is 2 volts and its resistance 1 ohm. Find the internal drop of potential when the circuit is closed by a wire of resistance (1) 2 ohms, (2) $\frac{1}{2}$ ohm.

12. In any cell or battery show that, if the external and internal resistances are equal, the internal drop of potential is half the E.M.F.

13. The E.M.F. of a battery is 20 volts and its resistance 15 ohms. Its terminals are connected by a wire of resistance 25 ohms. Find (1) the current, (2) the terminal potential difference, (3) the drop of potential along half the wire, (4) the potential at its middle point, supposing the negative terminal of the battery earthed.

14. Give an account of the distribution of potential in a cell (1) on open, (2) on closed circuit.

15. The poles of an insulated cell are connected by a wire. Will the strength of the current in the wire be altered if one of its poles be afterwards connected by another wire to the gas-pipe? Give reasons for your answer.

16. The resistance of a battery is 1 ohm, and the current through a wire A B, whose resistance is 5 ohms and which joins the terminals of the battery, is 1 ampere. If A B is replaced by another wire C D, the current is $\frac{1}{50}$ of an ampere. What is the resistance of C D?

17. A dynamo is employed to light an incandescent lamp which requires a potential difference of 100 volts to its terminals and a current of 2 amperes. The dynamo is some distance away from the lamp to which it is connected by a pair of feeders whose joint resistance is 4 ohms. Find the potential difference which the dynamo terminals must have in order that the lamp may be properly lighted.

18. Describe the action of a gold-leaf electroscope provided with a condensing plate, and explain how you would use it to detect the electrification of a large feebly electrified sphere.

19. You are supplied with two wires, and are required to determine which of them has the greater resistance; how would you proceed?

20. The coil of a tangent galvanometer consists of 8 turns of wire and has a mean radius of 20 cm. Find what current will produce a deflection of 45° if the horizontal intensity of the earth's field is ·18 C.G.S. unit.

21. The deflection of a galvanometer is 10° when connected directly to a small Daniell's cell. When a long coil of fine wire is placed in the circuit, the deflection falls to 3°. Compare the resistance of the coil with that of the rest of the circuit, assuming that the deflection is proportional to the current.

22. Equal currents pass through two tangent galvanometers, each of which consists of a single ring of copper, the radius of one ring being three times that of the other. In which of the galvanometers will the deflection be greater? If the greater deflection be 60° what will the smaller be?

23. It is desired to make a galvanometer for measuring large currents; should it be wound with many or with few turns of wire? Give reasons.

24. Calculate the specific resistance of steel rails per *inch* cube, and thence find the resistance of 20 miles of rail of 10 square inches sectional area (this is about the size employed in the Underground Railway, London).

25. The resistance of 1 metre of copper wire 109 square mm. cross-sectional area is ·000157 ohm. What length of wire 1 square mm. cross-sectional area will have a resistance of 1 ohm?

26. Two tangent galvanometers have coils of equal radius made of copper wire of the same diameter, but the coil of one has two turns and that of the other twenty. The galvanometers are connected with a battery, (i) in series, (ii) in parallel. Find the ratio of the tangents of the deflections in each case.

27. A battery of E.M.F. 1 volt and resistance 1 ohm is connected to a galvanometer of resistance 2 ohms. What is the current in the circuit? How is the current through the galvanometer affected by joining its terminals by a wire of 2 ohms resistance?

28. If *n equal* resistances be connected in parallel, show that the resistance of the combination is $\frac{1}{n}$th of that of a single one of them.

29. Fifty equal incandescent electric lamps are connected in parallel, and when their terminals are maintained at a P.D. of 240 volts, each lamp takes 0·8 ampere. Find the resistance of the entire set of lamps.

CHAPTER XII.

VOLTAIC CELLS AND BATTERIES.

175. Defect of the Simple Cell (§ 157).

Exp. 120. Set up a simple cell, connect its poles by pieces of copper wire to the terminals of a multiple coil tangent galvanometer and note the deflection. Allow the current to pass for a few minutes, the deflection gradually becomes less, and after ten or fifteen minutes will be practically *nil*. This clearly shows a falling off in the strength of the current. Now look at the copper plate; it will be found covered with minute bubbles of hydrogen. Take it out, clean it well so as to remove the remaining hydrogen, and put it back; the cell will now give as good a current as at first, but again it will soon fall off, and so on.

The accumulation of hydrogen therefore renders the cell inefficient. But how does the hydrogen act? It must do so either by increasing the resistance of the cell, or diminishing its E.M.F., or may-be both. Let us appeal to experiment.

Exp. 121. Set up a condensing electroscope (§ 160), take a freshly prepared simple cell and test its E.M.F. by the method of Exp. 112, noting the divergence of the leaves. Then detach the wires W and Z (Fig. 136), put the cell on closed circuit for a few minutes, then remove the wire connecting the poles and again test with the condensing electroscope. Compare the divergence obtained now with the previous one. It is much smaller, perhaps hardly detectable.

The cell has, therefore, *lost most of its E.M.F.* Doubtless its resistance has also increased at the same time owing to the presence of the hydrogen layer, but that is a small point—the loss of E.M.F. is the main thing. If we now take out the copper plate, clean it well, refit the cell, and again test by the electroscope, the E.M.F. will be found restored.

But *why* should the hydrogen layer destroy or diminish the E.M.F. of the cell? To answer this completely would necessitate the prior consideration why does a cell have any E.M.F. at all? Now, the whole theory of the action of a voltaic cell is very perplexing, and scientific authorities are by no means agreed

about it; we therefore purpose avoiding it as much as possible, but the following generally received view appears accurate as far as it goes:—Whenever there are immersed in a liquid two plates of different metals, one of which is capable of being acted on chemically by the liquid and the other is not, the plates acquire a difference of potential, so that the combination constitutes a voltaic cell, and *the plate that is acted on always constitutes the low potential or negative pole*: also (§ 165) the value of this potential difference when the cell is on open circuit constitutes the E.M.F. of the cell.

Moreover, the E.M.F. depends upon the degree to which the first plate is attacked; thus if in the simple cell we employ iron in the place of zinc we get a feebler E.M.F., because the iron is not attacked by the acid as much as the zinc is. Suppose now we employ two metals *both* of which can be attacked, but in different degrees—*e.g.* zinc and iron—then the action of the liquid on the zinc tends to make the potential of the zinc the lower, while its action on the iron tends to make the potential of the iron the lower, and on the whole there is a small balance in favour of the zinc being the lower, so that the combination constitutes a cell of feeble E.M.F. in which the iron is the high potential terminal. In fact the iron introduces a *back E.M.F.* which subtracts from that due to the zinc, the *actual* E.M.F. being the difference between the two.

Copper is also subject to attack by sulphuric acid, so that the true potential diagram for a simple cell is not that already given (Fig. 140), but

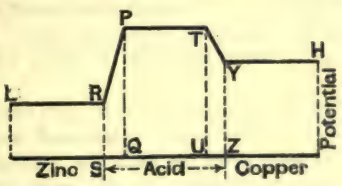


Fig. 158.

more like Fig. 158. Here T Y is the drop of potential from the acid to the copper plate. This occurs in a region close up to the copper plate, just as the drop P R occurs near the zinc plate (§ 166). The E.M.F. of the cell is the difference of the potentials at Y Z and R S.

Thus in the ordinary simple cell the copper plate produces a small *back E.M.F.*; but for certain reasons the explanation of which is too difficult for this book, the hydrogen bubbles which collect on the copper plate produce a much stronger back E.M.F., which ultimately becomes large enough to stop the current.

176. Polarisation. Smee's Cell. Other Kinds of Cells. When the hydrogen accumulates on the copper, platinum, or other H.P. plate of a simple cell, the plates are said to become *polarised*, and the back E.M.F. thus called into play is called the *E.M.F. of polarisation*. Numerous forms of cell have been devised in which polarisation is more or less successfully overcome, either by removing the hydrogen from the high potential plate or preventing its deposit. The simplest and oldest of these is the **Smee's Cell.** It is a simple cell in which the copper plate is replaced by one of "platinised silver"—that is, silver covered with finely divided platinum black so as to give it a rough surface; the roughness to some extent prevents the hydrogen adhering to the plate, but the action is not very satisfactory.

It will be observed that in Smee's cell the hydrogen is first allowed to form on the plate, and is then removed by *mechanical* means. In the other cells which we shall now proceed to consider, the removal is effected by *chemical* means, some substance being used in the cell which attacks the hydrogen and really prevents its ever forming on the high potential plate at all. The substance producing this effect is called the *depolariser*.

In some forms of cell, notably Daniell's, the E.M.F. remains practically unchanged, even though the circuit be kept closed for several hours: such are termed *constant* cells. In others, as the chemical action progresses, the E.M.F. falls with greater or less rapidity.

The student should be careful not to confuse between *the fall of E.M.F.* of a cell due to polarisation and modification of the chemical action as time goes on and the *internal potential drop* (§ 166); the latter is common to all cells and occurs immediately they begin to work, the former is a gradual change.

177. Grotthüs's Hypothesis. Now, there was a point respecting the simple cell left over in § 157 for future consideration—viz. *why does the hydrogen appear on the copper plate?* And we shall be the better able to understand the various other cells if we clear

this up.* We shall give first the explanation originally given by Grotthüs, and known as the *theory of alternate combination and dissociation*:—

Consider the cell freshly fitted up and on open circuit (Fig. 159). Between the zinc and copper plates there are many millions of

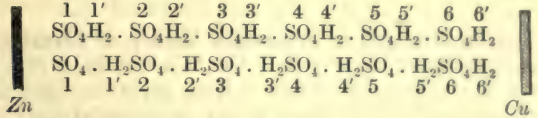


Fig. 159.

molecules of sulphuric acid, each having the composition of H_2SO_2 or SO_4H_2: let us for simplicity represent these by six only, arranged as in the upper row. Here the sulphion 1 is combined with the hydrogen 1′, the sulphion 2 with the hydrogen 2′, and so on. We now close the circuit, and chemical action sets in between the zinc and sulphuric acid in accordance with the equation—

$$Zn + H_2SO_4 = ZnSO_4 + H_2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The lower row now represents what Grotthüs conceived to occur. An atom of zinc combines with the sulphion 1, forming the group $ZnSO_4$, shown on the left, and liberating the hydrogen 1′. *But this hydrogen does not appear in the free state*; it combines with the sulphion 2, forming a new molecule of sulphuric acid $\overset{H_2SO_4}{1'\ 2}$. This liberates the hydrogen 2′, which combines with the sulphion 3, forming $\overset{H_2SO_4}{2'\ 3}$, and so on, right throughout the whole string of molecules, until we come to the one adjacent to the copper. Here the hydrogen 6′, liberated from its previous combination with the sulphion 6, finding no more sulphion to unite with, makes its appearance in the free state on the copper. Thus, in equation (1), the H_2 on the right-hand side is *not the identical H_2 belonging to the H_2SO_4 which attacks the zinc, but belongs to a different molecule altogether*. The hydrogen is

* A student may, with advantage, read § 189 at this stage.

conveniently said to "travel" from the zinc to the copper by alternate combination and dissociation.

The action according to the above view may be conveniently represented by the equation—

$$Zn + SO_4H_2 + SO_4H_2 + \ldots SO_4H_2 + SO_4H_2 = ZnSO_4 + H_2SO_4 + H_2 \ldots SO_4 + H_2SO_4 + H_2 \ldots (2).$$

178. The Dissociation or Ionisation theory of Clausius. This is an explanation of later date. On this theory a certain proportion of the molecules of any salt in solution dissociate, *i.e.* break up into smaller groups of atoms, or single atoms; and the weaker the solution the greater the amount of dissociation.

Thus the atoms or radicles of any one molecule are not always the same, but are continually changing, *i.e.* there is a continual interchange of atoms between the molecules and there are instants at which any given atom is passing from one molecule to another, *i.e.* it is **free** or **dissociated** from the molecules. Such a free atom is called an **ion**,* and it is further supposed that the metallic ions are positively charged and the non-metallic ions negatively charged.

Thus, if we take, for example, the case of a weak solution of sodium chloride (NaCl), there would at any instant be in the solution a large number of free sodium ions each bearing a positive charge. This charge is the same for all sodium ions and is the fundamental charge of electricity (§ 90). We will denote it by + E. Similarly there would be an equal number of chlorine ions in the solution. Since a neutral molecule of sodium chloride breaks up into two charged ions, it follows that the charge on the chlorine ion is exactly that which neutralises the charge on the sodium ion, *i.e.* it is − E. In the case of a weak solution of sulphuric acid (H_2SO_4) there will be twice as many hydrogen ions (H) as sulphions (SO_4), and while each hydrogen ion carries a charge + E each sulphion will bear a charge − 2E.

Again, when a metal is immersed in an acid it is supposed that it exerts a "solution pressure" analogous to a vapour pressure†

* This name was first used by Faraday. It is the Greek word *ion*, the present participle of *ienai*, "to go," and thus simply means *going*.

† Vapour pressure in the air over a liquid is due to particles of the liquid which travel into the air. Solution pressure in the acid round a metal is due to particles of the metal which travel into the acid.

over a liquid; so that the region around a zinc plate, say between the plate and A B (Fig. 139, No. 2), contains free zinc ions. These are positively charged, and their emission leaves the zinc plate negatively charged. According to the ordinary laws of electrostatics these ions should at once be attracted towards the zinc. Ordinary laws, however, do not act in this region. A similar state of affairs occurs at the copper plate, but the solution pressure of copper is less than that of zinc, so that the action there is less intense.

When the cell is on open circuit the effect of the positive metal ions discharged into the acid near the plates is to give the liquid a positive potential. The action being greater at the zinc plate the difference of potential there is also greater. See Fig. 158.

When, however, the circuit is closed Fig. 140, No. 2, represents the potential variation: except that there is a sudden drop of potential at the point T—that is to say, in the layer of acid next to the copper—due, of course, to the solution pressure of the copper.

Now the potential gradient in the acid, represented by P T in Fig. 140, No. 2, indicates an electric field which drives positive charges (and therefore positive ions) towards the copper plate and negative charges (and therefore negative ions) towards the zinc plate; see § 147.

It follows that, when the circuit is closed, the ions in the solution begin to move, the positive ions (zinc, hydrogen, and copper) towards the copper plate and the negative ions (sulphion) towards the zinc plate. When the hydrogen ions reach the copper plate they give up to it their positive charge and combine in pairs to form neutral molecules which collect on the copper plate and finally rise to the surface of the liquid. The sulphions travelling towards the zinc plate gradually lose all touch with their hydrogen partners and pass into the crowd of zinc ions. As long as the solution is weak this is all that happens. If, however, the solution gets strong, the zinc ion and the sulphion unite to form zinc sulphate. In any case, if the solution is evaporated to dryness, zinc sulphate is obtained.

The metals here used as the elements of the cell are copper and zinc, but any two substances which differ markedly in their attractions for oxygen or acid radicles may be employed—thus zinc and platinum, zinc and carbon, copper and carbon, zinc and

silver, iron and platinum, and many other pairs of substances act more or less efficiently as the plates of a voltaic cell. In fact if the metals are arranged in the order of their chemical affinity for oxygen or sulphion, and any two of the series are chosen as the elements of a cell, the electromotive force of the cell will be greater the further apart these metals are in the series. The solution pressures of metals thus vary in the same way as their affinities for oxygen and sulphion.

The following list is a list indicating decreasing affinity for oxygen or sulphion (or indicating decreasing solution pressures), and from it we see that a cell having plates of zinc and copper has less E.M.F. than one having, say, zinc and carbon as its elements—further, if a cell is made up of any two substances on the list, the higher on the list will always be the negative pole of the cell.

1. Magnesium.
2. Zinc.
3. Lead.
4. Tin.
5. Iron.
6. Copper.
7. Mercury.
8. Silver.
9. Platinum.
10. Carbon.

The order of this list is also the order in which the metals precipitate each other from solution (see Exp. 127).

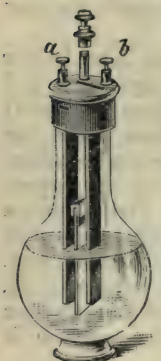


Fig. 160.

It may be mentioned in direct experimental support of the theories of Grotthüs and Clausius, that if, while the cell is working, a copper or platinum plate not connected with either of the cell-plates be put anywhere between them, the side of it facing the zinc becomes covered with bubbles of hydrogen.

We now proceed to describe the more common voltaic cells, and the methods by which in each case the polarisation of the positive plate is prevented.

179. The Chromic Acid Cell. The usual form of this cell is shown in Fig. 160. It consists of a glass bottle fitted with a wooden or ebonite cap; attached to the under surface of the cap is a strip of brass, to which are cemented two parallel plates of carbon (gas coke),

and one of the terminals, *a*, is connected with the brass, so that it constitutes the high potential pole of the cell. Through the centre of the cap is a brass tube *insulated from the plate beneath*, and in this slides a brass rod carrying at the bottom an amalgamated zinc plate: the terminal *b* is connected by a brass strip above the cap to the tube, and thus constitutes the low potential pole of the cell. The liquid employed is a mixture of dilute sulphuric acid and chromic acid,* the latter being the depolariser; and since this mixture attacks the zinc *even in open circuit*, the zinc plate is drawn up by means of the sliding rod when the cell is not in use.

The main action of the cell is the same as in the simple cell. The depolarising action consists in the chromic acid attacking the hydrogen, which would otherwise appear on the carbon plate, according to the equation—

$$\underset{\text{Hydrogen.}}{3H_2} + \underset{\text{Chromic Acid.}}{2H_2CrO_4} = \underset{\text{Chromic Oxide.}}{Cr_2O_3} + \underset{\text{Water.}}{5H_2O} \quad . \quad . \quad . \quad (1)$$

Thus no free hydrogen appears; it simply forms water along with the hydrogen belonging to the chromic acid. The chromic oxide dissolves in the excess of sulphuric acid present according to the equation—

$$\underset{\text{Chromic Oxide.}}{Cr_2O_3} + \underset{\text{Sulphuric Acid.}}{3H_2SO_4} = \underset{\text{Chromic Sulphate.}}{Cr_2(SO_4)_3} + \underset{\text{Water.}}{3H_2O} \quad . \quad . \quad (2)$$

Chromic sulphate is green, while chromic acid is red. As the materials become used up the colour of the liquid changes from red to brown, and from brown to green, by which time it is useless.

The chromic acid cell is a modern improvement on the old bichromate or **Poggendorf's cell**. In the latter, potassium bichromate was used in place of chromic acid, and chrome alum was formed as a waste product (instead of chromium sulphate), which crystallised on the plates and choked them up.

A freshly prepared chromic acid cell has an E.M.F. of about 2 volts. On closing the circuit and thus making the cell work, its E.M.F. retains this value for a short time, and then begins to run down. After using a few times, the chromic acid becomes exhausted, and in general the cell cannot be relied on for more

* 5 oz. chromic acid, 2 pts. water, 3 oz. sulphuric acid.

than about 1·7 volts, and that is liable to fluctuation. Size for size, its internal resistance is low compared with a good many other cells: and as very little trouble is involved in its use, it is a great favourite in laboratories and lecture-rooms when a fairly strong current is needed for only a minute or so at a time.

180. Daniell's cell. A common form of this cell is shown in Figs. 161, 162. It consists of a cylindrical copper pot, C (Fig. 162), in which is a concentrated solution of copper sulphate. In

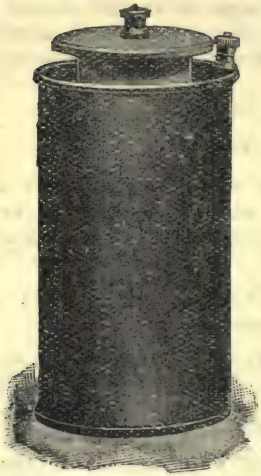


Fig. 161.

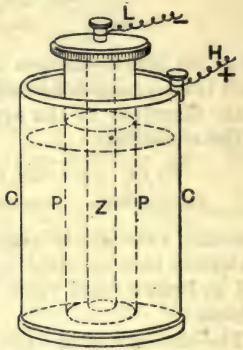


Fig. 162.

this stands a porous pot, P, *i.e.* a pot of unglazed earthenware through which liquids can slowly diffuse. In the porous pot is dilute sulphuric acid and in this a rod of amalgamated zinc, Z, supported by a heavily paraffin-waxed wooden lid which rests on the pot. Binding screws are attached to the zinc rod and copper pot, and constitute the low potential and high potential terminals of the cell respectively. Sometimes the copper pot is fitted near the top with a perforated shelf, on which rest

crystals of copper sulphate; these are covered by the copper sulphate solution, and as the latter becomes weakened by the action of the cell they dissolve, thus keeping the solution concentrated.

The action of the cell is as follows:—The acid and copper sulphate ionise, giving rise to positive hydrogen and copper ions and negative sulphions. The zinc exerts a solution pressure, its positive ions leave the zinc negatively charged. Under the influence of the electric field the zinc ions move through the acid in the pot, the hydrogen ions move through the acid into the pores of the pot and out into the copper sulphate, and the copper ions move through the copper sulphate to the copper plate where they are deposited and give up their charge. This charge then flows through the external wire to the negative zinc plate. The negative sulphions move throughout in the opposite direction. The action is represented in Fig. 163.

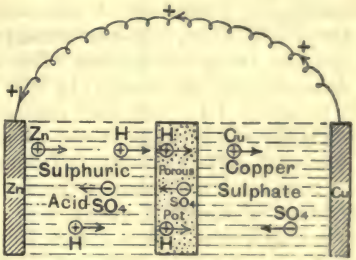


Fig. 163.

If the solution in the porous pot were evaporated to dryness, it would be found to contain zinc sulphate. The copper sulphate solution will be found to give an acid reaction, hence without pledging ourselves to any theory we could express the action by the two equations—

$$Zn + H_2SO_4 = ZnSO_4 + H_2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$H_2 + CuSO_4 = H_2SO_4 + Cu \quad . \quad . \quad . \quad . \quad . \quad (2)$$

When the cell has worked for some time the inner surface of the copper pot becomes coated with a dull red layer of finely divided but firmly adherent copper. *But no free hydrogen makes its appearance anywhere.* The depolariser is manifestly the copper sulphate, and the depolarising action is the action (2) which occurs at the porous pot, and consists in the substitution of copper for hydrogen.

Taking equations (1) and (2) together, it will be seen that the *total* action in a Daniell's cell consists in the replacement of Zn by Cu according to the equation—

$$\underset{\text{Zinc.}}{Zn} + \underset{\text{Copper-sulphate.}}{CuSO_4} = \underset{\text{Zinc-sulphate.}}{ZnSO_4} + \underset{\text{Copper.}}{Cu} \quad . \quad . \quad . \quad . \quad (3),$$

the H_2SO_4 playing on the whole no part whatever. The practical upshot of this is that we may do away with the latter, and in its place employ a solution of zinc-sulphate in the porous-pot; this is frequently done, and has the advantage that there is no danger of the zinc being eaten away when the cell is not in use.

As previously mentioned (§§ 164, 165) the E.M.F. of a Daniell's cell is about 1·1 volts, and remains remarkably constant, even though the cell be kept in use a long time; this is especially true of the zinc sulphate form. Taken size for size its resistance is high compared with that of most other cells; a pint size has a resistance of about 3 ohms, while that of a pint chromic-acid cell is only about $\frac{1}{4}$ ohm. But, like its E.M.F., it remains practically constant while working, so that it is the best cell we have for producing steady currents of no great strength. A specially constructed form is largely used in connection with the Post Office Telegraphs.

Daniell's cell is an example of "two-fluid" cells—that is, cells wherein two liquids are employed, separated by a porous partition. The simple and chromic-acid cells (§§ 157, 179), on the other hand, are "one-fluid" cells.

In practice it is found that the copper vessel often wears through. In the laboratory it is more convenient to make the external vessel of glass or earthenware and use a cylindrical plate of copper.

Exp. 122. *Set up a Daniell's cell.* The parts of a convenient laboratory form are shown in Fig. 164.

Z is the zinc plate, say 7″ by $1\frac{1}{4}$″ by $\frac{1}{8}$″. A hole is bored through it near one end just large enough to take a thick iron wire bent to the form *cdef*. This wire is soldered to the zinc as shown. While Z is still hot the top end should be covered with a thin coating of marine glue or bicycle cement. This prevents the acid creeping up to the joint.

P is a porous pot about 2″ wide, 7″ deep, and $\frac{1}{4}$″ thick. Z when placed within P is supported by the wire *cd*. The top of P for the first half inch may also be coated with marine glue.

C is a copper cylinder made out of sheet copper. The cylinder should be nearly closed, about 4″ in diameter and 6″ high. Solder to C a piece of thick copper wire, bent as shown, *ghk*.

J is an earthenware jar, about 5″ in diameter and 7″ high ; a 3-pound jam jar answers the purpose well.

(1) Make up some "Battery sulphuric acid." This consists of one of acid to ten of water by volume. Put sufficient water in a dish and gently

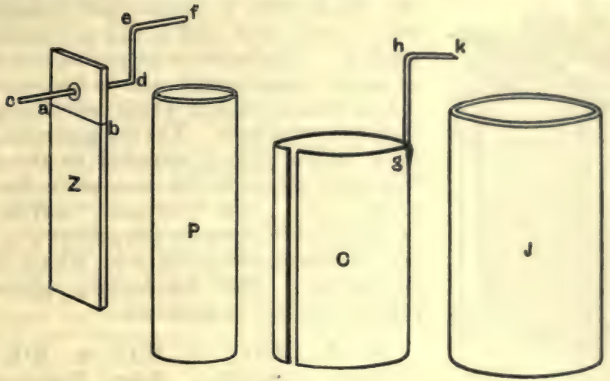


Fig. 164.

pour in the acid. Stir well. The mixture gets hot. Wait awhile, then pour in the rest of the acid.

(2) Make up the copper sulphate solution. Finely powder some copper sulphate crystals. Heat some water and stir in the powder. Stir well and leave it to cool.

(3) Clean by a file and emery paper the parts of the clamps and binding screws against which plates or wires are to be fixed.

(4) Amalgamate the zinc rod or plate. Three-quarters fill the porous pot with battery sulphuric acid, dip the zinc in until effervescence begins. Then remove it and dip the end into some mercury contained in a little dish. Amalgamation sets in. Take a piece of old duster and holding the zinc just over the dish, rub the mercury well over the zinc, and then rub off the excess. (Amalgamation should not be done too often ; it tends to make the zinc rotten.)

(5) Place now the zinc in the acid in the porous pot and stand the pot in the earthenware jar. Place the copper plate in the jar and fill up with the copper sulphate solution to half an inch from the top of the copper.

(6) Put connecting screws on the ends of the wires *ef* and *hk*. Fasten copper wires into these screws and the cell is now ready for use.

NOTE.—The constancy of a Daniell's cell during an experiment is better ensured if it is kept short-circuited for a little time before it is used.

When the cell is no longer required, it must be taken to pieces, the parts well rinsed with water (soak the porous pot for 12 hours), and set aside to dry.

181. Grove's Cell. This is shown in Fig. 165. It consists of a stoneware jar containing dilute sulphuric acid (one of acid to seven of water by volume), wherein is placed a bent plate of amalgamated zinc, Z Z. In the bend rests a porous pot containing strong nitric acid, and in this is a platinum plate, P. The latter is supported by a strip of wood* (not shown), and is fixed against it by a binding-clamp, which constitutes the high potential pole. The low potential pole is a similar clamp attached to the zinc. The main action is the same as in the preceding cells. The hydrogen "travels," as in §§ 177, 178, towards the porous pot, and here it meets the nitric acid, which is the depolariser. The depolarising action is represented by the equation —

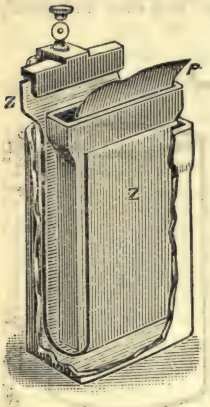


Fig. 165.

$$\underset{\text{Hydrogen.}}{H} + \underset{\text{Nitric acid.}}{HNO_3} = \underset{\text{Water.}}{H_2O} + \underset{\text{Nitric peroxide.}}{NO_2}.$$

The hydrogen is thus replaced by nitric peroxide. Now the latter is a gas, very soluble in strong nitric acid, and it accordingly dissolves therein, and does not appear on the platinum plate. After the cell has been in use some time more nitrogen peroxide is given off than the acid can dissolve. It escapes as dark brown fumes, which are very unpleasant.

The E.M.F. of a freshly fitted Grove cell is about 1·9 volts, and remains fairly constant for some time after closing the circuit. Its internal resistance is extremely low, that of the pint size being about $\frac{1}{8}$ ohm. The cell is, in fact, the best known for strong and fairly steady currents.

A Grove's cell should be taken to pieces at the end of a day's working, as the diffusion of the liquids through the porous pot would render the cell practically useless next day.

182. Bunsen's Cell. One form of this is shown in Fig. 166, but it is frequently made flat like the Grove's cell. It resembles a Grove's cell in all respects except its shape and in the fact that a rod of the hardest retort carbon is employed in the nitric acid instead of platinum. Its E.M.F. is 1·9 volts.

* Or, when a battery is used, by the zinc of an adjacent cell.

Exp. 123. Set up a Bunsen's Cell.

(1) Make some "Battery Nitric Acid." Strong nitric acid should be mixed with water until the specific gravity of the mixture is 1·38.

(2) and (3) same as (3) and (4) of Exp. 122.

(4) Put together the jar, porous pot, zinc and carbon plates. Fill, through a funnel, the porous pot, containing carbon, about three-quarters full, with strong battery nitric acid, and similarly the jar containing zinc with dilute battery sulphuric acid. Fix the binding screws. Fig. 169 shows a battery of four Bunsen cells joined in series.

As the working battery produces pungent gaseous oxides of nitrogen, it should be put in a fume closet away from the experimenter and apparatus.

(5) When the cell is no longer required, it must be taken to pieces, the parts well rinsed with water (soak the porous pot for twelve hours) and set aside to dry. Put the nitric acid back into the bottle labelled "Battery Nitric Acid"* and throw away the dilute sulphuric acid (it is contaminated with nitric acid). Put waste mercury from the battery vessels into the amalgamating dish.

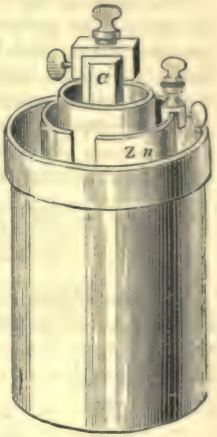


Fig. 166.

183. The Leclanché and Agglomerate Cells. These differ from all the preceding cells in the fact that the exciting liquid is not sulphuric acid, but a concentrated solution of ammonium chloride, so that the *main* action is different. The Leclanché is shown in Fig. 167. It consists of a glass bottle containing the ammonium chloride solution, and in this is a zinc rod (which need not be amalgamated) furnished with a copper wire and binding-screw (the latter not shown), which constitutes the low potential pole. The other plate is of gas coke, and the terminal attached thereto is the high potential pole. The depolariser is not a liquid, but a packing of black oxide of manganese and bits of carbon, the latter being added to prevent too great

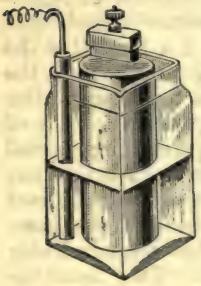


Fig. 167.

* When the Battery Nitric Acid has acquired a greenish blue colour it should be replaced by fresh acid.

compactness. The packing is contained in a cylindrical porous pot (preferably with a few small holes in its side), and the gas-coke plate is permanently embedded in it.

The agglomerate cell differs from the Leclanché in the absence of the porous pot; the depolarising mixture is made into compressed cakes, and the gas-coke plate is placed between two of these, which are pressed against it by india-rubber bands. This form of cell is superior to the Leclanché in having less resistance; in other respects it is electrically and chemically the same. The main action in either of the cells consists in the ammonium chloride attacking the zinc according to the equation—

$$\underset{\text{Zinc.}}{Zn} + 2\underset{\substack{\text{Ammonium}\\\text{Chloride.}}}{NH_4Cl} = \underset{\substack{\text{Zinc}\\\text{Chloride.}}}{ZnCl_2} + \underset{\text{Ammonia.}}{2NH_3} + \underset{\text{Hydrogen.}}{H_2} \quad . \quad . \quad . \quad (1).$$

The zinc chloride dissolves in the excess of ammonium chloride, forming a double salt; the ammonia dissolves in the water, while the hydrogen ion travels as far as the depolariser, when the following action sets in:—

$$\underset{\text{Hydrogen.}}{H_2} + \underset{\substack{\text{Black oxide of}\\\text{manganese.}}}{2MnO_2} = \underset{\substack{\text{Brown oxide of}\\\text{manganese.}}}{Mn_2O_3} + \underset{\text{Water.}}{H_2O} \quad . \quad . \quad . \quad (2).$$

The actual E.M.F. of a freshly prepared Leclanché or agglomerate is about 1·4 volts, and the internal resistance (especially of an agglomerate) is considerably less than that of a Daniell of the same size. On closing the circuit the E.M.F. rapidly falls, the reason being that the depolarising action (2) is feeble, and after a few seconds more hydrogen is liberated than the black oxide of manganese can effectually dispose of, thus permitting partial polarisation. On again opening circuit and allowing a little time, the action (2) goes on until the excess of hydrogen is all oxidised off and the cell is ready for a fresh start. These cells are therefore quite useless for long-continued currents, but are admirably adapted for intermittent ones of short duration. They have the great practical advantage that, when once fitted up and used for intermittent work only, they will last for many months (as many as 18) without attention. They are of great service for electric bells and telephone calls.

Dry Cells are modifications of the Leclanché, moist pastes being used instead of solution of ammonium chloride.

184. Primary and Secondary Cells. All the foregoing cells are called *primary* because they furnish the current, so to speak, direct from the resources of Nature without having to be previously "charged," as is the case with *secondary* cells (§ 198). There are many other primary cells besides those above described, but for operations on a large scale, such as electric lighting and traction, all primary cells are useless on account of their expense. The invention of a *really good and cheap primary cell* would be as important an event in the world's history as that of the steam-engine.

185. Batteries. The term *battery* is commonly applied to any combination of two or more cells. There are three classes of batteries—viz. *series, arc* (or *parallel*), and *compound circuit* batteries—which differ in the mode of arrangement of the cells. We shall deal only with the two former, of which the first is the most important; in fact, when a "battery" is spoken of it is in general understood to be a series battery.

We have already encountered the term series and parallel in relation to the arrangement of wires (§ 171), their meaning as regards batteries is similar, the cells of which the battery is composed taking the place of the wires.

186. Series Battery. The arrangement in this case will be

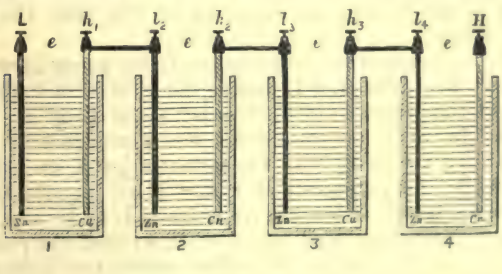


Fig. 168.

seen from Figs. 168 and 169, which represent a series battery on open and closed circuits respectively. The cells are depicted as simple cells in Fig. 168 and as Bunsen's cells in Fig. 169.

but it must be distinctly understood that the diagrams are *typical*: the cells may be of any kind whatever, and all our observations perfectly general. In practice it is usual to have all the cells alike, and we shall suppose this the case. In the

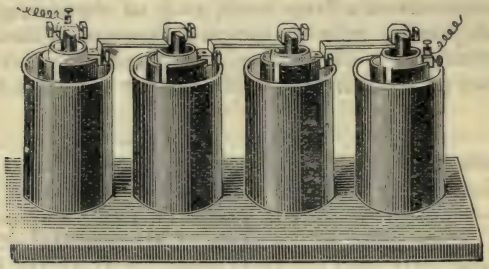


Fig. 169.

diagrams four cells (1, 2, 3, 4) are represented, but there may be any number. The connections are as shown. The H.P. pole h_1 of No. 1, Fig. 168, is connected by a short thick wire or any convenient clamp to the L.P. pole l_2 of No. 2, the H.P. of No. 2 to the L.P. of No. 3, and so on. This leaves the end poles, L and H, of the first and last cells free and these constitute the *terminals of the battery*, to which as in Fig. 169 the external circuit is joined.

Now, to understand the advantage of the series arrangement, let us study Fig. 168. Let E denote the E.M.F. of each separate cell. Then the potential of h_1 exceeds that of L by E. Since no current is flowing the potentials of h_1 and l_2 are equal by Poisson's Principle. In the same way the potential of h_2 exceeds that of l_2 by E, while the potentials of l_3 and h_2 are equal; and so on. Hence, as we travel in imagination from L to H, the potential makes four jumps up, each equal to E, so that the potential of H is 4E above that of L. In other words the E.M.F. of the battery is 4E. And in general if there be n cells, each of E.M.F. E, the E.M.F. of the battery is n E. This is a most important point, and is the one to which the practical value of a series battery is due. *We get increased electromotive force.* For example, with a single Grove, we can at best get only 1·9 volts,

but with twelve such series we get $1{\cdot}9 \times 12$, *i.e.* just on 23 volts.

The increase of E.M.F. obtained by connecting cells in series may be well shown by means of the condensing electroscope as follows:—

Exp. 124. Take a cell, and connect the terminals to the condenser as explained in Exp. 112: on lifting the earth plate, very slight if any divergence will be observed. Now join up two cells in series and repeat the operations; the divergence will be fairly distinct. Repeat again with three cells, the divergence will be still greater, and so on, until with five or six cells the divergence will be as great as if the cap of the electroscope had been beaten with fur, and a small spark may be obtained from it.

Suppose we now close the circuit of a series battery by a wire. The current flows along the external circuit, from the high potential pole to the low potential pole, and back again through the cells and connections as indicated by the arrows. In so doing it encounters the external resistance, and also the resistance of all the cells in succession. If B denote the resistance of each cell, and n the number of them, the resistance of all of them is clearly n B, and adding to this the resistance R of the external circuit we obtain—

$$\text{Total resistance} = \mathrm{n\,B} + \mathrm{R}.$$

Now we have already seen that if E denote the E.M.F. of each cell,

$$\text{Total E.M.F.} = \mathrm{n\,E}.$$

Hence if C denote the current we have by Ohm's Law

$$\mathrm{C} = \frac{\mathrm{n\,E}}{\mathrm{n\,B} + \mathrm{R}} \quad \ldots\ldots\ldots (1).$$

EXAMPLES:—1. Twelve cells are connected in series. The E.M.F. and resistance of each are 1·5 volt and 2 ohms respectively, and the circuit is completed by a wire whose resistance is $\frac{1}{100}$ ohm. Find the strength of the current, and compare it with that given by only one of the cells with the same external resistance.

(*a*) Total E.M.F. $= 12 \times 1{\cdot}5 = 18$ volts.

Battery resistance $= 12 \times 2 = 24$ ohms.

$\therefore$ Total resistance $= 24 + \frac{1}{100} = 24{\cdot}01$ ohms.

$\therefore$ Current $= \frac{18}{24{\cdot}01} = {\cdot}75$ ampere, very nearly.

(*b*) When a single cell is used the E.M.F. in the circuit = 1·5 volt and total resistance = $2 + \frac{1}{100} = 2{\cdot}01$ ohms.

$\therefore$ Current = $\frac{1{\cdot}5}{2{\cdot}01} = {\cdot}75$ ampere, very nearly.

2. Work the foregoing question, substituting an external resistance of 100 ohms for the resistance of $\frac{1}{100}$ ohm.

(*a*) Total resistance now = 24 + 100 = 124 ohms.

$\therefore$ Current = $\frac{18}{124} = 1{\cdot}45$ amperes, very nearly.

(*b*) Total resistance now = 2 + 100 = 102 ohms.

$\therefore$ Current = $\frac{1{\cdot}5}{102} = {\cdot}015$ ampere, very nearly.

After working these questions, the student will be struck with the fact that in the first example, where the external resistance is small, the current given by twelve cells is practically no stronger than that from a single cell, while in the second example, where the external resistance is big, it is very much stronger. Now, this is not a mere accident of the particular figures given, but a general principle; it can readily be deduced algebraically from (1), and the student should prove it in this way as an exercise. Practically, however, it is more instructive to view the matter thus: Comparing a single cell with the battery, when we connect n cells in series we multiply the internal resistance by n. If now the external resistance is very small, *it* makes very little difference to the total resistance of the circuit, so that we have very nearly *multiplied the total resistance by n*. But we have also multiplied the E.M.F. by n. Hence the ratio of the two is very nearly the same, and therefore by Ohm's Law the current is very nearly the same.

But if the external resistance is big, it is the chief resistance in the circuit; the internal resistance is a mere trifle in comparison with it, and whether it be multiplied by n or not is of little importance. Hence putting the cells in series *leaves the total resistance very nearly the same as if there were but one cell.* But it still multiplies the E.M.F. by n. Hence the ratio of the latter to the former is nearly n times as great, that is by Ohm's Law the current is nearly n times as great.

The general conclusion, therefore, is that *with small external resistance it is of no practical advantage to use a series battery, but with big external resistance it is a great advantage.*

The truth of this conclusion may be shown experimentally thus:—

Exp. 125. Take a battery of cells, and connect its poles to the terminals of the high and low resistance galvanometer of § 167. Set the galvanometer-switch on the button L, so as to throw the *low-resistance* coil into circuit. Then join up one cell, and notice the pointer-reading. Afterwards add on, in succession, two, three, four, etc., cells: *the reading will be pretty much the same in each case.* Now disconnect all the cells, set the galvanometer-switch on the button H, so as to put the *high-resistance* coil into circuit and repeat. *For every additional cell used there will be a decided increase in the pointer-reading.*

Fig. 170.

The attention of the student is now recalled to the conventional method of representing a voltaic cell. Fig. 170 shows nine cells in a series battery, H and L are the battery terminals and these are joined by a wire. The direction of the current is indicated by the arrows. Diagrams of this kind are easily drawn and are very useful.

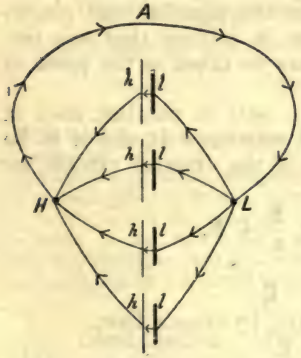


Fig. 171.

187. Arc or Parallel Battery. In this the cells must be all alike. Their arrangement is shown in Fig. 171, which represents a four-cell battery of this kind, the cells being drawn in the conventional manner. All the high-potential plates *h* are, by means of wires or suitable clamps, brought to a common terminal, H, and all their low-potential plates to a common terminal, L; H and L then constitute the terminals of the battery to which the external circuit H A L is joined. The course of the current is shown by the arrows; from H it travels through the external circuit, and on arriving at L divides, part going through each cell and the parts re-uniting at H. In an arc battery nothing is

gained in the way of E.M.F., *the* E.M.F. *of the battery is simply that of a single cell*; this may be shown as follows:—

In Fig. 171 imagine the wire H A L absent, and the other wires removed *temporarily*; we thus have four similar cells separate from one another. The potentials of all their high-potential poles are equal, and therefore, by Poisson's Principle, when the wires connecting them to H are restored nothing happens, and the potential of the common junction H remains equal to that of either of the high-potential poles to begin with. The same remark applies to L. Hence the potential difference between H and L is equal to that of h and l in either separate cell, *i.e.* the E.M.F. of the battery is equal to that of a single cell.

The only advantage of the arrangement is that it *decreases internal resistance*; if each cell has resistance B and there are n of them, the resistance of the whole battery is $\frac{B}{n}$. (See § 171.)

The n cells are thus by this arrangement practically converted into one cell of n times the size of a single cell.

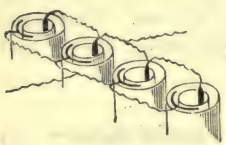


Fig. 172.

Fig. 172 shows four Daniell's cells joined up in parallel. Since all similar poles are connected together and practically made one, it follows that *any* two unlike poles may be taken as the poles of the battery.

If we have n cells in parallel, each of E.M.F. E and resistance B, and if R be the resistance of the external circuit, we have, therefore,

$$\text{Total Resistance} = \frac{B}{n} + R,$$

while, as already seen,

$$\text{Total E.M.F.} = E.$$

Hence, if C denote the current, we have, by Ohm's Law,

$$C = \frac{E}{\frac{B}{n} + R} = \frac{E}{B + nR} \quad \ldots\ldots\ (1).$$

The student should compare this with equation (1), § 186, and work through the following example, which makes use of the same cells and resistances as the worked examples on pp. 285, 286.

EXAMPLE.—(1) Twelve cells are connected in parallel. The E.M.F. and resistance of each cell are 1·5 volts and 2 ohms respectively, and the circuit is completed by a wire of resistance $\frac{1}{100}$ ohm. Find (*a*) the strength of the current in the wire and compare it with (*b*) that given by only one of the cells through the same external resistance.

(*a*) Total E.M.F. = 1·5 volts.

$$\text{Battery resistance} = \frac{2}{12} \text{ ohm.}$$

$$\therefore \text{ Total resistance} = \frac{2}{12} + \frac{1}{100} = \cdot 167 + \cdot 01 = \cdot 177 \text{ ohm.}$$

$$\therefore \text{ Current} = \frac{1 \cdot 5}{\cdot 177} = 8 \cdot 5 \text{ amperes, very nearly.}$$

(*b*) When a single cell is used

$$\text{Current} = \frac{1 \cdot 5}{2 \cdot 01} = \cdot 75 \text{ ampere, very nearly.}$$

2. Work the foregoing question, substituting an external resistance of 100 ohms for the resistance of $\frac{1}{100}$ ohm.

(*a*) Total resistance now $= \frac{2}{12} + 100 = 100 \cdot 17$ ohms.

$$\therefore \text{ Current} = \frac{1 \cdot 5}{100 \cdot 17} = \cdot 015 \text{ ampere, very nearly.}$$

(*b*) Total resistance now $= 2 + 100 = 102$ ohms.

$$\therefore \text{ Current} = \frac{1 \cdot 5}{102} = \cdot 015 \text{ ampere, very nearly.}$$

Note that the results found are the reverse of those found in the series arrangement.

Thus in the parallel arrangement, where the external resistance is small the current given by twelve cells is much stronger than that from a single cell, while, where the external resistance is big, the two are much about the same. Again, this is not a mere accident of the particular figures, but a general principle; the student should reason it out on lines similar to those adopted for the series battery.

188. Local Action. We have already seen (§§ 156, 157) that pure or amalgamated zinc is not attacked by dilute sulphuric acid unless it forms part of a voltaic cell on closed circuit, whereas common zinc dissolves in the acid at once. The explanation of the latter action is that the metallic impurities in the common zinc really constitute a large number of plates corresponding to the copper in the simple cell, so that we get a lot of little cells on short circuit. The energy furnished by the chemical action

is doubtless first converted into electrical energy, but this all fritters away as heat, and is unavailable in any other form.

When the zinc is amalgamated the impurities (iron, arsenic, etc.) remain undissolved, get loose, and become detached from the plate.

Suppose, now, we fit up any kind of cell, using impure unamalgamated zinc; the E.M.F. and external current are practically the same as if the zinc were pure or amalgamated, and to produce a given amount of electrical energy in the main circuit the same amount of zinc is dissolved. But in addition to the energy of the main circuit, a large amount of useless energy is produced in the small local circuits, and this entails the consumption of zinc to no useful end. The reason for amalgamating the zinc is, therefore, *not to augment the E.M.F. or main current*, but to save the zinc that would be wasted in producing local currents.

SUMMARY.—CHAPTER XII.

1. A simple cell is very inefficient; there is an accumulation of hydrogen on the copper (or platinum, etc.) plate, which produces a *back* E.M.F., thus causing the effective or actual E.M.F. to fall off. This phenomenon is called polarisation (§§ 175, 176).

2. Attend carefully to the ionisation theory (§ 178).

3. Attend carefully to the depolarising action of the several cells described in the text, and in particular to the chemical action in Daniell's cell (§§ 179-183).

4. Batteries, series, and arc. The object of a *series battery* is to obtain *increased* E.M.F. If E denote the E.M.F. of each cell, that of n cells in series is nE (§ 186). A series battery is a great advantage when working with *high external resistances*.

The only use of an *arc* battery is to obtain *diminished internal resistance*, its E.M.F. being merely that of a single cell. It is of advantage when working with *low external resistances* (§ 187).

5. Local action. The reason why common zinc dissolves in dilute sulphuric acid is on account of local electric circuits (§ 188).

EXERCISES XII.

1. What is approximately the E.M.F. of a series battery of 20 Leclanché cells?

2. Show that, if a number of cells of different kinds be connected in series, the E.M.F. of the resulting battery is the sum of those of the separate cells.

3. Four cells, each of E.M.F. e, are joined in series, and by accident one of them is put in the wrong way, that is its H.P. terminal is connected to the H.P. terminal of its neighbour. What is the E.M.F. of the whole battery?

4. Draw a diagram of six simple cells in series. What happens if the terminals of the arrangement be connected by a wire and left so for a while? How would you prove experimentally the truth of your statement?

5. A six-cell battery is taken on open circuit, the condensing electroscope experiment performed with it, and the divergence noted. The circuit is then closed by a wire of moderate resistance, and the experiment repeated. What difference, if any, will there be in the behaviour of the leaves? Explain your answer.

6. Find the strength of the current when a battery of E.M.F. 24 volts and resistance 36 ohms has its poles connected by thick wires to the terminals of a galvanometer of resistance 12 ohms. Find also the potential difference of the galvanometer terminals.

7. Three cells all alike are connected in series, but one of them is put in the wrong way. The battery is then short-circuited. Compare the strength of the current with that given by one of the cells only (supposed also on short circuit).

8. You have given two batteries, A and B. You place A in circuit with a galvanometer, and get a certain deflection. You then substitute B for A, and get a smaller deflection. How would you find out whether this was because B's E.M.F. was less than A's, or because its resistance was greater?

9. It is required to produce a current of 2 amperes in a wire of resistance 10 ohms. How many cells (E.M.F. 1·5 volt, internal resistance 0·5 ohm) must be placed in series to give this current?

10. How would you prove *experimentally*, by means of the condensing electroscope, that the E.M.F. of an arc battery is no greater than that of one of its cells?

11. Five cells, each of E.M.F. 2·1 volts and resistance 1 ohm, are arranged in parallel, and the poles of the battery thus formed are joined by a wire of 1 ohm resistance. Find the current in the wire, the current in each cell, and the difference of potential between the terminals of the battery.

12. How many cells, each of E.M.F. 1·5 volts and resistance 0·5 ohm, must be arranged in parallel in order to give a current of 6 amperes through an external resistance of 0·2 ohm?

13. A strip of platinum and a strip of zinc dip into a vessel of acidulated water. How would you show that the two copper wires, fastened one to the zinc and the other to the platinum, are in different electrical states?

14. How would the action of Daniell's cell be modified if the copper pot and copper sulphate were replaced by a zinc pot and zinc sulphate?

15. With a battery made up of many cells, how would you show that the electrical conditions of the terminals differ from each other, and that the extent of the difference is greater the greater the number of cells?

16. You have two batteries, each of six Leclanché's cells. In one the cells are small, in the other large. You try them each on open circuit with the condensing electroscope. Will there be any difference in the behaviour of the electroscope? If so, what? You next try them with a low-resistance galvanometer. What difference, if any, will there be in the pointer-reading? You lastly try them with a high-resistance galvanometer. Again, what difference, if any, will there be? Give reasons for all your answers.

17. Ten voltaic cells, each of E.M.F. 1·75 volts and resistance ·75 ohm, are joined up in series, and the circuit completed by a wire of resistance 12·5 ohms. Find (1) the strength of the current, (2) the terminal potential difference of the battery.

18. The poles of a voltaic cell are connected by a long wire. The wire is then cut and a galvanometer introduced into the circuit. Will it make any difference in the deflection of the needle according as the cut is made near the H.P. pole, near the L.P. pole, or at about the middle of the wire? Give reasons for your answer.

19. A simple cell is put in circuit with a multiple-coil tangent galvanometer, whose needle is observed to be deflected with its north pole eastward. It is left till the needle comes nearly back to the magnetic meridian. The zinc plate is then removed (care being taken not to disturb the copper), and replaced by one of platinum, and the circuit again completed. How will the needle behave? Give reasons.

20. Describe Leclanché's cell. Why does its E.M.F. diminish when it is short-circuited?

21. Describe the zinc sulphate form of Daniell's cell, and explain its action.

22. Two batteries A and B are made up of cells of the same kind arranged in series, A being composed of sixty and B of thirty cells. If the plates of a condensing electroscope are touched successively by the terminals of the two batteries and then separated, will the divergence of the leaves depend upon whether the order in which the two pairs of contacts are made is A B or B A? Give reasons for your answer.

23. Describe how you could make the leaves of an electroscope diverge from voltaic sources without the use of a condenser.

24. A battery of five cells, each having an E.M.F. of 2·1 volts and resistance $\frac{1}{2}$ ohm, is arranged in series and thoroughly insulated, the extreme plates being unconnected. What are the potentials of the free terminals (i) when the extreme zinc is earth-connected, (ii) when the extreme platinum is earth-connected, (iii) when the zinc of the middle cell is earth-connected, (iv) when there is no earth-connection?

CHAPTER XIII.

CHEMICAL EFFECTS OF THE CURRENT.

18°. Chemical Preliminaries:

(i) **Atoms and Molecules.** When a substance is reduced by any mechanical method, such as pulverisation, etc., it is supposed that the particles cannot be made smaller than a certain definite size. The smallest particles which might be so obtained are called molecules.

Each molecule is in its turn supposed to consist of one or more *atoms*, which may be atoms of the same kind, in which case the substance is called an element, or may be atoms of different kinds, in which case the substance is called a compound. Thus a molecule of silver consists of two atoms of silver, and silver is an element; a molecule of water consists of two atoms of hydrogen and one of oxygen, and thus water is a compound.

An atom of a substance is formally defined as being the smallest portion of that substance which can take part in a chemical (as distinct from a mechanical) operation.

The atomic weight of an element is the ratio of the weight of one of its atoms to that of an atom of hydrogen. Thus when we say that the atomic weight of zinc is 65, we mean that an atom of zinc is 65 times as heavy as an atom of hydrogen. It is not an easy matter to obtain the actual weight of a single atom, but it is a comparatively easy matter to find the ratio of the weight of an atom of one element to that of an atom of another element.

Each element has a recognised symbol; that for hydrogen is H, for oxygen O, for sulphur S. Each compound also has a symbol; that for water is H_2O, which signifies that *two* atoms of hydrogen (H_2) are combined with one of oxygen to make a molecule of water. A molecule of sulphuric acid consists of two atoms of hydrogen combined with one of sulphur and four of oxygen, and thus its symbol is H_2SO_4.

(ii) **Acids and Salts.** There are three acids of great importance in chemistry,* *viz.* :—

Sulphuric acid H_2SO_4, Nitric acid HNO_3, Hydrochloric acid HCl.

It will be seen that in each of these acids the molecule consists of one or two atoms of *hydrogen* united with an element or group of elements. This element or group of elements is called the *acid radicle*; in the case of hydrochloric acid the acid radicle is the element *chlorine* (*Cl*), in sulphuric acid it is the group SO_4 called *sulphion*, and in nitric acid the group NO_3 called *nitrion*. Neither sulphion nor nitrion can be obtained in the free state.

The characteristic property of these acids is that it is always possible either directly or indirectly to replace the hydrogen in them by almost any metal, and the resulting compound is called a *salt*. Salts derived from sulphuric acid are called *sulphates*, those from nitric acid *nitrates*, and those from hydrochloric acid *chlorides*. Subjoined is a list of the salts of chief importance in electricity:—

Zinc sulphate	$ZnSO_4$	Sodium Chloride (Common salt)	NaCl
Copper sulphate	$CuSO_4$	Mercuric Chloride (Corrosive sublimate)	$HgCl_2$
Ferrous ,,	$FeSO_4$	Ammonium Chloride Sal Ammoniac	$(NH_4)Cl$
Ferric chloride	$FeCl_3$		
Silver nitrate	$AgNO_3$		
Sodium sulphate	Na_2SO_4		

Zinc sulphate is derived from sulphuric acid by replacing H_2 by Zn, ferric chloride from three molecules of hydrochloric acid (3HCl) by replacing three atoms of hydrogen, 3H, by one of iron, Fe, and so on. *Each salt clearly consists of a metal combined with an acid radicle.* Salts are generically named according to the metal they contain; thus we speak of "zinc salts," "mercury salts," etc. *An acid may be regarded as a salt of hydrogen*, the hydrogen playing the part of a metal.

All the salts mentioned above are *soluble salts*, that is, they dissolve in water. There are some (*e.g.* barium sulphate) which do not, but with such we shall have no concern.

* There are also a vast number of others. The remarks of this chapter apply with modifications to all of them, but it is these three only that are of practical concern in relation to electricity.

(iii) **Chemical Equivalents.** If zinc is dissolved in sulphuric acid the zinc replaces the hydrogen so as to form zinc sulphate ($ZnSO_4$), and the hydrogen is expelled from the liquid in bubbles. This reaction is represented by the "equation"

$$Zn + H_2SO_4 = ZnSO_4 + H_2.$$

We here notice that one atom of zinc replaces two atoms of hydrogen.

An atom of some substances such as silver (Ag.) can only replace one atom of hydrogen. The number of atoms of hydrogen that can be replaced by an atom of any element is called the *valency* of that element.

The *chemical equivalent* of an element is the number of grams which are chemically equivalent to one gram of hydrogen, that is —it is the number of grams which will combine with or replace one gram of H in any of its compounds. It is evidently equal to its atomic wt. divided by its valency.

The following table should be learnt:—

Substance.	Hydrogen.	Oxygen.	Chlorine.	Silver.	Zinc.	Copper.
Symbol	H	O	Cl	Ag	Zn	Cu
Atomic Wt.	1	16	35	108	65	63
Valency	1	2	1	1	2	2

190. Chemical Action of the Current. Voltameters. Electrolysis. Look at Fig. 173. PQ Q′P′ is a jar or bottle containing a solution (in water) of *any salt*. In it are two platinum plates, A*h*, K*l*, fitted with binding screws.* These are connected to the high and low potential plates of a battery—a storage battery of two or three, or in some cases only one cell, answers admirably. The jar with plates, etc., is called a **voltameter**, and the plates are called **electrodes**, of which the one, A*h*, connected to the high potential terminal of the battery, is called the **anode**, or *positive*

* In practice it is convenient to fit a cork in the bottle and to have the platinum plates attached to brass rods passing through the cork, the binding screws being fixed on top of the rods. The cork should have a small hole in it for the escape of gas.

electrode, and the other, K*l*, attached to the low potential pole, the **kathode**, or *negative electrode*. When the circuit is complete as shown, a current of electricity flows from H through the leads and voltameter in the direction of the arrows, and so round again to L.

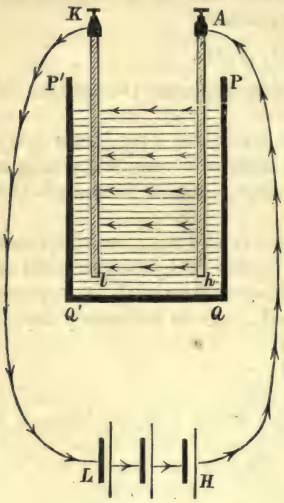


Fig. 173.

In the voltameter there is a down-gradient of potential through the liquid from the anode to the kathode, and the electricity flows down this gradient.* Now it is observed that when the current passes through the voltameter the dissolved salt through which it passes undergoes chemical decomposition, *the metal belonging to it being deposited on the kathode, or negative electrode.* For example, if we use a solution of copper sulphate we get metallic copper on the kathode, with zinc sulphate we get metallic zinc, with silver nitrate metallic silver, and so on. If we employ an acid, then hydrogen gas appears on the kathode, the hydrogen thus playing its part as a metal (§ 189). The positive carriers or ions which travel towards the kathode are sometimes called **kations** and the negative carriers or ions which travel towards the anode are called **anions**.

The process of thus decomposing a salt or acid by means of an electric current is called **electrolysis**, and the solution decomposed is called an **electrolyte**.

Exp. 126. *To electrolyse lead acetate.* Make a strong solution of lead acetate (sugar of lead) and immerse in it two platinum wires attached to the poles of a Grove's cell. Note the beautiful tree-like growth of lead on the kathode and the evolution of gas (oxygen) on the anode.

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* This may enable the students to remember to which plates the names *anode* and *kathode* are respectively applied. The term kathode is derived from the Greek κατά, meaning "down"; the electricity flows *down the gradient towards the kathode.*

Exp. 127. *To electroplate a wire with silver.* Repeat the above experiment, using a strong solution of silver nitrate, a silver wire as anode, and a copper wire as kathode. After a few minutes the copper becomes silvery-white, due to a deposit of silver. This is the principle of **electroplating,** discovered in 1800, and now a large industry.

Exp. 128. Dip the blade of a knife into a solution of copper sulphate. Immediately the blade becomes coated with a red deposit of copper. The action is—

$$\underset{\text{Sulphate of copper.}}{CuSO_4} + \underset{\text{Iron.}}{Fe} = \underset{\text{Copper.}}{Cu} + \underset{\text{Sulphate of iron.}}{FeSO_4}.$$

This is similar to the depolarising action of the Daniell's cell.

The deposition of metal or hydrogen on the kathode is in some respects the most important feature of electrolysis. But, as shown by Exp. 126, something happens at the anode as well. If the salt is a sulphate or nitrate, and the electrodes are of platinum, bubbles of gas appear on the anode, and these when collected and examined are found to be oxygen. Taking the case of a sulphate, there is every reason to believe that what is *first liberated* on the anode is the acid radicle sulphion, but that this reacts in presence of water according to the equation—

$$\underset{\text{Water.}}{H_2O} + \underset{\text{Sulphion.}}{SO_4} = \underset{\text{Sulphuric acid.}}{H_2SO_4} + \underset{\text{Oxygen.}}{O} \quad . \quad . \quad . \quad (1);$$

so that what *actually appears* is simply oxygen. This view is borne out by the fact that free H_2SO_4 gradually accumulates in the liquid.

In the case of a nitrate the action is similar: the salt first splits up into the metal and nitrion, and the latter then reacts with the water, forming nitric acid and oxygen thus:—

$$\underset{\text{Water.}}{H_2O} + \underset{\text{Nitrion.}}{2NO_3} = \underset{\text{Nitric acid.}}{2HNO_3} + \underset{\text{Oxygen.}}{O} \quad . \quad . \quad . \quad . \quad (2).$$

If a *chloride* be employed, free chlorine appears at the anode, but this gas being chemically very active attacks the platinum plate, forming platinum chloride; for the electrolysis of chlorides an anode of hard carbon (gas coke) should be employed.

Taking the case of copper sulphate, the entire action including the liberation of oxygen may be written:—

$$CuSO_4 + H_2O = Cu + H_2SO_4 + O \quad . \quad . \quad . \quad (3).$$

The appearance of the metallic constituent of a salt on one of the electrodes and of the acid radicle on the other is explicable by the aid of the ionisation theory (cf. § 178). Thus, consider the electrolysis of copper sulphate. The solution of copper sulphate contains undissociated copper sulphate molecules and also copper ions and sulphions which have resulted from the dissociation of copper sulphate molecules. The copper ions are positively charged and the sulphions negatively charged. Under the influence of the electric field due to the electrodes these ions are set in motion, the copper ions moving down the gradient towards the kathode and the sulphions moving up the gradient towards the anode. Arriving at the kathode the copper ion gives up its charge and is deposited on the surface. The sulphion arriving at the anode yields up its charge and then reacts with the water, forming sulphuric acid and liberating oxygen as shown in equation (1).

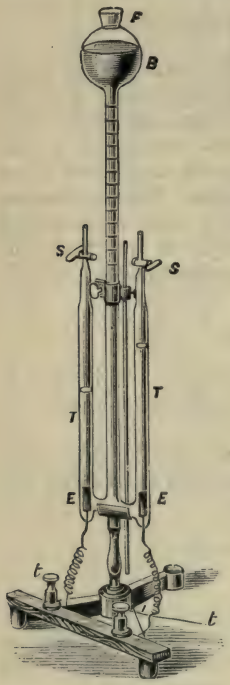


Fig. 174.

191. Electrolysis of Dilute Sulphuric Acid (or of Water). Fig. 174 shows a form of voltameter (Hofmann's) commonly employed for the electrolysis of dilute sulphuric acid, a case of electrolysis which is of special importance.

Two vertical glass tubes, T, T, communicate with one another near the bottom by a cross piece, and from this springs a longer tube furnished at the top with a reservoir, B. The reservoir and central tube being filled with acid, the cocks S are opened, when some of the acid rises in the tubes T, T, filling them and expelling the air. The cocks are then closed, and the electrodes E, E, connected by means of the terminals *t*, *t*, to the poles of a Grove's battery of two or three cells. The result of the electrolysis of sulphuric acid is that hydrogen appears at the kathode and oxygen at the anode; these gases collect in the respective

tubes. When a fair quantity of each has been obtained, if either cock be opened the weight of the acid in the central tube drives the gas out at the jet: the hydrogen can be identified by its property of burning when a light is applied, and the oxygen by re-kindling a glowing match or taper.

The explanation is as follows:—When the strong sulphuric acid is mixed with water a large number of the sulphuric acid molecules dissociate into positively charged hydrogen ions and negatively charged sulphions. The hydrogen is therefore given off at the kathode. At the anode the sulphion reacts as explained in § 190 and oxygen is given off as described above.

The effect of dissociation is

$$H_2SO_4 = H_2 + SO_4 \quad . \quad . \quad . \quad . \quad . \quad (1).$$

At the anode we get

$$SO_4 + H_2O = H_2SO_4 + O \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Combining (1) and (2) we get

$$H_2SO_4 + H_2O = H_2SO_4 + H_2 + O,$$

or deducting the H_2SO_4 molecule from each side we get

$$H_2O = H_2 + O,$$

so that the result of the two actions together is simply to produce the separate gases H_2 and O in place of H_2O, *precisely as if we had decomposed a molecule of water*; and accordingly the proportions of hydrogen and oxygen obtained in the tubes are the same as exist in water, two volumes of hydrogen to one of oxygen. [This is shown in the diagram; the left-hand tube contains the kathode, the right the anode.] On this account the electrolysis of dilute sulphuric acid is sometimes spoken of as the "electrolysis of water." But it can only be regarded as such in a fictitious sense, for if the sulphuric acid be omitted and we try the experiment with pure water no decomposition takes place. *Pure water is practically not an electrolyte.*

Exp. 129. *To decompose water by the electric current.* Join up four or five Bunsen cells in series. Connect platinum wires to the terminals and dip these wires into a beaker of water acidulated with sulphuric acid (1 of acid to 5 of water by volume). Note the evolution of gas at the surface of each wire: oxygen at the anode, hydrogen at the kathode.

Fig. 175 shows a form of voltameter very suitable for this experiment. E, E are platinum electrodes* connected by wires to the binding screws *t*, *t*. These are joined by copper wires to the battery, and the gases are collected by displacement in inverted test-tubes, T, T, held in place by a cross-piece C. (It is convenient to have these graduated into parts of equal volume.) Note that the volume of hydrogen is practically twice that of the oxygen. The identity of the gases may be verified by showing that the hydrogen burns with a pale blue flame if a lighted taper is held near the reinverted tube, while the oxygen rekindles a glowing match held similarly.

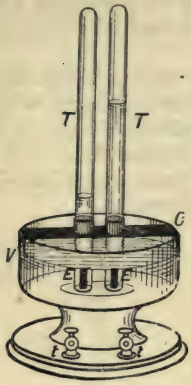


Fig. 175.

192. The Action at the Anode. The electrodes of a voltameter are not necessarily of platinum, although this metal is usually preferred because it is not attacked either by sulphion, nitrion, or free oxygen. But it is attacked by chlorine; hence, in electrolysing chlorides an anode of carbon is employed. The material of the kathode is of less importance because the metal liberated there is simply deposited on it, but even here platinum is practically the best for most purposes. When the anode is of a substance which is attacked by the acid radicle or by oxygen the action is then modified according to the nature and extent of the attack.

A simple case is afforded by the electrolysis of a solution of zinc sulphate with a pure zinc anode; here no oxygen is formed at all, the SO_4 simply attacks the zinc directly (either the zinc of the plate itself or the zinc ions which exert the "solution pressure," see § 178) and forms $ZnSO_4$, a molecule of $ZnSO_4$ being thus formed at the anode for every one decomposed at the kathode. Thus *the strength of the solution remains unaffected*, the kathode receives deposits of zinc and the anode is gradually eaten away, the ultimate result being just the same as if zinc had been bodily transferred from the anode to the kathode.

* To make an electrode take a strip of platinum (1″ by $\frac{3}{8}$″) and weld it to a piece of platinum wire 1″ long. Place the foil on a smooth thick iron plate resting on a brick, lay the end of the wire on the foil, direct the flame of a foot blowpipe on the foil and end of wire, and when these are white-hot hammer them together with a small hammer. The joint is easily made.

The same occurs in the case of copper sulphate with a copper anode *provided the copper on the latter be in a finely divided state*, in which case it is readily attacked by SO_4. The latter fact is well seen by electrolysing a solution of $CuSO_4$ with platinum electrodes in the ordinary way, and then reversing the current: the first part of the action deposits a finely divided layer of copper on the kathode; on reversing, the original kathode becomes the anode, and this finely divided layer of copper is eaten off, while a corresponding layer is deposited on the kathode, thus giving the appearance of bodily transference.

If an *ordinary* plate of copper is employed as the anode the action is a good deal more complicated; it consists mainly in some of the oxygen converting the copper into copper oxide, CuO, which forms a blackish coat on the plate.

193. Measurement of Current. Faraday's Laws of Electrolysis. There are various methods by which currents of electricity may be measured. Thus we may build up three scales of current dependent on

(1) the deflection of a magnetic needle (cf. Exp. 108),

(2) the amount of chemical action in a voltameter (cf. Exp. 109),

(3) the amount of heating in a specified wire (cf. Exp. 111),

and there is no *a priori* reason why these scales should be identical. In practice it has been found most convenient to use No. 2 as our basis.

Faraday, who was the first to investigate the quantitative measurement of current, set up a number of water voltameters in series, and passed a current through them for some time. He showed that the amount of gas liberated was the same in all the voltameters, even though they had plates of different metals of different sizes and shapes and solutions of different strengths. He then split the circuit at one point and made it go through two voltameters arranged in parallel. Here again he found that the sum of the quantities of gas evolved in the two voltameters in parallel was equal to the quantity of gas evolved at any of the voltameters in the series. From these he deduced the law that the amount of decomposition in a voltameter is proportional to the amount of electricity which passes through it. Faraday

showed next that the law was also true in the cases of metallic decomposition.

He next proceeded to join up different voltameters in series and showed that the amounts of the different elements deposited at the kathodes, or liberated at the anodes, were proportional to the chemical equivalent weights of these elements, *i.e.* the amounts of, say, hydrogen, oxygen, chlorine, zinc, silver, were proportional to 1, 8, 35, 32·5, 108. He then gathered his results into one law as follows:—

Faraday's Law of Electrolysis. *The quantity of a substance which separates out at an electrode of a voltameter is proportional to the quantity of electricity that has passed through the voltameter and to the chemical equivalent of the substance.*

So far Faraday has only dealt with quantity of electricity. He now defines a uniform current as a current which carries past any point in the circuit equal quantities of electricity in equal times, however small these times may be, and assumes that the intensities of uniform currents are proportional to the rates at which they liberate substances at the electrodes of voltameters.

Thus if a uniform current A deposits 1 gm. of copper in a given time, and a uniform current B deposits 2 gms. of copper in the same time, B shall be said to be twice A.*

The unit of current is the **Ampere**, and it is defined (§ 164) as the unvarying electric current which, when passed through a solution of silver nitrate in water, of certain strength, † etc., deposits silver at the rate of 0·00111800 gm. per second.

The unit of quantity of electricity is the *Coulomb*, and is the amount of electricity carried past any point of a circuit by a current of one ampere passing for one second. In §§ 161, 169 we used a tangent galvanometer to measure current. Having defined equal currents by the electrolytic method we shall now have to prove that the magnetic field at the centre of the tangent galvanometer coil is proportional to the current, or the tangent galvanometer method of measuring currents will not be accurate.

* This accords with the idea given in § 178 that the ions of any one element carry charges of the same magnitude and that the ions of different elements carry charges which are proportional to the valencies of those elements.

† The strength, etc., does not greatly alter matters, but it is better to be definite on such a point. The details will be found in the Report of the International Congress (see § 164).

The simplest way to do this is to connect a copper voltameter and tangent galvanometer in series with a battery and adjustable resistance. Adjust the resistance to keep the current so that the deflection of the needle remains constant say at an amount δ_1 and allow the current to flow for a given time. Weigh the deposit of copper, say w_1. Repeat the experiment, keeping the deflection constant at an amount δ_2, and passing the current for the same time. Let the amount of copper deposited be w_2. Then if

$$\frac{w_1}{w_2} = \frac{\tan \delta_1}{\tan \delta_2}$$

we are justified in our assumption that the magnetic field is proportional to the current which produces it.

Another even simpler way of proving this is to have three exactly equal tangent galvanometers and join them up as in Fig. 176 and pass a current. Since B and C are equal in all respects the current going through A is equally divided at D, half going through B and half going through C. If then δ_1 is the deflection of A, and δ_2 that of B (and C), and we find that $\tan \delta_1 = 2 \tan \delta_2$, we know that the magnetic field is proportional to the current which produces it.

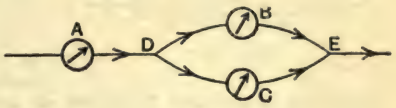


Fig. 176.

The weight of a given substance liberated per second by one ampere is called the **electrochemical equivalent** of the element. The electrochemical equivalent of silver is 0·001118 gm. per ampere-sec. or per coulomb. The electrochemical equivalents of some of the commoner elements are given in the following table. The reader should prove that the figures given are in accordance with Faraday's Laws.

Hydrogen . . .	·0000104
Oxygen . . .	·0000828
Chlorine . . .	·000368
Zinc . . .	·000336
Copper . . .	·000330

On our theory of electrolytic conduction the electricity passing through the liquid in the voltameter is actually carried by the

ions of the elements liberated. The second part of Faraday's Law is therefore another way of saying that the ionised atoms of all elements carry a charge proportional to their valency.

Faraday's Law holds for solvents other than water, also for fused solids and for gases.

Exp. 130. *Show that if a constant current is maintained through a water voltameter the volume of hydrogen or oxygen produced is proportional to the time during which the current flows.*

Join in series a two-cell Grove's or secondary battery (see §§ 184, 198), S (Fig. 177), a water voltameter, V, having graduated collecting tubes, a carbon rheostat, CR, and plug key, K (plug out). * It is advisable though not absolutely necessary to insert a tangent galvanometer in the circuit. It will indicate any variations in the current, and these can be counteracted by the rheostat.

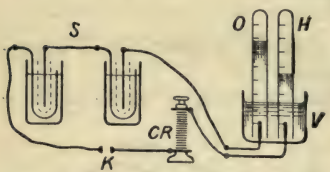


Fig. 177.

(1) Complete the circuit by pushing in the plug of the key, adjust the carbon rheostat until there is a rapid production of gas (say 5 cu. cm. of hydrogen per minute). Break the circuit. Refill the tubes with acidulated water.

(2) Place a watch where the seconds hand can be readily seen. Start the current through the voltameter just when the seconds hand is beginning a minute. Note the number of seconds that have elapsed when, say, 10, 15, 20, 25, 30, 35 cu. cm. of hydrogen have been collected, and 6, 9, 11, 14, 16, 19, 21 cu. cm. of oxygen † (If a tangent galvanometer is being used adjust CR to keep the deflection constant.)

Tabulate (i) volumes of hydrogen, (ii) time in seconds ; (iii) volumes of oxygen, (iv) time in seconds. Plot on the same sheet of squared paper (1) volumes of hydrogen with regard to time, (2) volumes of oxygen with regard to the same time axis. A graph like Fig. 178 will be obtained.

In each case the graph will be practically straight, and for any abscissa the ordinate of the hydrogen graph is twice that of the oxygen. Each graph passes through the origin. The results are thus in agreement with Faraday's law of electrolysis.

Exp. 131. *To show the effect on the current of conductors of different resistances.* 1. Join in series two Bunsen's or secondary cells, a water-voltameter, and plug key (plug out). Make a mark on a collecting tube five or six inches from the top. (Bind a piece of fine copper wire round it,

---

* Note that the direction of current through the circuit may be inferred by identifying the gas liberated at either electrode.

† The readings for oxygen can thus be taken between those for hydrogen.

or a graduated tube may be used.) Fill the voltameter and collecting tube with the electrolyte (water, 5 by vol., sulphuric acid, 1). Place the tube over the kathode. Start the current through the voltameter at a definite time. Note the number of seconds taken for the tube to fill to the mark with hydrogen. Then stop the current.

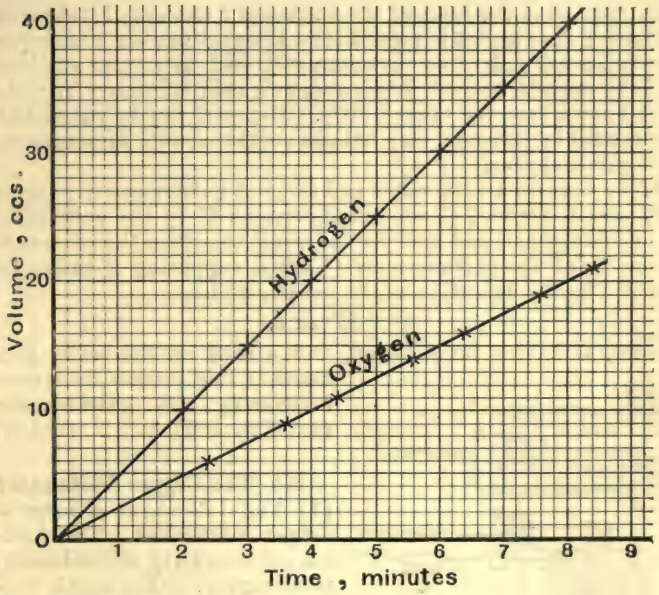


Fig. 178.

2. Refill the hydrogen tube with the electrolyte. Place a length of German silver wire (say a yard of No. 20 S.W.G.) in series with the battery, voltameter, etc. Again observe the time taken to collect the same amount of hydrogen as before.

3, etc. Repeat with wires of equal length and diameters of platinoid, manganin, etc.

The times taken to collect the same quantity of hydrogen will be different; the shorter the time the greater the current. The experiment shows that the strength of the current depends on the conductors in the circuit, some of these offer more resistance than others and less current flows.

194. Faraday's Law applied to the Measurement of Current. If a current of C amperes pass for t seconds through a solution of a salt of a metal whose electro-chemical equivalent is w, and if W denote the weight of the metal liberated, we have clearly

$$\mathbf{W = C\,w\,t.}$$

This relation is the basis of the voltametric method of measuring currents; the current is passed for a known time, say 30 minutes, through a solution of a convenient salt, such as copper sulphate, and the deposit washed, dried, and weighed; this gives W; also w is known (see the table § 193). Hence C can be calculated: the process is, however, very tedious compared with measurement by a galvanometer.

If we connect a voltameter and tangent galvanometer in series and pass a current, C, we can determine C by the voltameter, as just explained, and may use the result to obtain the reduction-factor, K, of the galvanometer (§ 169), supposing it unknown. For if α be the deflection of the needle, we have

$$C = K \tan \alpha,$$

whence, C having been found, and α read on the scale, K is known: this process is termed *calibrating* the galvanometer (see Exp. 145).

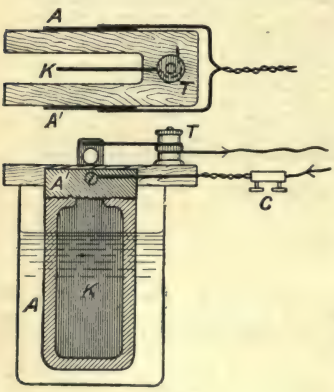


Fig. 179.

195. The Copper Voltameter. (1) Fig. 179 shows a form of copper voltameter. The kathode, K, is a strip of moderately thin copper. The anode consists of two thick copper strips, A, A', fixed to a wooden fork. A copper wire is soldered to each strip, the ends are bent round the fork and twisted together. A connector, C, joins the anode with the + ve pole of the battery. A stiff wire is also fixed to the kathode; the end of this is clamped by the terminal, T, so that K is supported between A and A'. (In the lower figure A' is only partly drawn in order to show the position of K.) The wooden fork rests on

the top of a wide-mouthed earthenware jar or glass, into which the electrolyte is put. The copper plate is rounded at the corners and edges.

(2) The *copper plates.*—(i) Before use thoroughly clean* the copper plates with glass paper. Then dip them for, say, half a minute in moderately strong nitric acid (conduct the operation in a fume closet), lift out and immerse in a jug of tap water, finally rinse in distilled water. Do not touch the plates with the fingers; always interpose a doubled strip of paper when it is necessary to handle them. Hold the kathode in the warm air two or three feet above a Bunsen burner till dry; then allow it to cool and weigh it. (ii) When the kathode is removed from the copper sulphate solution, plunge it at once into water very slightly acidulated with sulphuric acid (this prevents oxidation of the copper), then immerse it in a jug of tap water, finally rinse with distilled water, drain, and lay it on filter paper. Dry and weigh again.

The area of the kathode surface must not be less than 25 sq. cms. (it is well to allow 50 sq. cms.) per ampere passing; if it is less, the copper adheres to the plate poorly.

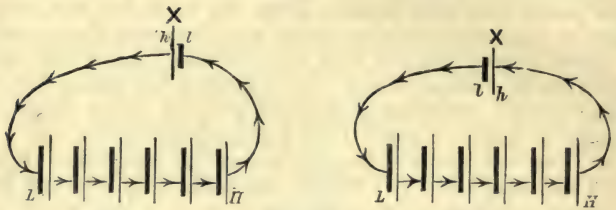
(3) *The electrolyte* is prepared by adding one per cent. by volume of strong sulphuric acid to a solution of density 1·18 (about) of recrystallised copper sulphate in tap water.

The copper voltameter is often used to find the constant of a tangent galvanometer (see Exp. 145, which may be performed at this stage).

196. Battery Cells in Opposition. In Fig. 180, No. 1, the cell X is in series with the battery, the high potential terminal of the latter being connected to the low potential terminal of X, and *vice versa.* But now consider No. 2: here X is said to be in *opposition* with the battery, the connections being made the other way—that is, the high potential plates are joined together, and likewise the low potential. When X is in opposition its E.M.F. goes *against* that of the battery, and not only so but *the natural action of* X *is reversed*; it behaves not like a cell, but like a voltameter. If, for example, it were a simple cell with zinc and platinum plates, and if there were already some $ZnSO_4$ in the solution, instead of

* Unless well cleaned the copper deposit will not adhere. There should be no trace of brown oxide on either side.

the sulphuric acid dissolving more zinc, some of the $ZnSO_4$ would become electrolysed, metallic zinc appearing on the zinc plate (the kathode), and oxygen on the platinum (the anode). Or if the



No. 1. Fig. 180. No. 2.

cell X were a Daniell of the "zinc-sulphate form" (§ 180), copper would be eaten off the copper pot and zinc deposited on the zinc rod.

If in place of a single cell X we have another battery, the same kind of thing occurs; the battery of higher E.M.F. performs its natural action, and the one of lower E.M.F. its reversed action. If both batteries have the same E.M.F. no action occurs in either of them.

197. The Voltameter as a Storehouse of Energy. In a voltaic cell the chemical action resulting in the *formation* of the zinc sulphate or other salt is a *source* of electrical energy. In a voltameter precisely the reverse is the case: the *decomposition* of the salt entails an *absorption* of energy from the electric circuit which is converted, not into heat, but into chemical potential energy; in other words, a voltameter is a *sink* of electrical energy. Now we should expect that the energy thus sunk in the voltameter might again be utilised to produce an electric current, and that this is so the following experiment will prove:—

Take the voltameter, shown in Fig. 173, and electrolyse a solution of some salt, say copper sulphate; the chemical action which occurs is

$$CuSO_4 + H_2O = Cu + H_2SO_4 + O \quad . \quad . \quad . \quad . \quad (1).$$

After the operation has been going on for a short time there will be on the kathode a deposit of finely divided copper, and on

the anode some minute bubbles of oxygen. Now detach the battery and connect the voltameter to a delicate galvanometer. The needle will indicate a current in the reverse direction to the arrows of Fig. 173; the voltameter, that is to say, is now *acting as a voltaic cell*, in which what was the anode is the high potential plate, and what was the kathode is the low potential plate, the chemical action which occurs being

$$Cu + H_2SO_4 + O = CuSO_4 + H_2O \quad . \quad . \quad . \quad (2),$$

which is precisely the reverse of (1).

It thus appears that *a voltameter is a storehouse of energy*; during electrolysis energy is absorbed from the current, and stored in the potential form, and afterwards, when the voltameter is used as a cell, the chemical action is reversed and the stored energy given back in the form of the electric current.

198. Storage Cells. An ordinary voltameter, though, as just explained, a storehouse of energy, is a very inefficient one, because nearly all the oxygen escapes as fast as it is generated. But in the year 1860 M. Gaston Planté devised an arrangement whereby the storage could be rendered much more effectual, and his invention has since developed into the modern "accumulator," or "secondary," or "storage" cell, which is a valuable adjunct to the electrical engineering industry. There are many types of such cells, but they are all essentially the same, and it will suffice to describe one, *viz.* the so-called L type manufactured by the Electric Power Storage Co.

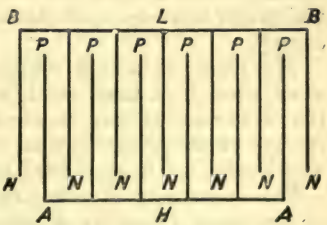


Fig. 181.

In this type of storage cell the plates consist of leaden grids, each measuring about $8\frac{1}{2}$ by $9\frac{1}{2}$ inches, and $\frac{3}{16}$ inch thick; and as they issue from the factory the apertures of the grids are filled with a firmly adherent paste of litharge (PbO) or red lead (Pb_3O_4). The arrangement is shown in horizontal section in Fig. 181. A set of grids, P P P P P P, are electrically connected into one nest by a leaden lug, A A, with a terminal, H, attached, and another set,

N N N N N N N, are similarly connected by another lug, B B, with a terminal, L, attached; the former set is designed to form the positive, the latter the negative plate of the cell.* The two sets are interlocked as shown, the P's being insulated from the N's by strips of ebonite or other suitable material, and the whole immersed in a vessel of dilute sulphuric acid. The reason for this arrangement in preference to a pair of simple plates is that the resistance of the cell is rendered much lower—with the larger cells it is as little as $\frac{1}{500}$ ohm.

In order to prepare the cell for use it has to be first treated as a voltameter, and this operation is known as "charging" it. The terminal L is connected with the negative, and H with the positive pole of a dynamo,† when the sulphuric acid undergoes electrolysis just as in the apparatus, Fig. 174. But the hydrogen liberated on the plates N, instead of escaping, reduces the PbO or Pb_3O_4 to the condition of *metallic lead in a spongy state*, while the oxygen, instead of escaping from the plates P, oxidises them to the condition of *peroxide of lead* (PbO_2). The cell can now be detached from the dynamo and put away, and as long as it is kept on open circuit nothing further happens. But when the circuit is closed chemical action takes place between the materials on the two plates in presence of the sulphuric acid, and this action is accompanied by the production of an electric current. The dynamo thus charges the cell with chemical potential energy ‡ which can be stored and utilised for the reproduction of electrical energy.

The chemical action which produces the current is not the exact reverse of that which was produced by the current from the dynamo. It may, however, be regarded as *essentially* the reverse of the earlier action, though certain complications arise which need not be discussed here.

The operations of alternately charging and discharging may be continued a number of times until the pastes break away from the grids, or the latter themselves fall to pieces.

* The actual number of grids in each nest depends upon the size of the cell, but there is always one more negative than positive.

† A dynamo is a machine which produces very powerful electric currents: see § 238. For most commercial purposes batteries of primary cells are useless as current producers.

‡ The student should guard against the popular error that the cell is charged with *electricity*.

The E.M.F. of these cells is about 2 volts each, and on account of their extremely low resistance they can be made to furnish powerful currents and possess high efficiency; in practice, however, it is not advisable to work them too hard, or there is danger of the active material being forced off the grids by the violence of the chemical action, and of the plates buckling and short-circuiting: the cells are then said to be "over-discharged."

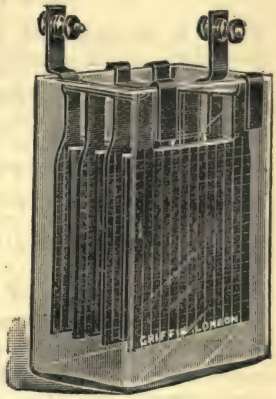


Fig. 182.

Fig. 182 shows a form of secondary cell in common use. The outer vessel is made of glass or earthenware. The positive terminal (attached to the more highly oxidised or dark brown plate) is painted red, the negative terminal (attached to the slate-coloured plate) is painted black.

The fact that secondary cells can be charged and discharged many times gives them a great advantage over primary in respect of cheapness, but their weight and liability to damage by jolting render them unsuitable for many purposes.

Exp. 132. *Make a secondary cell.* Immerse the lower ends of two strips of lead in dilute sulphuric acid.* Join the upper ends to the terminals of a battery (two of Bunsen's cells in series). Note that as the current passes the anode oxidises (it acquires a brownish appearance), and hydrogen bubbles off from the other plate. So far the arrangement acts like a voltameter. Now disconnect the Bunsen battery from the cell and connect the latter up to a galvanometer. Observe the deflection of the needle and deduce the direction of the current. It will be found that the current from the secondary cell passes around the external circuit from the oxidised plate to the unoxidised one. Thus the current from the secondary cell on discharging passes through the cell in an opposite direction to that of the charging current from the Bunsen battery.

199. Back E.M.F. in a Voltameter. As pointed out in § 191, the decomposition of the salt in a voltameter entails an absorption of energy. This absorption is accompanied by an E.M.F. tending to oppose that of the applied battery; this E.M.F. is called the *back E.M.F. of the voltameter.*

* 1 volume of strong sulphuric acid to 5 volumes of distilled water.

For example, consider the electrolysis of zinc sulphate with platinum electrodes. The main action *in a voltaic cell* is

$$Zn + H_2SO_4 = ZnSO_4 + H_2 \quad . \quad . \quad . \quad . \quad (1)$$

which is accompanied by a *production* of energy. Now the *electrolytic* action consists of two parts, *viz.*

$$ZnSO_4 = Zn + SO_4$$

and

$$SO_4 + H_2O = H_2SO_4 + O,$$

and these are together equivalent to the action

$$ZnSO_4 + H_2O = Zn + H_2SO_4 + O \quad . \quad . \quad . \quad (2)$$

Now, the action (2) may be regarded as consisting of two items, *viz.*:—

(*a*) The severance of SO_4 from Zn, and its union with H_2. This is the exact reverse of (1), and entails an absorption of energy equal to that which (1) would produce.

(*b*) The severance of H_2 from O. This entails an additional absorption of energy.

The item (*a*) corresponds to a back E.M.F. equal to the direct E.M.F. of a freshly fitted up simple cell on open circuit, which is about ·8 volt. The item (*b*) corresponds to an additional back E.M.F. of 1·47 volts. Thus the total back E.M.F. of a zinc sulphate voltameter with platinum electrodes is about 2·27 volts.

Similar reasoning applies to all voltameters, though the amounts of energy corresponding to the several actions are different and the back E.M.F.'s are different. The back E.M.F. of a copper sulphate voltameter is about 1·17 volts and of one with dilute sulphuric acid 1·47 volts.

Next consider a zinc sulphate voltameter with zinc electrodes. The action at the kathode is

$$ZnSO_4 = Zn + SO_4 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This entails an absorption of energy, and produces a back E.M.F.

The action at the anode, as explained in § 192, is

$$SO_4 + Zn = ZnSO_4.$$

This is the precise reverse of (3); it produces just as much energy as (3) absorbs, and is accompanied by an equal *direct* E.M.F. On the whole, therefore, such a voltameter has *no*

E.M.F.; it absorbs no energy other than in the conversion into heat. Thus, while voltameters in general are "alive," a zinc-sulphate voltameter with zinc electrodes is "dead,"—it merely offers *resistance* just as a wire would do.

Analogous actions occur in a copper-sulphate voltameter with copper plates, provided the metal on the anode be distributed in a finely divided form. With an ordinary copper anode the attack of the SO_4 on the Cu is (§ 192) incomplete, and there is an uncertain back E.M.F. which, however, is much less than with a platinum anode. By adding sulphuric acid to the solution, as advised in § 195, the copper oxide is more effectively dissolved to copper sulphate, and this comes to the same thing as if the SO_4 attacked the Cu directly; the back E.M.F. is in this way diminished, and at the same time the strength of the solution is more equably preserved.

But suppose now the battery has a less E.M.F. than the voltameter. Unlike the back E.M.F. of the cell X in Fig. 180, No. 2, that of a voltameter has no independent existence, and can never exceed the direct E.M.F. which calls it into play. But we have just seen that the back E.M.F. of a given kind of voltameter is a certain number of volts. What, then, is the meaning of the apparent contradiction? Simply this:—When we say that the back E.M.F. of a zinc-sulphate voltameter is 2·27 volts, we mean the back E.M.F. *when the electrolysis is going on.* It might appropriately be called the "maximum back E.M.F." If, now, we apply to a voltameter a battery whose E.M.F., E, is less than this maximum back E.M.F., E′, then the whole of E′ is not called into play. We might, however, expect that sufficient back E.M.F. would be called into play to balance E, and that no current would start until E was so far increased as just to exceed E′. And it is perfectly certain that there is no *electrolysis* until this is the case.

Thus the back E.M.F., E′, of a zinc-sulphate voltameter with platinum electrodes is 2·27 volts, and if we put on to it one Daniell (E = 1·1 volt), we get not a trace of copper deposited. In like manner with one Daniell on a dilute H_2SO_4 voltameter (back E.M.F. = 1·47 volt), we get no liberation of gas. But, notwithstanding this, if we put a delicate galvanometer in the circuit we find that when the cell is first put on there is a direct current (*i.e.* a current in the direction the cell would naturally send if the voltameter were absent). The current lasts only for

a few seconds, during which it rapidly weakens, and soon becomes very nearly *nil*. This somewhat perplexing phenomenon has been investigated and explained by Helmholtz; it is, however, needless to discuss it here: the point of real importance is that stated above—*viz.* that *no electrolysis* can occur unless the E.M.F. applied to the voltameter exceed its "maximum back E.M.F."

If several voltameters be joined up in series, the back E.M.F. of the whole arrangement is the sum of their separate back E.M.F.'s; hence to start electrolysis it would be necessary to employ a battery of E.M.F. greater than the said sum.

An electromotor* also possesses a back E.M.F., but this follows altogether different laws from that of a voltameter: it depends upon the speed at which the machine is driven. At very slow speeds it is very small, and theoretically the very least applied E.M.F. would start the motor; practically, however, this is by no means true, on account of the load and of friction.

SUMMARY.—CHAPTER XIII.

1. When a salt undergoes electrolysis, *the metal always appears at the negative electrode or kathode*—that is, the one connected with the L.P. pole of the battery; what appears at the anode is in general *oxygen*, the latter being the result of a secondary action between the electro-negative radicle (*e.g.* SO_4) and H_2O (§ 190).

The equation for the electrolysis of copper sulphate including the secondary action is

$$CuSO_4 + H_2O = Cu + H_2SO_4 + O. \qquad (\S\ 190.)$$

2. Electrolysis of water (§ 191).

3. Measurement of current by the voltameter. Faraday's Laws of Electrolysis (§§ 193, 194, 195).

4. When two cells or batteries are joined in opposition the one of higher E.M.F. performs its natural, and the one of lower its reversed, action, and the total E.M.F. is the *difference* of the separate E.M.F.'s (§ 196).

5. Construction and action of Storage cells (§ 198).

6. Back E.M.F. in a voltameter (§ 199).

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* See § 238.

EXERCISES XIII.

1. Write down the equation for the electrolysis of dilute sulphuric acid in a form similar to the equation on p. 314 for copper sulphate.

2. An electric current (which is the same in all the parts of the trough) flows horizontally in a trough filled with a solution of copper sulphate. A rod of copper is then supported horizontally in the trough, with its length parallel to the direction in which the current is flowing. How will the rod be affected by the current?

3. It being given that the chemical equivalents of silver, copper, and iron (ferrous) are 108, 31·5, and 28 respectively, calculate their electro-chemical equivalents.
Also calculate the electro-chemical equivalent of zinc and of ferric iron.

4. A current of 1·5 amperes is passed for 10 minutes through a solution of zinc sulphate; find how many *grammes* of zinc are liberated.

5. A current is passed through a solution of copper sulphate, and in $\frac{1}{4}$ hour 3 grammes of copper are liberated; find the strength of the current.

6. There are three voltameters, A, B, C. A contains silver nitrate, and a current of $\frac{1}{2}$ ampere is passed through it for 10 minutes. B contains copper sulphate, and 1 ampere is passed for 15 minutes. C contains dilute sulphuric acid, and 4 amperes are passed for 5 minutes. Compare the weights of silver, copper, and hydrogen liberated.

7. An electric current traverses a tangent-galvanometer and a copper voltameter arranged in series. The deflection of the galvanometer needle is 30°. To what extent would this deflection and the rate of deposition of copper in the voltameter be affected, if the current were made three times as strong?

8. The platinum and copper plates of a Grove's and a Daniell's cell are connected by a wire. Would there be a current if the zinc plates were also connected, and if so in what direction would it flow?
What reason have you for your answer?

9. A single Grove's cell is connected in opposition with four Daniell's cells in series. State and explain what happens.
Also taking the E.M.F. of a Grove's cell as 1·9 volts, and that of a Daniell's cell as 1·1 volts, and the resistance of the whole circuit as 10 ohms, find the strength of the current.

10. A storage cell with a single pair of plates connected by a wire of 0·8 ohm resistance gives the same current as a similar cell with plates twice as broad, twice as deep, and twice as far apart, which are connected by a resistance of 0·9 ohm. Find the resistance of each cell.
Why are the plates of a storage cell usually of large surface?

11. Six zinc sulphate voltameters are joined up in series. Find the least number of (1) Daniell's cells, (2) Grove's cells (in series) that would be capable of effecting electrolysis in them.

12. An electromotive force increased gradually from 0·25 to 2·5 volts is applied between two platinum wires dipping into a vessel filled first with mercury and afterwards with dilute sulphuric acid. Describe the experiments you would perform to show the difference between the phenomena observable in the two cases.

13. The zinc and copper terminals of a Daniell's cell are connected to two platinum plates which dip in a solution of copper sulphate. Describe carefully what happens at each of the platinum plates, and give the chemical equation for the action.

14. In what way will the action in the preceding question be modified (if at all) if the two plates be of copper?

15. You have access to the terminal wires of a hidden battery. Explain how you would tell which wire was connected with the zinc and which with the platinum pole of a battery, by observing what happened when the wires were connected to the terminals of a voltameter containing a solution of silver nitrate.

16. Two copper plates of the same weight are connected, one with the positive (*i.e.* H.P.), and the other with the negative (L.P.) pole of a voltaic battery, and immersed side by side in a solution of copper sulphate. If after some time the plates are removed, dried, and reweighed, they are found no longer to weigh alike. Account for this, and explain how by the continued action of the current the equality of weight could be re-established.

17. A vertical partition of porous earthenware is fitted into a tumbler, and dilute sulphuric acid is poured into each compartment. Rods of *common* zinc and copper are placed respectively in the two compartments, and connected by a wire. State what will be observed with regard to the evolution of gas, and how the observed phenomena will be modified when copper-sulphate solution is poured into the compartment containing the copper rod.

18. It is intended to set up one hundred Grove's cells in series, and by mistake ten cells are arranged in opposition to the rest: what is the relation of the potential-difference of the terminals on open circuit to what would have been obtained if the mistake had not been made?

19. Two galvanic cells are made of exactly the same materials, but in one cell the plates are much larger than in the other. What would be the effect of introducing both into a circuit so that they tend to send currents in opposite directions? Give reasons for your answer.

20. Two cells of E.M.F.'s 1·4 and 2·1 volts, and resistances 1 and 2 ohms respectively, are to be arranged in series so as to send a current through a tangent-galvanometer of 4 ohms resistance. Make a diagram showing all connections. Find the current. What would have been the value of the current if one of the cells had been wrongly connected up?

21. Describe the chemical processes that occur during the action of a Daniell's cell.

If the dilute sulphuric acid usually in the cell were replaced by a strong solution of sulphate of zinc, what would be the effect of driving a current through the cell in the direction opposed to its E.M.F.?

22. Describe some form of secondary battery. State (1) how you would charge it, (2) which would be the positive pole.

What are the advantages and disadvantages of such a battery as compared with a Leclanché's cell?

CHAPTER XIV.

ELECTROMAGNETISM.

200. Preliminary Geometrical Considerations. Before entering upon the subject of this chapter there are two points of a purely geometrical character which it is necessary to understand thoroughly; these we shall deal with in the next two articles.

201. Rotation: Clockwise and Counter-clockwise. It is frequently necessary in dealing with rotary motion to distinguish *the way a thing goes round.* When it does so like the hands of a clock, the movement is said to be *clockwise,* and when the opposite way, *counter-clockwise.* It should be noted that these terms imply that we are looking at the thing from some agreed point of view, *e.g.* in Fig. 137 the current as indicated by the arrows is going round clockwise when we view the instrument *from the front* as shown, but if viewed from the back it would be going counter-clockwise.

The student is cautioned against the phrases "right to left" and "left to right," sometimes employed to distinguish rotation, as they are very confusing and usually quite meaningless; the hands of a clock, for example, go from left to right over the top of the face, and from right to left over the bottom; in fact, these phrases, *when applied to rotation,* have no meaning whatever.

202. Spirals: Right-handed and Left-handed. The term spiral or helix is applied to a piece of wire or other material coiled after the manner of an ordinary spring. Now, there are two kinds of spirals distinguished by the above names and possessing an essential difference.

Exp. 133. *Make a right-handed spiral.* Hold a pencil, A B (Fig. 183), upright in the left hand and taking a piece of thin copper wire press one end between the thumb of the left hand and the bottom front, A, of the pencil. Then holding the rest of the wire with the other hand move the hand first to the right, then around behind the pencil and then across the front to the right hand, completing one turn of the spiral, the free part of the wire now being above the fixed end. Proceed in the same way to wind the spiral (Fig. 183); the arrow shows the direction of the winding. Finally withdraw the pencil.

Exp. 134. *Make a left-handed spiral.* Hold A B as before, but now press the end of the wire to the back of the bottom of the pencil and move the hand holding the free end of wire to the right, front, left, back, right, etc., in succession, making as below the second turn above the first. Proceed for some distance. A left-handed spiral results (Fig. 184).

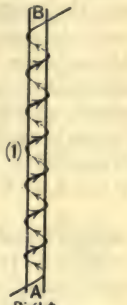

Fig. 183. Fig. 184.

The first thing to be noticed is that each spiral possesses its own individuality; from whichever end a right-handed spiral is viewed it retains the same appearance, and the same is true of a left-handed spiral; we cannot make a left-handed spiral into a right-handed one by altering our point of view. Now, to see what is the essential difference between the two, imagine a worm to start at the end A and crawl along the wires of the respective spirals as shown by the arrows, until he reaches the end B; and let us stand at A, and looking along A B watch it during its journey. Then on the right-handed spiral it will be seen to be moving clockwise, and on the left-handed spiral counter-clockwise. The distinction, therefore, is that if anything travel from the near to the far end of the spiral, it must, if the spiral be right-handed, move clockwise, and if left-handed, counter-clockwise.

This may be put into other forms of words all amounting to the same thing, but probably the most serviceable statement is that *in a right-handed spiral clockwise rotation is associated with a push, and in a left-handed one with a pull*; it is impossible to forget this if one bears in mind that all ordinary carpenters' screws are right-handed spirals, and that in driving them into the wood the screw-driver has to be rotated clockwise. The ordinary corkscrew is another example of a right-handed spiral.

203. Electromagnetism. We have already had in the electromagnet and galvanometer instances of the production of magnetic effects by the current. The branch of the subject which deals with these effects is termed Electromagnetism, and is one of the most important especially from the point of view of the electrical engineer. The fundamental fact of electromagnetism is that *an electric current produces a magnetic field in the region round about*; this field *exists* whether or not there is any iron or other magnetic material in the region, though on account of its high permeability the tendency of such material is greatly to *intensify* the field.*

204. Arago's and Oersted's Experiments. The study of electromagnetism dates from 1820, when Arago of Paris and Oersted of Copenhagen independently discovered the existence of a magnetic field in the neighbourhood of a wire carrying a current.

Exp. 135. *Repeat Arago's experiment.* Bore a hole through a sheet of smooth cardboard or glass, and pass through it a stout straight copper wire (Fig. 185), carrying a current from a battery of four or five Grove's cells arranged in parallel. Hold the board horizontal and the wire vertical. Sprinkle iron filings over the board and gently tap. The filings at once arrange themselves along the lines of magnetic force, taking the form of concentric circles round the wire as their common centre. Tap the board more; the filings move broadside towards the wire, the nearer ones actually adhering to the wire while the current is passing.

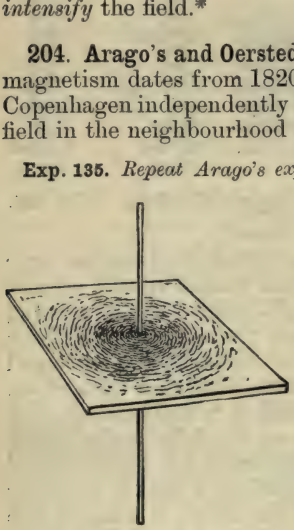

Fig. 185.

We know that these curves map out the lines of force, but the experiment does not tell us which is the positive direction of the lines of force. To settle this point it is only necessary to place a magnetic needle near the wire and note how it acts.

* It should be noted that a mere electrostatic charge, however great or however strong its potential, produces no magnetic field whatever. A charged copper ball or wire will neither magnetise iron nor deflect a compass-needle; it is only when the electricity is *in motion* that we get these effects.

Exp. 135*a*. While the current is flowing in Exp. 135 place a tiny compass needle on the card sufficiently near the wire for the action of the current to be pronounced. Move the compass-needle around the wire. Record in your notebook the relation between the directions of the current in the wire and the lines of force around the wire.

Exp. 136. *Repeat Oersted's experiment.* Hold a wire carrying a strong current pivoted over a magnetic needle (Fig. 186) with its length parallel to the axis of the needle. The poles of the needle at once experience the magnetic force due to the current, and the needle is deflected out of

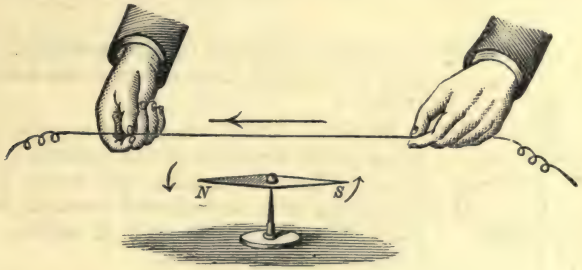

Fig. 186.

the magnetic meridian until it reaches a position of equilibrium, where the moment of the couple due to the magnetic field of the current is balanced by the moment of the couple due to the magnetic field of the Earth. If the direction of the current be from north to south, then the north pole of the needle is urged eastwards ; but, if the current flow from south to north, the north pole is urged towards the west. The south pole is urged in an opposite direction in each case. Now hold the wire under the needle and note the direction of movement of the needle.

Enter your results thus :—

Wire placed	Current from	N-end of compass moves
Above the compass	North to South	
Below ,, ,,	North to South	
Above ,, ,,	South to North	
Below ,, ,,	South to North	

Now arrange a portion of the wire vertically. Adjust one end of the compass-needle close to the wire and note whether on completing the circuit the N-end is deflected east or west.

Wire against	Current	N-end of compass moves
N-end of compass	Up wire	
S ,, ,, ,,	,, ,,	
N ,, ,, ,,	Down ,,	
S ,, ,, ,,	,, ,,	

It is unnecessary to commit to memory the above results. They may be obtained from either of the following rules:—

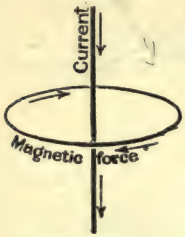

Fig. 187.

Ampère's Rule. Imagine a man swimming in the direction of the current to turn so as to face the needle; the north pole of the needle will be deflected towards his left hand (the south pole will, of course, move towards his right hand).

Maxwell's Rule. Imagine an ordinary right-handed screw to lie along the wire and to be twisted so as to move in the direction of the current, then a north pole tends to go round the wire in the same direction as the head of the screw turns. (This is often called the **corkscrew** rule.)

Only one of these rules need be remembered, and a figure (such as Fig. 187) is often of greater help to the student than a rule.

Exp. 137. You are provided with a cell enclosed in a box so that only wires from the terminals protrude. Find by using the above rules which is the positive and which the negative terminal. Remember that the current flows from the positive terminal through the external circuit.

205. The Circular Coil: Law of Electromagnetic Polarity.

Exp. 138. Bend a piece of stout copper wire into a circle of about 20 cm. diameter. Bend out the two free ends and put binding screws on them. Support the coil vertically. Take a piece of cardboard about 30 cm. square, cut out two narrow slots in it as shown in Fig. 188 so that the ends of the slots are just 20 cm. apart. Fit the cardboard to the coil

and support it horizontally. Scatter iron filings over the card and then pass a strong current through the coil from a 5-Grove's battery (a current of 5-10 amperes gives good results). The filings take up definite positions. Record the position by a diagram in your notebook.

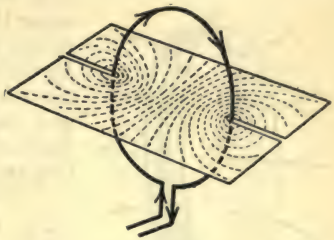

Fig. 188.

The student will notice that around the two parts of the coil cutting the cardboard there are lines of magnetic force similarly placed to those in Fig. 185. The curves, however, deviate from the circular form as we leave the wires, and at the centre of the coil the two systems coalesce and give a line of force through the centre perpendicular to the plane of the coil. On either side of the centre the lines of force are approximately parallel. The field is practically uniform for a small space around the centre. Note how this result was used in the treatment of Tangent Galvanometers.

Place the coil now with its plane in the magnetic meridian and place a small compass-needle at the centre of the coil. The arrangement is now a simple tangent galvanometer. Note the relation between the direction of the current in the coil and the direction of the magnetic field at the centre of the coil (cf. footnote, p. 235).

The result may be stated thus:—When the current goes around the coil clockwise the lines point away from the observer, and when counter-clockwise, towards the observer.* This law may be obtained by a slight extension of Ampere's or Maxwell's Laws and is called the **Law of Electromagnetic Polarity**. This law is very useful, the student should remember it not so much in words as in mental picturing, always associating CLOCKWISE rotation with arrows pointing AWAY FROM him, and *vice versa*. The carpenter's screw may here again be helpful; the direction of the current is the way the screw-driver goes round, and the lines of force point the way the screw moves.

* Since the magnetic lines are closed curves it follows that when they have gone some distance from the plane of the coil they must curve back around the outside to the coil to get ready to march through the coil again (see Fig. 188).

Suppose, now (Fig. 189), we have a current circulating clockwise round a copper wire or strip, A B C D E, and introduce an iron core K;* the lines of force clearly enter the core at the end or face nearest to us and emerge at the other; hence, by the above law and § 35 the far end of the core acquires northern, and its near end southern polarity; if the current be made to go counter-clockwise the polarities are of course reversed.

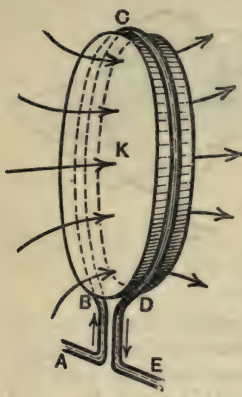

Fig. 189.

Exp. 139. *Map the magnetic field through a circular coil when carrying a current.* The bobbin of a tangent galvanometer (high resistance preferable) may be used. Arrange it with its plane vertical and in the magnetic meridian. Fix a sheet of paper to a board by pieces of stamp paper at each corner. Arrange the board so that the paper lies horizontally and passes through the centre of the hoop.

(1) Trace the magnetic lines of force by the method of Exp. 32, when no current passes through the hoop.

(2) Substitute a fresh sheet of paper for that used in (1). Join the terminals of the hoop to a voltaic cell (Daniell's or a secondary) and rheostat. Adjust the rheostat so that a compass-needle placed at the centre of the hoop is deflected at 40° or 50° from the meridian. Then map the field thoroughly.

Results. The map obtained in the first case will be that of the earth's field alone and will therefore consist of a number of parallel lines. If this is not found to be the case it will indicate disturbances of the earth's field due to iron pipes, etc., in the neighbourhood.

The map obtained in the second case will be not unlike Fig. 188 as regards the general disposition of the lines, but they will lie obliquely to the plane of the coil, not perpendicular to it as in Fig. 188, which shows the lines of force in the coil when the field of the earth is negligible in comparison to the field of the coil. The map obtained now is that of the resultant field due to (1) the current in the coil, (2) the earth. The obliquity of the lines at the centre of the coil indicates the angle through which a compass-needle would have been deflected by the current if it had been placed in that position.

---

* If the core is as big as the coil it must be wrapped round with a piece of cloth or velvet to prevent its touching the copper unless the wire bears an insulating covering, otherwise the current would short-circuit *through* the core instead of passing *round* it.

Exp. 140. *Find the laws connecting the magnetic field at the centre of a circular coil with number of turns in the coil, the radii of the turns and the current through the coil.* Take a wooden hoop H (Fig. 190), 20 cm. in diameter (obtainable at a toy or fancy shop), and support it in a vertical

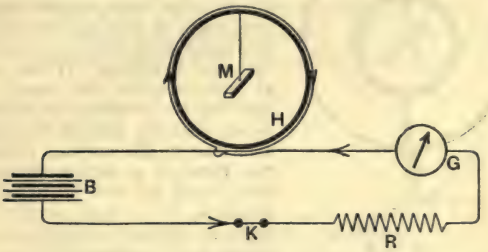

Fig. 190.

position with its plane perpendicular to the magnetic meridian. Place a small suspended or pointed magnetic needle M at the centre of the hoop. Take a constant battery B (5 Daniell's cells), and put in circuit with it a long piece of cotton-covered copper wire, and a key K. It is also preferable, though not essential, to have in the circuit a galvanoscope G and a rheostat R for the purpose of testing and arranging that the current is constant during the experiment.

(i) *The relation between the field and the number of turns of wire.* First with the key out make the needle vibrate and find the frequency of vibration of M as in § 60. Then wind a turn of wire over the hoop, making the wire go upwards on the western side of the hoop so that the field due to the current is northward at the centre of the hoop. Complete the circuit. Again find the frequency of vibration of M. Wind another loop on H. Again find the frequency of M. Repeat until several loops of wire are wound on. Let f_1, f_2, f_3, f_4 . . . be the successive frequencies. Then by § 59 the strengths of the magnetic field at the centre of the hoop due to the current around the hoop are as $f_2^2 - f_1^2$, $f_3^2 - f_1^2$, $f_4^2 - f_1^2$, . . . If these numbers are in the ratio 1 : 2 : 3 : . . . it follows that the field at the centre of a circular coil due to a current in the loops around the hoop is proportional to the number of turns.

(ii) *The relation between the field and the current in the coil.* When there are n turns on the wire the effect is obviously that of a current of n times the strength flowing around one turn. Hence the field at the centre of a circular coil—other things being the same—is proportional to the current around the coil.

(iii) *The relation between the field and the radii of the coil.* Modify the arrangement given above as shown in Fig. 191. The coils are now placed *in the meridian* and as it is not necessary now to keep the current constant dispense with G and R. The hoop H is the same as before, and one loop of wire is wound on it. The wire is now continued into two loops of wire of twice the radius of the inner hoop and going around in the opposite direction. When the needle is at rest complete the circuit. You will find that the needle is not deflected. This means that the field at the centre of the coil due to the current is zero. Hence one turn at a certain distance gives a field equal to two turns at twice the distance, *i.e.* a field double that of one turn at twice the distance. Hence—other things remaining the same—the field at the centre of a circular coil is inversely proportional to the radius of the coil.

Fig. 191.

We may combine the above three results in the formula

$$\text{Field} \propto \frac{n\,\text{C}}{r}.$$

In § 169 the formula was

$$\text{Field} = \frac{n\,\pi\,\text{C}}{5\,r}.$$

The reason why the factor $\frac{\pi}{5}$ is introduced cannot be explained here. It depends upon the definitions of unit field and unit current.

206. Application of the Law of Electromagnetic Polarity to Spirals. Unless the current be extremely strong it is only possible with a single turn of wire to obtain a feeble magnetic field. To get strong effects with weak or moderate currents spirals are employed, and the effects of the several turns then add together as explained in dealing with the multiple-coil galvanometer (§ 167). The most powerful effects are obtained by winding the spirals with their coils close together and with numerous laps, in which case the wire must be "insulated," *i.e.* covered with cotton or some other insulating material, otherwise

the current would short-circuit from one end to another and not go round the coil at all.

The law of electromagnetic polarity can easily be applied to spirals, the special point to notice being that *everything depends upon the way the current goes round*, whether clockwise or counter-clockwise; the question whether the spiral is right- or left-handed (§ 202) being merely incidental. Thus in Figures 183 and 184, if the current viewed from the ends A be clockwise,

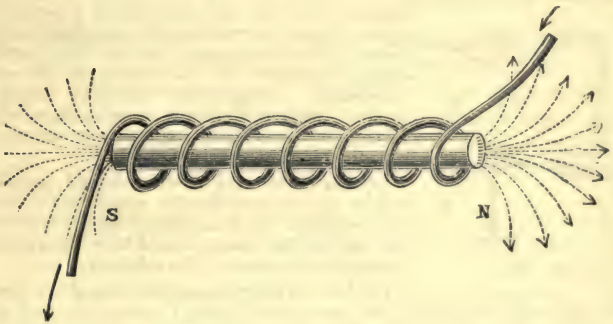

Fig. 192.

the lines of force (following their positive direction) will run through the spirals from the bottom to the top of the page, and if iron cores be placed in them, so as to make electromagnets (§ 207), the ends B will become north poles; the only difference will be that in R the current will work its way from A to B, and in L from B to A.

Fig. 192 will repay examination. The spiral is a left-handed one, the current viewed from the right-hand end is going counter-clockwise, as indicated by the two big arrows, and in accordance with the law of electromagnetic polarity the lines of force point to the right, as shown by the small arrows.*

* The student is strongly recommended to draw spirals as in Figs. 183, 184, rather than as in Fig. 192; diagrams of the latter type, unless done in good perspective, are apt to be very confusing. On the other hand, by drawing (as in Figs. 183, 184) the upper parts of the coils thick and the lower thin, the nature of the spiral is obvious at a glance.

It should be noted that even in the absence of a core, a current-carrying spiral behaves in all respects like a magnet, the end where the lines of force emerge from its interior being its north "pole"; when suspended so as to be free to turn horizontally, it sets itself in the magnetic meridian, and its poles are attracted or repelled by the poles of a magnet or of another spiral according to the law of § 6.

Exp. 141. *Test the polarity of a solenoid.** (1) Wind a piece of insulated copper wire (No. 18, say) round a pencil or a glass, brass, or cardboard tube (half-inch diameter, six inches long), so that the turns lie closely together (tie with string at the ends if necessary). Hang from a wooden clamp and connect with a Bunsen's or secondary cell, S. It is convenient to introduce a commutator, K, into the circuit (Fig. 193).

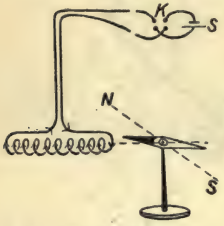

Fig. 193.

(2) Place one end of the solenoid against an end of the compass-needle, complete the circuit, note whether there is attraction or repulsion of the pole of the compass, and thus determine the polarity of the end of the solenoid.

(3) Move a finger along the wire of the solenoid in the direction in which the current is flowing: look at the end of the solenoid tested in (2): note if the finger appears to move round the solenoid in the same way as the hands of a clock (clockwise), or the reverse (counter or anti-clockwise).

(4) Similarly test the other end of the solenoid.

(5) Reverse the current and again test both ends.

(6) Confirm the results by testing the ends of the suspended solenoid with the poles of a bar magnet. Observe which pole of the magnet repels one end of the solenoid.

The observations will show that when the current is circulating round the solenoid it acts like a weak bar magnet, and exhibits a north pole at one end, a south at the other. The polarity is reversed when the current is reversed. The *relation between direction of current and polarity* is given by the following RULE: If, when looking at one end of a solenoid and following the current with the finger, the finger passes round (i) clockwise,

* A *solenoid* is an electrical term for the spiral obtained by winding a wire round a cylinder. The direction of a current may be inferred from the polarity imparted to a solenoid,

then the end is a south end, (ii) counter-clockwise, then the end is north.

Confirm the results by rewinding the solenoid in the reverse direction: *e.g.* if wound right-handedly at first, rewind left-handedly.

Exp. 142. *Test the polarity of a solenoid with an iron core.* Rewind the solenoid of Exp. 141 round an iron rod. Repeat the tests of Exp. 141. Observe that the iron considerably increases the strength of magnetisation, but does not alter the nature of the poles.

207. Electromagnets. These have already been considered (§ 9), and it chiefly remains (1) to accentuate the fact (§ 44) that their great power depends upon the high permeability of the iron employed for their cores, which enables them when under the influence of the current to acquire a very heavy magnetic flux, and (2) to point out certain facts regarding their winding.

In the case of a straight core wound with a single lap of wire, it is obvious that the spiral must be of the same kind all along it, otherwise there will be consequent poles (§ 24) at the points when it changes from right- to left-handed or *vice versâ*. This of course means that the wire must be wound the same way throughout—it must not kink back.

Fig. 194 shows the proper way of winding a horseshoe-magnet with a single lap; here again the essential thing is that the spiral must be of the same kind throughout. In the figure it is right-handed, and if the current enter at H and leave at L, it will (when viewed from the free ends) travel clockwise round the limb A A′, and counter-clockwise round B B′, so that A will become a south and B a north pole. It is practically immaterial whether the central part A′ C B′ be wound or not; and it is usual not to do so, but to bring the wire across from A′ to B′, and it will be seen that the wire in passing from A′ to B′ must cross from the front to the back of the horseshoe.

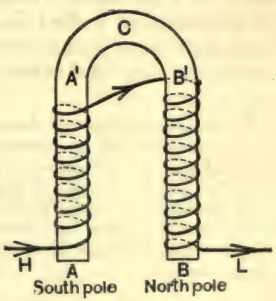

Fig. 194.

When an electromagnet has more than one lap of wire, care

must be taken to wind the consecutive laps so that their electromagnetic effects co-operate with one another. Thus, if the core be straight the current must circulate the same way round all the laps, and since the current travels from one end A to the other end B along one lap and back from B to A along the next, this means that the spirals must be alternately right- and left-handed; we must therefore wind the first lap one way from A to B, and then not kink back but continue winding the second lap the same way from B to A, and so on until all the laps are completed.

Steel may be permanently magnetised by passing a current through a spiral which surrounds it; indeed (cf. § 25), this is by far the best way of making large permanent magnets.

With a given current a much more powerful magnet can be obtained while the current passes by using a soft iron core instead of a steel core; such magnets are greatly used in practical electrical work. Smaller electro-magnets are also much used in automatic appliances, where rapid magnetisation and demagnetisation are produced by making and breaking the circuit in the coils.

The internal lines of force of a spiral either with or without an iron core, especially when closely wound, are practically straight lines or curves parallel to the central line of the spiral, so that they run through it, with more or less lateral leakage, very much like those of a permanent magnet (cf. Fig. 15). One of the best ways of magnetising a steel ring circumferentially is to wind it with a close spiral and pass a current; the internal lines of force then run round the ring and cause its molecules to set as in Fig. 17.

For many purposes of electrical engineering the leakage of lines of force along the sides of an electromagnet is a serious matter on the score of economy, and to minimise it the "Ironclad" type of magnet is employed, in which the windings are enclosed in a cylinder of soft iron, which by its high permeability largely prevents the lines from straying into the external field. Some of the best ironclad magnets have a leakage of only about 15 per cent., whereas in an ordinary open wound one it may amount to 50 per cent.

Exp. 143. *Excite an electro-magnet.* Join a Bunsen or secondary battery of two or more cells with the terminals of a horseshoe electromagnet. Include a plug key in the circuit. Note that the armature or keeper is easily pulled off before the current is started, but not when the current is flowing. Also that the keeper when separated from one pole readily

adheres to the other along one of its edges, and turns like a hinge about this edge.

Excite the magnet by the current, place the keeper across the poles, stop the current. Observe that the keeper now sticks to the poles. Pull it off, then replace it across the poles: it is no longer held on by them. Repeat the experiment. Also repeat it with a sheet of paper between the keeper and the poles: after the circuit is broken the keeper is found to be held with much less force than before. Repeat with double and treble thicknesses of paper.

The *experiment shows* that (i) when the keeper is in contact with the poles the residual magnetisation left after stopping the current is considerable. A large proportion of this, however, disappears when the keeper is pulled off. (ii) When there is a thin gap (the thickness of a sheet of paper) between the keeper and polar surfaces, the residual magnetisation is much less than when the keeper is in metallic contact with the poles.

208. Electric Bells. One of the simplest applications of electromagnets is to be found in electric bells. Electric Bells are of two typical kinds, viz. *Tremblers*, which ring continuously as long as the current passes, and *Strikers*, which give only a single sound each time contact is made. Fig. 195 shows diagrammatically the construction of a trembler. On a wooden base (not drawn) is mounted a small horseshoe electromagnet LMNR. The end L of the wire which is wound on it is permanently attached to one of the terminals T of the instrument, and the other to a nut K. Fixed to K is a steel spring S carrying a soft iron armature L, to the other end of which is fitted an arm terminating in the clapper C. The spring S is so adjusted that, when no current passes, the armature rests lightly against the point of a brass screw E adjusted in a brass bearing H, and from this bearing a wire proceeds and is permanently attached to the other terminal T'. In the external circuit are a battery B, and bell-push or contact-maker P.

When contact is made a current passes via the route BPT'HISKRNMLT. This magnetises the core of the electromagnet, draws I towards its poles, and makes the clapper C strike the gong G. Contact being thus broken between the point of the screw E and the armature, the core becomes demagnetised, and the armature, by reason of the elasticity of the spring S, falls back against the said point. This re-establishes

contact, the core is again magnetised and the armature attracted again immediately to fall back, and so on, the result being a continuous tintinnabulation.

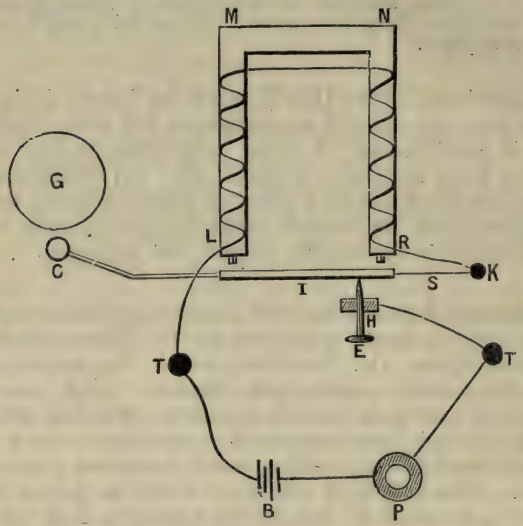

Fig. 195.

In *strikers* the screw E and bearing H are absent, the terminal T′ being connected directly to the nut K. Thus as soon as contact is made by the push P the armature is attracted and the clapper gives one stroke, but as there is nothing now to break the circuit automatically the armature does not fall back until the finger is taken off the push.

Exp. 144. *Arrange a Leclanché's cell to ring an electric trembling bell.* Note the essential parts of an electric bell, viz. the gong, the electromagnet, the soft iron keeper of the magnet to which the striker is attached, and the arrangement called a *make and break* for starting and stopping the current.

Join the terminals of the cell to the terminals of the bell. Note the attraction which ensues at once between the magnet and the piece of soft iron and the consequent striking of the bell. This breaks the circuit, and the spring forces the soft iron back again to the terminal of the bell and so completes the circuit, when the whole action is repeated. Finally, make a diagram of the bell.

209. Astatic Galvanometers. In order that a galvanometer may be delicate, it is necessary to arrange matters so that the needle may be affected as *much* as possible by the current, and as *little* as possible by other influences. One way of promoting these

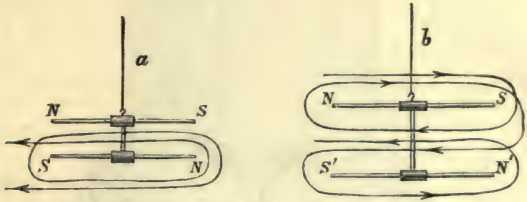

Fig. 196.

ends is to employ many turns of wire, and another is to suspend the needle by a fine silk thread so as to avoid the friction of a pivot. Finally, we must abolish as much as possible the earth's action, for which purpose we employ an astatic needle (§ 58); we thus get an *Astatic Galvanometer*. Fig. 196 shows the method of winding.

The simplest arrangement is the one indicated in the left-hand diagram, *a*; here the wire encircles the lower needle only. As drawn, the current relative to the lower needle is counter-clockwise, so that its north pole is urged towards us; at the same time the current relative to the upper needle is clockwise, and its north pole is therefore urged away from us: both agencies therefore urge the needle as a whole the same way. A better mode of winding is shown in the right-hand diagram, *b*; the wire encircles both needles, but in opposite ways, so that the current flows clockwise round one and counter-clockwise round the other; the effect is similar to that in *a*, but stronger, because, other things the same, the upper needle is in a more powerful magnetic field.

Fig. 197 exhibits Nobilis' Astatic Galvanometer. The needle is suspended by a fine silk thread from an adjustable screw; the

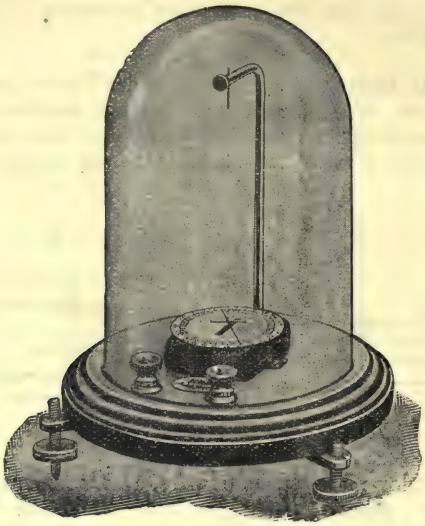

Fig. 197.

coils encircle the lower needle only, and the upper carries a pointer, which travels over a cardboard scale.

210. The Mirror Galvanometer. A common form of Lord Kelvin's astatic or mirror galvanometer is shown in Fig. 198. The coil is wound on a bobbin R enclosed in B, a cylindrical brass box with a glass front. The upper needle of the instrument is a short carefully magnetised strip of steel attached by shellac or cement to the back of a small concave mirror *m*, suspended by a single silk fibre* at the centre of the coil. The

* The advantage of silk fibre is that, when twisted, it has little tendency to untwist. To minimise what there is the fibre should be long and fine. Fibres, however, are easily broken.

lower needle is of the same construction and is attached to a vane which hangs below the coil. The upper and lower needles are rigidly connected by a piece of aluminium wire.

The deflection of the mirror is measured by the "lamp and scale" method. This is illustrated in Fig. 199. The galvanometer stands on a firm shelf, and in front of it is placed a paraffin lamp and graduated scale. When no current is passing, the mirror faces the scale. A narrow beam of light shines through a slit below the scale, and, following the direction of the lower dotted line, is reflected along the upper, and falls on the middle, or "zero," of the scale. If a current is now sent through the galvanometer, the mirror swings either to the right or left, the "spot of light" travels along the scale, and unless the current be excessively weak goes off the scale altogether.

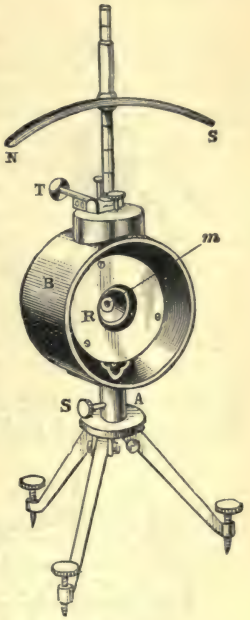

Fig. 198.

The large magnet N S supported above the coil has important uses. If it is removed the needle of the galvanometer sets in the meridian, and the magnetic field in which it lies is that due to the horizontal component of the earth's magnetic field. If now the magnet is replaced with its length in the magnetic meridian and its *south* pole pointing northwards (Fig. 200) the magnetic field which it produces at the centre of the coil will be added to that of the earth. It will now be more difficult to deflect the needle, and therefore the galvanometer will be less sensitive. The nearer the magnet is to the coil the greater will the effect be, hence by lowering the magnet on its support the sensitiveness may be very considerably diminished.

If the magnet is placed with its *north* pole pointing north (Fig. 201) the field it produces at the centre of the coil tends to neutralise that due to the earth, and a certain point can be found where the field of the magnet exactly balances that of the earth.

For this position of the magnet the needle is unstable and remains at rest in any position. If the magnet is lowered below this

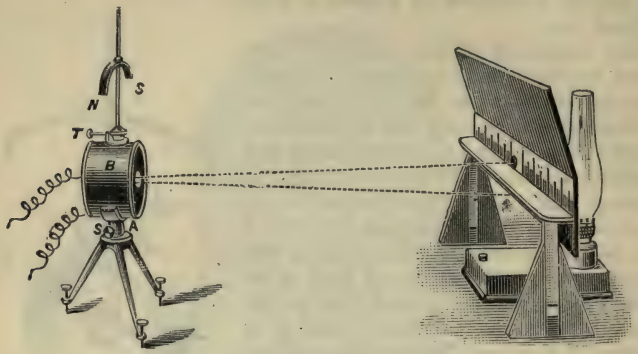

Fig. 199.

position the direction of the resultant field at the position of the needle is reversed and the needle tends to turn around through

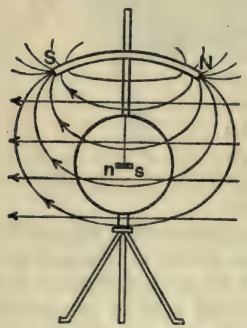

Fig. 200.

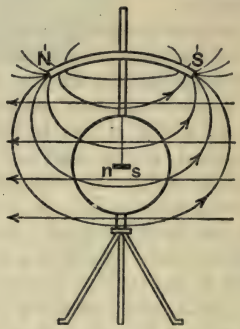

Fig. 201.

180°. With the magnet just above this position the resultant field is very small and the instrument is extremely sensitive.

So far we have considered the galvanometer placed with the plane of its coil in the magnetic meridian, but this is not an

essential adjustment; by turning the control magnet on its support the needle may be made to set in almost any required plane and the coils are then set parallel to it by rotating the body of the instrument round a central axis at A.

In adjusting the galvanometer for use it is first levelled so that the mirror M hangs truly at the centre of the coil. The position of the magnet N S is then adjusted so as to secure the required sensitiveness, and to set the needle in a position convenient for work. The coil is then set parallel to the plane of the needle and fixed in position by the screw at S. The scale must now be arranged with its length parallel to the plane of the coil, and the line joining the centre of the mirror to the zero of the scale should be approximately at right angles to this. On adjusting the lamp and scale to the proper height the spot of light reflected on to the scale will now come to rest somewhere near the centre of the scale and the spot may be adjusted exactly to the centre mark by slightly rotating the magnet N S by means of the tangent screw T. The galvanometer will now be ready for use, and Fig. 199 shows how it is connected in circuit, but the student can only realise the entire process of adjustment by going through it for himself.

This form of galvanometer enables us to detect very minute currents, say of an order of one-millionth of an ampere.

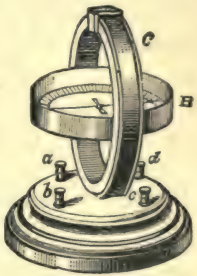

Fig. 202.

211. Tangent Galvanometer. The theory of this galvanometer has been already given in § 169. A simple form of tangent galvanometer is shown in Fig. 202. The wire is wound on a circular bobbin. The magnetic needle which may be pivoted or suspended at the exact centre of the coil is very short,* and carries a light pointer of wire or aluminium fixed perpendicular to itself. The ends of this pointer move over a circular scale graduated in degrees. The needle and scale are enclosed in a shallow cylindrical box, B, by a glass cover. A strip of mirror is usually fitted to the base of the compass box to avoid parallax error. To get a correct

* The length of the magnet must not be greater than one-tenth the diameter of the coil.

reading the eye must be vertically over the mirror: and this will be the case if the needle exactly covers its reflection in the mirror.

The ends of the wire of the coil are brought out under the base of the instrument and connected to screw terminals fixed to the base-board. Most instruments, however, are usually fitted with several coils of different lengths wound on the same frame, but the ends of each coil are connected to the terminal screws in such a way that any one coil may be used separately, or any two or more combined into one by choosing the proper screws as terminals. Thus in Fig. 202, if the screws a and b are taken as terminals the current passes only through the coil joining them; but if the screws a and c are taken the current passes through the coil joining a and b and also through that joining b and c. Similarly if a and d are taken the current passes in succession through the coils a to b, b to c, c to d. Three coils are sufficient for ordinary purposes:—one consisting of a single turn of thick copper wire, a second of a number of turns of wire of medium thickness, and a third composed of a large number of turns of thin wire.

As shown in § 169, the formula used with a tangent galvanometer is

$$C = \frac{5Hr}{\pi n} \tan \alpha$$

or

$$C = K \tan \alpha, \text{ where } K = \frac{5Hr}{\pi n}.$$

K may be calculated from the dimensions of the coil and the value of H, or may be found as in the following experiment:—

Exp. 145. *Find the constant of a tangent galvanometer by electrodeposition.* (1) Adjust the tangent galvanometer. Level the instrument and see that the magnet swings over the centre of the coil. Turn galvanometer around until plane of coil is in the magnetic meridian. This is so when the pointer which is fixed at right angles to the magnet reads zero on the circular scale. A better test now is to send a current one way, note the deflection, reverse the current, and note the deflection again. If the two deflections differ by more than 1° turn the coils a little more one way or the other until the two readings agree to within 1°.

Join in series a constant cell B (Daniell's), carbon rheostat R, copper voltameter V (described in § 195), commutator C, and tangent galvanometer G (low resistance). Connect as shown in Fig. 203 and carefully adjust the galvanometer. Before starting the experiment proper let the current flow around

the circuit and adjust the rheostat until the deflection is somewhere near 45° (the galvanometer is most sensitive when the deflection is 45°). See that the connections are such as to make the proper plates of the voltameter anode and kathode. Prepare the kathode and weigh it. Connect up, note

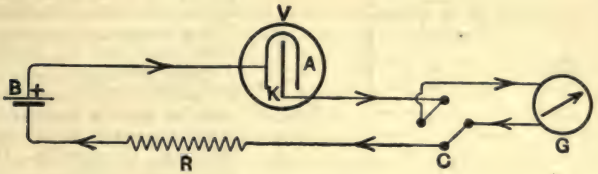

Fig. 203.

the time, and insert the plug in the key. Run the current for 30′, keeping the galvanometer deflection constant. Then quickly reverse the commutator, so that the current runs in the opposite direction through the galvanometer, and run for another 30′. In each case read both ends of pointer so that the angle finally stated in your notebook should be the mean of four angles. Then break circuit, remove kathode plate, and weigh. From the formula $W = C\,wt.$ find C, and then from $C = K \tan \alpha$ find K.

Note.—If 60′ is not a long enough time for running the experiment try a longer time, or try more or better cells in the battery.

K being now found, the formula $K = \frac{5Hr}{\pi n}$ may now be used to find H.

EXAMPLE.—A steady current from 3 Daniell's cells which passed through a copper voltameter and a tangent galvanometer and a rheostat in succession deposited ·169 gm. of copper in 30 minutes and gave a deflection of 45°. Find the constant of the galvanometer. The mean radius of the coils was 8·15 cm. and there were 8 turns. From the constant and these dimensions find the horizontal component of the earth's magnetic field.

We have

$$W = C\,wt.$$

$$\cdot 169 = C \times \cdot 000328 \times 1800,$$

$$\therefore\ C = \frac{1690}{18 \times 328} = \cdot 286 \text{ ampere.}$$

Now $$C = K \tan \theta = K \tan 45° = K,$$

$$\therefore\ K = \cdot 286.$$

Also $$K = \frac{5Hr}{\pi n}, \quad \therefore\ H = \frac{\cdot 286 \times \pi \times 8}{5 \times 8 \cdot 15} = \cdot 177 \text{ dyne per unit pole.}$$

Exp. 146. *Use a tangent galvanometer to compare the electromotive forces of a Daniell's and a Bunsen's or Secondary cell (Wiedemann's method).* Arrange the circuit as in Fig. 204 where E_1, E_2 are the cells whose E.M.F.'s are to be compared, G a tangent galvanometer, *a b c d* a commutator, and R B a resistance (which, however, may not be necessary). Arrange the commutator so that the cells act, 1st, in *conjunction*, that is, send a current in the same direction round the circuit; 2nd, in *opposition*, that is, tend to send currents in opposite directions round the same circuit. The value of the resistance (R B) should be arranged so that suitable deflections of the galvanometer needle are obtained. It must not be altered in value during the two parts of the experiment.

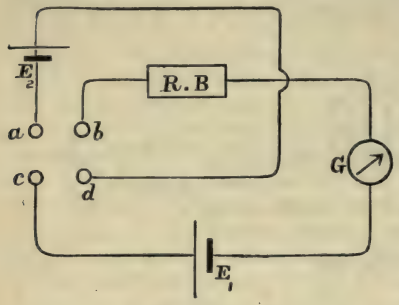

Fig. 204.

Theory of the method. Since the resistance $(R + G + B_1 + B_2)$ is the same in both parts of the experiment, the currents produced will be proportional to the actual E.M.F. acting. The value of this when the cells are in conjunction is $(E_1 + E_2)$, and $(E_1 - E_2)$ when in opposition. Hence, if α_1, α_2 are the respective deflections,

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{C_1}{C_2} = \frac{\tan \alpha_1}{\tan \alpha_2},$$

$$\therefore \frac{E_1}{E_2} = \frac{\tan \alpha_1 + \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2}.$$

EXAMPLE.—In an experiment to compare the E.M.F. of Bunsen's and Daniell's cells by Wiedemann's method a deflection of 66° was observed when the cells were in conjunction and a deflection of 32° when they were in opposition. Assuming the Daniell's cell to have an E.M.F. of 1·1 volts, find that of the Bunsen's.

From tables we find that $\tan 66° = 2{\cdot}25$, $\tan 32° = {\cdot}62$.

$$\therefore \frac{\text{E.M.F. of Bunsen's cell}}{\text{E.M.F. of Daniell's cell}} = \frac{2{\cdot}25 + {\cdot}62}{2{\cdot}25 - {\cdot}62} = \frac{2{\cdot}87}{1{\cdot}63} = 1{\cdot}76,$$

$$\therefore \text{E.M.F. of Bunsen's cell} = 1{\cdot}1 \times 1{\cdot}76 = 1{\cdot}94 \text{ volts}.$$

212. Galvanometers and Galvanoscopes: Calibration. The magnitude of the *deflection* or angle through which the needle of a galvanometer is turned depends partly on the strength of the current through the instrument. The greater the current

strength the larger the deflection. There is not, however, always a simple relation between the angle of deflection and current strength. The current strength in the case of the mirror galvanometer is proportional to the displacement or deflection of the index when small. In the tangent galvanometer the current strength is proportional to the tangent of the angle of deflection. Those instruments in which the relation rule or *law* connecting deflection and current is simple are spoken of as *galvanometers*, in distinction to others, called *galvanoscopes*, that are detectors or indicators of the existence of current.

A galvanoscope may, however, be converted into a current measurer or galvanometer by *calibration*. To effect this, currents of known value or ratio are passed through it, and the resulting positions of the pointer on the scale of the instrument noted. The observations are plotted, currents as ordinates, scale readings as abscissae. The graph obtained is called the *calibration curve*. By means of the curve the instrument may be made *direct-reading*, that is, have its scale marked to indicate amperes. This may be done by finding from the calibration curve the deflections corresponding to currents 1, 2, 3, etc., amperes, then making marks on and numbering the scale of the instrument at the values of deflection so determined. The scale is not necessarily one of equal parts.

213. Telegraphy. Suppose that we have a battery in London connected by wires to a sensitive galvanoscope in Plymouth, and that the circuit can be alternately closed and opened by a "key." When a person in London presses the key, and thus closes the circuit, the current flows, and the needle of the galvanoscope at Plymouth is deflected. When the key is released, the needle swings back to its normal position. If a code be drawn up embodying relations between the letters of the alphabet and the time the key is down, it can at once be seen that the messages can be transmitted from London to Plymouth. This is the essence of *telegraphy*.

214. Detection of Electrostatic Currents. Whenever two conductors in an electrostatic state, but at different potentials, are connected, a momentary current flows. If, as in an electric machine, we *maintain* a P.D., the current becomes continuous. Whether continuous or not, such currents may be appropriately termed "electrostatic currents," to distinguish them from the "electrodynamic currents" furnished by batteries, dynamos, etc. In

the same way, potentials obtainable by electrostatic methods may be called "electrostatic potentials," while those furnished by batteries, etc., are called "electrodynamic potentials."

Electrostatic potentials are vastly stronger than electrodynamic; a metal ball beaten with fur may readily attain a potential of 50,000 volts, and a good plate machine one of 100,000, while a Grove's cell gives less than 2. The difference between the currents is precisely the reverse—electrostatic currents are vastly *weaker* than electrodynamic. Thus, while a moderate-size Grove's cell on short circuit may easily furnish eight or ten amperes, it is much if a good plate machine will give $\frac{1}{50000}$ ampere. A Leyden jar will give more. A charged metal ball may give something under $\frac{1}{500000}$ ampere.

It is therefore impossible to detect an electrostatic current by any ordinary galvanometer: even a big Leyden jar will produce not the least movement of the needle. But the detection is quite easy with Lord Kelvin's astatic galvanometer. If we connect one of its terminals, A, with the positively charged prime conductor of even a bad plate machine, and the other, B, with the rubber or the gas-pipe; then on slightly moving the handle, the spot of light travels along the scale in the same direction as if A had been connected to the copper and B to the zinc plate of a simple voltaic cell. If, instead of the machine, we employ a charged metal ball, the spot will move slightly, the movement being in one or the other direction, according as the charge on the ball is positive or negative. If we place a charged Leyden jar on an insulating stool, and then connect its outer coat to one terminal of the galvanometer and its knob to the other, the "spot" will move very decidedly; in fact, unless the jar be very feebly charged, there is some danger of damaging the instrument.

The student should note carefully that these experiments, as well as those with the condensing electroscope (§ 160), establish beyond doubt the essential identity of the things with which we deal in electrostatics and electrodynamics.

215. Mutual Magnetic Action in Terms of Magnetic Flux: Maxwell's Law. Consider the following simple experiment:—

Exp. 147. Float a cork on water, place on it a small bar magnet or magnetic needle, A, and hold another bar maget, B, horizontally, with its north pole near A. Then A first turns round, so that its south pole faces the north pole of B, and then moves up to and touches it.

Of course this follows from the elementary law of § 6. But there is another way of looking at it. When the movable magnet, A, has attained its final position, its own lines of force point in as nearly as possible the same direction as those of the fixed magnet, B, and it is also in the part of the external field where B's flux is densest, so that as much as possible of B's flux passes through A. Now we have here a principle which applies to all kinds of magnetic action, whether due to magnets or

electric currents, or both, and which covers cases hardly within the domain of the elementary law. This principle is known as **Maxwell's Law**, and may be thus stated:—

When two circuits mutually act on one another, they tend so to place themselves that:—*

(1) *Their respective lines of force point as nearly as possible in the same direction.*

(2) *Each circuit encloses as much as possible of the other's magnetic flux.*

To these clauses should be added:—

(3) That if the circumstances be such as to compel the two sets of lines of force to point more or less in OPPOSITE directions, each circuit tends to embrace as LITTLE as possible of the other's flux.

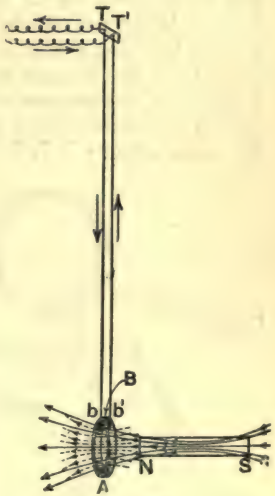

Fig. 205.

For example, in the foregoing experiment if we place the movable magnet, A, with its north pole facing the north pole of B, and then by some means prevent its turning round, while still leaving it free to move bodily, it will recede from B into the part of the field where B's flux is lightest.

Let us now consider some other phenomena in the light of Maxwell's Law:—

In Fig. 205, A B is a flat spiral of fine copper wire whose ends are suspended by long thin wires from terminals T, T′ on a wooden stand. A bar magnet is placed horizontally and pointing towards the spiral; supposing its north pole towards the left, the lines of force also point towards the left as shown, and the flux is densest at the centre of the magnet, more or

* The word circuit is here used broadly to denote either a conductor carrying a current, or a magnet. See § 231 for the so-called Amperian currents which exist in a magnet.

less lateral leakage (§ 42) occurring between the centre and the ends. We now pass a current round the spiral. Suppose that when viewed from the right it goes clockwise, then *its* lines of force also point to the left as shown by the dotted lines, so that clause (1) of Maxwell's Law is *already* satisfied, and it simply remains for the spiral to obey clause (2), which it accordingly does by shooting itself along the magnet and coming to rest at the middle, where it embraces as much as possible of the magnet's flux.

If (re-starting in the position shown in the figure) we now reverse the current, the lines of force of the spiral will point opposite to those of the magnet, and in compliance with clause (1) the spiral will first twist round* so as to face the other way and then shoot on as before. But if it be forcibly prevented from twisting round while still leaving it free to move to and fro, then it becomes a case of clause (3), and the spiral moves as far away from the magnet as it can get, so as to embrace as *little* as possible of the magnet's flux.

Fig. 206 illustrates a somewhat simpler apparatus for showing the same effect; it is known as **De La Rive's floating battery**. A zinc and copper plate are fitted in a cork and connected to the ends of a flat spiral, the cork is then floated in dilute sulphuric acid; a current circulates round the coils of the spiral, and, when a magnet is held near it, it behaves like the spiral in Fig. 205. Also if we place two such "batteries" in the same vessel, they will set themselves with their coils parallel and their currents circulating in the same direction.

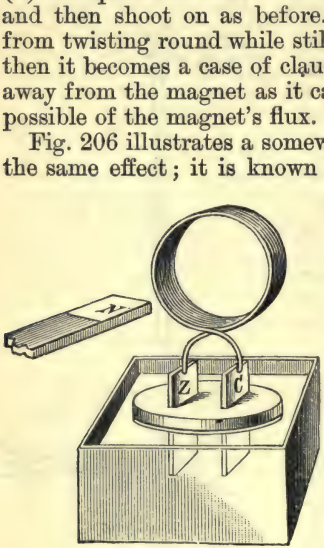

Fig. 206.

It should be noted that the fact that in any assigned case the flux through a circuit can be different in different positions implies that as the circuit changes from one position to another it *cuts* the lines of force; in fact, the change of flux is commonly

* Care should be taken that the two suspension wires do not touch in this process, or short circuiting will occur.

spoken of by engineers as the "number of lines of force cut." In Fig. 205 the lines of force cut between the end and middle of the magnet are manifestly due to lateral leakage: if there were no such leakage the spiral would move up to N, but no farther. In a circumferentially magnetised ring there is no leakage, and the spiral would remain quite stationary whether threaded on one of its limbs or placed so as to be capable of sliding over the entire ring.

Clause 1 of Maxwell's Law is of great importance in electrical engineering, as being the principle on which the action of moving coil galvanometers and all ordinary electromotors depends.

Exp. 148. *Use a De La Rive coil to find the direction of the magnetic meridian.* Provide the floating coil with a long spiral instead of the flat one of Fig. 206. Place the cork in the centre of the vessel and allow it to set itself as it desires. A line drawn through the axis of the spiral is in the magnetic meridian. Place two wooden uprights in line with this axis, one to each end of the spiral. Remove the dish and test the accuracy of the position of the meridian line by a compass-needle.

Exp. 149. *Repeat Jamieson's experiment.* Make the apparatus shown in Fig. 207. A B C D is a wooden frame. On the base are two terminals, T, T′, and a copper wire T′ H *t* runs from T′ to another terminal, *t*, on the upper arm. T and *t* are connected by a long loose strip of gold thread or tinsel, *a b c d*, and near this is fixed vertically a cylindrical bar magnet, N S, whose north pole we will suppose is uppermost; its lines of force (*i.e.* following their positive direction) then point upwards. The terminals, T, T′, are connected to a battery fitted with a current reverser.

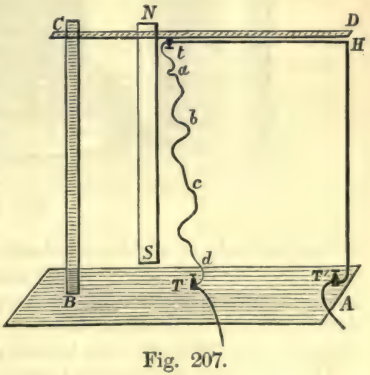

Fig. 207.

Suppose now we let the current enter at T, so that it travels *up* the thread; then what we observe is that the latter instantly winds itself on the magnet in the form of a right-handed spiral. Now, why is this? The lines of force of the magnet point *upwards*. Hence, by clause (1) of Maxwell's Law, the thread will, if it can, arrange itself so that its lines of force will also point upwards. To achieve this it must (§ 205) take such a form that the current viewed from below shall be going round clockwise. And since it is at the same time travelling upwards, that is, from the near to the

far end, it must (§ 202) travel along a right-handed spiral, and accordingly the thread winds itself in that form. Moreover, by clause (2) of Maxwell's Law, it must enclose as many of the magnet's lines of force as possible, which accords with the fact that it *coils itself round* the magnet.

If now we switch back the current reverser so as to make the current travel *down* the thread, the latter will at once untwist itself and then re-wind in the form of a left-handed spiral: the student should think out in detail the reason of this behaviour.

216. Moving Coil Galvanometers. If a coil such as A B, Fig. 205, hangs so that no lines of magnetic force pass through it before the current is sent around it, it will on completing the circuit turn so as to enclose more lines. The extent to which it

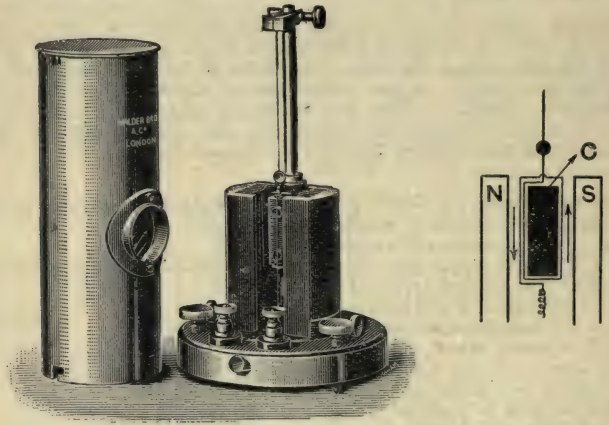

Fig. 208.

turns depends upon the constants of the coil, the strength of the field, the rigidity of the suspension and the magnitude of the current. With the same coil, suspension, and magnetic field the deflection of the coil is directly proportional to the current, so that the arrangement can be used as a galvanometer.

The Despretz D'Arsonval galvanometer shown in Fig. 208 was the first of this type. It consists essentially (see the right-hand figure) of a strong horseshoe magnet N S fixed vertically as shown in the figure and a coil suspended between the poles of the magnet so as to be capable of deflection round the suspension as an axis. The coil is small and rectangular; it is suspended

either by two fine wires as in the left-hand figure (Fig. 208), or by a single wire as in the right-hand figure (Fig. 208), so as to hang in the field between the poles of the magnet. If two wires are used the current is led down one wire and up the other; this is called a bifilar suspension. If only one wire is used the current is led by it down to the coil and out of the coil, by a lower wire attached to a spiral spring joined to the other terminal of the instrument. In this way the suspension wire is kept tightly stretched and the coil, while free to be deflected, is kept in position between the poles. In order to intensify the field in the space in which the coil moves, a cylinder of soft iron C which nearly fills the coil is fixed inside the coil with its axis in the line of the suspension wire.

The coil is adjusted in position, so that with no torsion on the suspension it hangs with its plane parallel to the direction of the field. When the current passes it tends, as described in Maxwell's Law, to turn so as to enclose as many lines of force as possible, and the deflection increases until the moment of the deflecting couple is balanced by the moment due to the torsion of the wire. The deflection of the coil is noted and measured by the usual lamp and scale method, the small mirror being attached to the coil as indicated at *m* in the figure.

This form of galvanometer has many advantages over the old suspended magnet galvanometer. The field in which the coil moves is so intense that any alteration in magnitude or direction of the earth's field has no effect on it. It may consequently be set facing in any direction.

217. Ammeters and Voltmeters. The word *ammeter* is an abbreviation for ampere-meter, and denotes an instrument for the measuring of currents graduated in amperes. The galvanometer also measures currents, but as a rule its resistance is fairly high, so that when it is included in the circuit the total resistance is increased and therefore the current diminished. The ammeter is essentially a low resistance galvanometer which can be included in the circuit without sensibly affecting the current.

The *voltmeter* is an instrument for measuring the potential difference between two points of a circuit and is graduated in volts: it is essentially a high resistance galvanometer. It is not included in the circuit, but its terminals are placed in contact with the two points of the circuit whose P.D. is to be measured.

The voltmeter now forms an alternative path for the current between these two points, but as the resistance is high only a very small portion of the current passes through it, and the original current is practically undisturbed.

By Ohm's Law the current through the instrument is proportional to the P.D. of its terminals, *i.e.* to the P.D. of the two points with which its terminals are in contact: hence the instrument can be graduated to indicate this P.D. in volts.

Both these instruments are largely used by electrical engineers.

218. Mutual Action of Currents. If we place two separate coils of wire, each carrying a current and having suitable freedom of movement, a short distance apart, they will approach or recede from one another in accordance with Maxwell's Law, and the same will be true of two straight currents or of two portions of current-carrying wire, not necessarily complete coils. Bearing in mind the law of electromagnetic polarity (§ 205), we

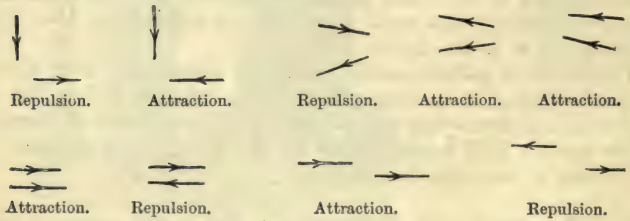

Fig. 209.

can therefore determine the relative behaviour of any two given currents. But we may also refer the behaviour *directly to the currents*, without troubling about their magnetic fields. This was first investigated experimentally by Ampère, who established the following laws:—

(1) *Two* PARALLEL *currents* attract or repel one another according as they flow in the same or in opposite directions.*

* That is current-carrying wires.

(2) *Two* NON-PARALLEL *currents* attract one another if both approach or both recede from the point of meeting of their directions, while they repel one another if one approach and the other recede from that point.*†

These laws are illustrated in Figs. 209 and 210, which the student should carefully study. In Fig. 210 the vertical current, Q P, attracts the portion A M of the horizontal one, and repels the portion M B; but since A M is longer than M B there is, on the whole, attraction; if M were the middle point of A B the total

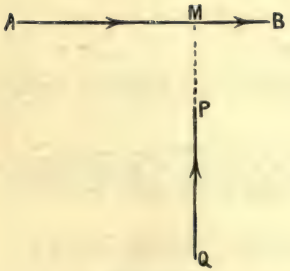

Fig. 210.

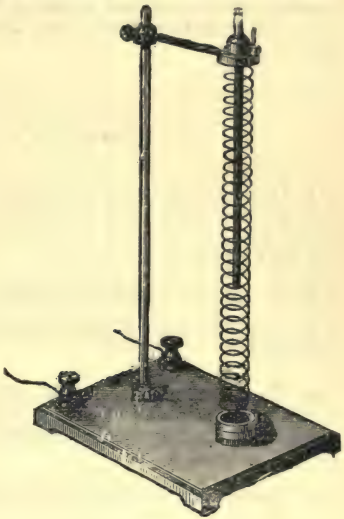

Fig. 211.

force would be *nil*. In any case, however, Q P is propelled to the left, since it is attracted by A M and repelled by M B.

The experiments whereby Ampère's Laws are established merely consist in having two circuits, one fixed and the other movable, placing them in various relative positions and passing

* That is current-carrying wires.

† If the currents be not in the same plane the words "point of meeting of" should be replaced by "common perpendicular to."

currents in sundry directions, when the movable circuit is observed to behave in accordance with the said laws.

Exp. 150. *Set up and work a Roget's vibrating spiral.* Take some copper wire, No. 22, and coil it into a spiral of about 30 turns of 25 mm. in diameter. Suspend the spiral by its upper end from a fixed support, and allow its lower end to dip into a small pool of mercury (Fig. 211). One pole of a battery of 5 cells is joined to the upper end of the spiral and the other pole is joined to the mercury. A current of about 2 amperes is necessary. When the current flows the adjacent turns of the spiral attract one another and the spiral pulls itself up. This breaks the lower contact and stops the current, when the spiral drops by its own weight. The action is then repeated. The introduction of a bar magnet into the coil greatly augments the action.

SUMMARY.—CHAPTER XIV.

1. Distinction between clockwise and counter-clockwise rotation (§ 201), and between right- and left-handed spirals (§ 202). In a right-handed spiral clockwise rotation is associated with a push, and in a left-handed one with a pull.

2. Arago and Oersted's experiments (§ 204).

3. Ampère's and Maxwell's Rules. Application to straight circuits (§ 204).

4. The circular coil. Law of Electro-magnetic Polarity, *viz.* that when a current goes round **clockwise** the lines of force point **away from** the observer, and when **counter-clockwise, towards** him (§ 205). Application of this to spirals (§ 206).

5. Proper way of winding electromagnets (§ 207). Electric bells (§ 208).

6. Construction and delicacy of Astatic Galvanometers (§ 209). The mirror galvanometer (§ 210).

7. The tangent galvanometer. Determination of its constant (§ 214).

8. Two ways of proving the identity of the things dealt with in Electrostatics and Electrodynamics, *viz.* (1) by the condensing Electroscope, (2) by the Astatic Galvanometer (§§ 160, 214).

9. Maxwell's Law, especially its first clause (§ 215). Moving coil galvanometers (§ 216).

10. Ampère's Laws of the mutual action of currents (§ 218).

EXERCISES XIV.

1. A B is a left-handed spiral. The platinum terminal of a Grove's cell is connected to the end A and the zinc to the end B, and the end A is then held a little distance west of a compass-needle. How will the needle behave?

2. A coil of wire is wound on a glass tube in the same direction as the turns of a right-handed corkscrew. How will a compass-needle be affected if placed (1) outside, (2) inside the tube near the end where the current leaves the coil?

3. A vertical copper wire is mounted in a frame, and a compass-needle placed a little to the (magnetic) north of it. A current is now passed along the wire from bottom to top. How will the needle behave?

4. A straight wire carrying a current is placed over a thin rod of iron (1) parallel to the iron, (2) at right angles to it. Is the iron in either case magnetised? Give reasons.

5. A current flows down a vertical wire, and is of such strength that at a distance of one foot from it its magnetic field is equal to the horizontal field of the earth. Indicate in a diagram the directions in which a freely suspended compass-needle would set if carried round the wire at a distance of one foot, when the needle is N., N.E., E., S.E., S., S.W., W., and N.W. of the wire.

6. Describe experiments whereby it is proved that the terminals of a voltaic cell differ electrically in the same way as the prime conductor and rubber of a frictional machine.

7. If you were asked to show that, when the two coatings of a charged Leyden jar are connected by a wire, a current of electricity of the same nature as that produced by a voltaic cell passes through the wire, how would you proceed?

8. A compass-needle is placed at the centre of a vertical ring of wire. When the plane of the ring lies east and west the needle is not necessarily deflected when a current is sent round the ring, but when the plane of the ring lies north and south the needle is deflected violently. Explain this.

9. A rod of soft iron is placed upright on a table. Its upper end is surrounded by a coil of insulated wire which does not touch the rod. When a strong current goes through the wire, the iron rises in the coil. Explain why.

10. A gutta-percha-covered copper wire is wound round a wooden cylinder, A B, from A to B. How would you wind it back from B to A, (1) so as to increase, (2) so as to diminish, the magnetic effects which it produces when a current is passed through it? Illustrate your answer by a diagram drawn on the assumption that you are looking at the end B.

11. A strong electric current flows through a copper wire which passes through the centre of an iron ring, and is at right angles to the plane of the ring. Describe the magnetic state of the ring.

12. Describe an experiment which shows that, in the charging of a condenser by contact of its plates with the poles of a strong battery, there is a current of electricity. Explain why this current is only momentary.

13. Two equal-sized magnets are firmly attached at right angles to a wire which is suspended so that the magnets can rotate in a horizontal direction. The magnets are parallel and like poles are turned in the same direction. You are supplied with a flexible wire through which an electric current is passing, and are required to determine (without touching the magnets) whether they are of equal strengths. Describe and explain your procedure.

14. A long straight wire is stretched on a table in the direction of the magnetic meridian, and a dip circle, with its plane parallel to the magnetic meridian, is placed on the table near to the wire and on the west side of it. Will the dip of the needle be altered when an electric current is passed along the wire from south to north? and if so, how? Give reasons.

15. Insulated copper wire is closely wound on a thin brass rod from end to end in such a way as to form a long spiral, and the ends of the rod are bent round until they touch each other. If an electric current is passed round the wire, what will be the effect on a compass-needle placed close to the spiral? Would the effect be different if the rod were of iron instead of brass? Give reasons for your answer.

16. Describe the construction and method of using a reflecting galvanometer. What are the special advantages of such a galvanometer?

17. A powerful current flows from magnetic east to west in a cable. Supposing the experiment to be performed in England, how would a dipping-needle be affected if placed (i) above, (ii) below, (iii) N., (iv) S. of the cable? Represent the results in a diagram.

18. Sketch the lines of magnetic force produced by a current flowing (*a*) in a straight wire, (*b*) in a wire coiled into a circle. If a circular coil of wire carrying a steady current is free to move in the earth's magnetic field, how will it set itself?

19. The N pole of a long magnet is placed at the centre of a circle of wire conveying an electric current. In what direction is it urged by the current? How does the magnitude of the force depend upon the radius of the circle? Describe an experiment by which you might measure the force in grammes weight.

20. Given a sensitive galvanometer of low resistance, connecting wires, and two voltaic cells made of the same materials but of different sizes, how would you show that they have the same E.M.F.?

Since this is so, what advantage does a large cell usually have over a small one? Can you imagine a case in which it would have no advantage?

21. How would you proceed, and what data would you require in order to measure a current in amperes by means of (1) a tangent galvanometer, (2) electrolysis of copper sulphate? What do you consider to be the relative advantages and disadvantages of the two methods?

22. A narrow glass tube is wound with a spiral of wire and a current passed. A neutral knitting-needle is then placed with one end just inside the tube. On letting go it shoots right in, and stops when its middle coincides with the middle of the tube. Explain this.

23. A De la Rive's floating battery is allowed to come to rest. A pole of a bar magnet is then presented to it sideways at a little distance. Supposing the "battery" to have plenty of room to move about, state and explain how it will behave.

24. Two squat spirals each carrying a current are arranged so that one is quite free to move while the other is fixed, ard are a little distance apart. State and explain the behaviour of the movable one when (1) their faces are parallel and their currents going round the same way, (2) their faces are parallel and their currents going round opposite ways, and (3) their faces are at right angles.

CHAPTER XV.

ENERGY AND HEATING EFFECTS OF THE CURRENT.

219. Preliminary Considerations; the Coulomb and Ampere-hour. In § 158 we saw that the electric current is analogous to the flow of water in a pipe. Now, the water current in a pipe is estimated by the amount of water that passes any cross-section of the pipe in a given time, say the number of gallons per second. In like manner we may conceive of the electric current as estimated by the amount of electricity that passes any cross-section of the circuit per second. The amount that so passes where the current is one ampere constitutes the practical unit of quantity, and is called a *coulomb*; an ampere is thus a *coulomb per second*, or, in other words, a coulomb is an *ampere-second*. Practical electricians frequently employ the *ampere-hour* as the unit of quantity—it is obviously equal to 3,600 coulombs.

EXAMPLE.—A house is fitted with an electric light installation taking 10 amperes, and kept going on an average two hours a day all the year round. Find how many ampere-hours and also how many coulombs are taken per annum.

Ampere-hours taken per day $= 10 \times 2$,

$\therefore$,, ,, ,, annum $= 10 \times 2 \times 365 = 7300$,

$\therefore$ Coulombs taken per annum $= 10 \times 2 \times 3600 \times 365 = 2{\cdot}63 \times 10^7$.

220. Work in any portion of an Electric Circuit. Consider any dead portion of any electric circuit, *e.g.* A B, Fig. 138, suppose a current of C amperes flowing in it, and let V volts be the potential difference between its extremities A and B. Then in one second C coulombs enter this portion of the circuit at A and leave it at B, so that they fall through V volts.

The fall of electricity through a potential difference is analogous to the fall of water or other material substance from a height, and in both cases work is done. We saw in Chapter X., § 149, that the work done by an electrostatic unit quantity of electricity falling down a unit electrostatic potential difference is one

erg. It is shown in more advanced text-books that one coulomb, the practical unit of quantity of electricity, is equal to 3×10^9 or 3,000,000,000 electrostatic units of quantity. Also that a potential difference of one volt is equal to $\frac{1}{300}$ of an electrostatic unit of potential. From which it follows that the work when one coulomb drops a potential difference of one volt is equal to $\frac{3 \times 10^9}{300}$ or 10^7 ergs. Now by definition* a joule is equal to 10^7 ergs, hence the work done when one coulomb of electricity falls down a potential of one volt is equal to one joule. Hence if C coulombs pass through a portion of a circuit and the potential drop is V volts the electrical energy spent is C V joules, or, in other words, the rate of work is C V joules per second.†

Now the rate at which work is done is termed *power*, and the particular rate of one joule per second is termed a *watt*; a watt is thus a unit of power.* We may therefore state our result, thus:—

When a current of C amperes flows through a portion of a circuit the P.D. of whose extremities is V volts, it delivers energy to that portion at the rate of V C watts. For shortness this may be written:—

$$\text{Rate of work} = \text{V C watts} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or still more simply:—

$$\text{Watts} = \text{volts} \times \text{amperes} \quad . \quad . \quad . \quad . \quad (2)$$

the latter being the form usually quoted by electrical engineers, to whom the relation is of frequent service.

The English unit of power is the power which does 33,000 foot-pounds of work per second. This is called a horse-power. The watt is the power of a machine which does one erg of work per second. It is a matter of arithmetic and a knowledge of the units to show that

$$\text{one horse power} = 746 \text{ watts},$$

* Appendix, § 244.

† If the portion A B be alive the same result is true, the only difference is that, if dead, the heat generated in A B is the exact equivalent of this work, which otherwise it is not; *e.g.* if A B contains an electromotor geared to a tram-car, part of the C V joules go to drive the car and only part to heat the coils of the motor.

so that corresponding to equation (2) we have

$$\text{Horse-power} = \frac{\text{volts} \times \text{amperes}}{746} \quad . \quad . \quad . \quad . \quad (3).$$

EXAMPLE.—An overhead trolley wire is at a potential of 500 volts, and a car is taking 12 amperes, find the h.-p. supplied to the car. Also if the motor utilises 85 per cent. of the energy supplied, find the h.-p. at which it is actually running.

The current leaves the car by the rails at potential zero, and therefore the fall of potential is 500 volts. Hence by (3) :—

$$\text{H.-P. supplied} = \frac{500 \times 12}{746} = 8 \text{ about.}$$

The h.-p. at which the car is running will be 85 per cent. of this, which is 6·8.

221. The Heating Effect of the Current.

Exp. 151. Join the poles of a Bunsen's or a Grove's cell by a short piece of iron wire about half a millimetre in diameter. A current flows through the wire. Hold the wire between the fingers; note that it gradually becomes hot, and before long it may become too hot to touch.

The experiment proves that one of the effects of passing a current through a wire is to heat it. By varying the experiment we may learn something more of this **heating effect** of the current.

Exp. 152. Join two wires—one of fairly thin copper, one of platinum of the same diameter—end to end, and the ends of this string to the terminals of a Bunsen's or Grove's cell. Note that the platinum becomes much hotter than the copper. This shows that the heat developed increases with the resistance of the wire.

Exp. 153. Repeat Exp. 152, using now two cells in parallel. The rise of temperature is much faster than before, thus showing that the heat developed varies with the current used—the stronger the current the greater the amount of heat developed.

If we regard a current in a wire as the flow of electrons through the material of the wire we may regard the development of heat by a current as a consequence of the friction which the electrons undergo as they thread their way through molecule to molecule.

The **practical applications** of the heating of wires by currents are numerous:—The surgeon uses a platinum wire made white-hot by a current for cauterising, the miner uses a similar wire for firing his charge, and the naval gunner for firing his torpedo and submarine mine. Iron wires heated by a current are used now for cooking purposes, and carbon filaments and rods made white-hot by the current are extensively employed in the familiar incandescent and arc electric lamps (§ 225).

222. Heat developed by a Current in a Wire. Joule's Law. If a current C amperes passes through a wire, and the resistance of the wire is such that the potential difference between the ends of the wire is V volts, it follows from § 220 that since the wire is merely a dead portion of a circuit the electrical energy turned into heat energy in the wire is V C joules per second. It is, however, more usual to express the heat energy in terms of the resistance and the current. If R is the resistance of the wire, we have, by Ohm's Law, $V = C R$ and therefore $V C = C^2R$.

Our result is, therefore,

$$\left.\begin{array}{l}\textbf{Heat generated per second in a wire}\\ \textbf{of resistance R ohms by a current}\\ \textbf{of C amperes.}\end{array}\right\} = C^2R \text{ joules} \quad . \quad . \quad (1)$$

This is known as *Joule's Law*; from it, it is clear that *for the same current the heat developed in a given time is proportional to the resistance*, while *for the same resistance it is proportional to the square of the current.*

It should be carefully noted that the heating effect in a circuit is in no way dependent upon *which way* the current goes, whether from A to B or B to A; in this respect it strongly contrasts with the chemical and magnetic effects considered in previous chapters.

It is more usual to express heat in terms of calories, a calorie being the heat required to raise one gram of water 1° C. In text-books of heat* it is shown that

1 calorie = 4·2 joules or 1 joule = ·24 calorie, hence it follows that

$$\left.\begin{array}{l}\text{Heat generated per second in a}\\ \text{wire of resistance R ohms by}\\ \text{a current of C amperes}\end{array}\right\} = \cdot 24\ C^2R \text{ calories} \quad . \quad . \quad . \quad (2)$$

* See *Senior Heat*, § 174.

Exp. 154. *Make a Joule's calorimeter and verify Joule's Law.* Take a thin copper calorimeter* and fit it with a cork (Fig. 212. In the figure the calorimeter is made transparent in order to show the interior). Bore four holes through the cork, two quite small, to admit lightly thick copper wires, and two larger, one to admit an accurate thermometer† and the other a stirrer. To the lower ends of the thick copper wire solder the ends of a spiral of silk-covered German silver‡ wire, enough wire being taken for the coil to have a resistance of 10 to 20 ohms. The resistance should be accurately known. To the upper end of the copper leads attach binding screws and connecting wires. Mount the calorimeter in a heat shield as in specific heat work.§ Make up a circuit as in Fig. 213, where G represents a tangent galvanometer whose constant is known, B a battery of 5 Grove's cells, C Joule's calorimeter, K a commutator, R a carbon rheostat. Put a weighed quantity of water into the calorimeter, fit the cork and take the temperature of the water. Now complete the circuit, and adjust R to get a reasonable deflection of the galvanometer and keep the current steady at this mark. The temperature of the water gradually rises. Stir well. Note carefully the time it takes to rise successive degrees.

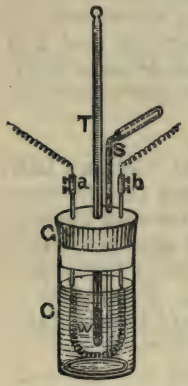

Fig. 212.

While the calorimeter is colder than the air it is gaining heat from outside. When warmer than the air it is losing heat. We shall get our best result if we take the rate of rise of temperature when the temperature of the calorimeter is equal to that of the air. To get this plot a graph between temperature and time, smooth out irregularities, and then calculate the correct slope of the graph when the temperature is equal to that of the air. Calculate the total water equivalent of calorimeter and its contents. Then if M is this water equivalent in gms., and θ is the rise of temperature in degrees Centigrade per t seconds, calculated as explained above,

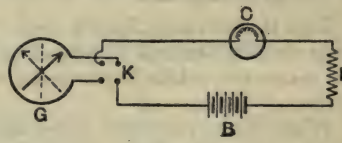

Fig. 213.

$$\frac{M\,\theta}{t} = \text{heat in calories developed per second.}$$

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* A pot made of fairly thin copper regularly used in experiments on heat.

† It should be graduated to fifths or tenths of a degree.

‡ The resistance of German silver does not alter with temperature (§§ 170, 224).

§ See *Senior Heat*, § 65.

Obtain C from the reading of the tangent galvanometer and evaluate C^2R. You should find that

$$\frac{M\theta}{t} = \cdot 24\, C^2R.$$

EXAMPLE.—A steady current was passed through a resistance of 15 ohms and in 2·0 minutes it raised the temperature of 240 gms. of water 5° C. Find the magnitude of the current.

Applying Joule's law, we get

Heat developed per second $= C^2R \times \cdot 24$.

$$\therefore \quad \frac{240 \times 5}{2 \times 60} = C^2 \times 15 \times \cdot 24,$$

$$\text{or } C^2 = \frac{10}{15 \times \cdot 24} = \frac{1}{\cdot 36},$$

$$\text{or } C = \frac{1}{\cdot 6} = 1\tfrac{2}{3} \text{ ampere.}$$

223. Rise of Temperature of a Wire due to the passage of a Current. In the preceding article we have seen that if the same current be passed through two wires the *rate of production of heat* in them is proportional to their resistance. But a little consideration will show that the *temperature they attain* cannot follow the same law. For example, suppose a current to pass through a uniform wire a foot long; the last eleven inches of it have eleven times the resistance of the first inch, and yet of course the wire attains the same temperature throughout.

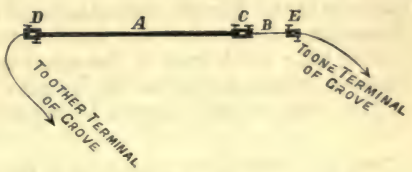

Fig. 214.

Again, consider the arrangement in Fig. 214. A and B are two platinum wires of *equal* resistances, A being long and thick, and B short and thin. They are joined together end to end by a binding-screw C, the ends D and E are joined by binding-screws and copper wires to the poles of a Grove or of a storage battery of three or four cells. In both wires A and B the current is the same, and their resistance is the same, and therefore the rate at which heat is generated in them is the same. *But if you carry out the experiment you will find that the wire B becomes white hot, while A remains merely warm.*

Now, why is this? When we first send the current through any wire it produces heat, which raises the temperature of the wire. But at the same time heat escapes from the sides into the surrounding air by radiation, and as the temperature rises this escape becomes more rapid. After a short time the heat escapes by radiation *as rapidly as it is produced by the current*, and *the temperature then rises no more* however long the current may be kept going; the wire has thus attained its final temperature, which remains unchanged so long as the current continues steady.*

Referring now to Fig. 214, when each wire has attained its final temperature,† since the rate of escape of heat from each is equal to its rate of production, and the latter is the same for both, *the rate of escape must be the same for both.* Now, it is explained in books on heat that in order that the rate of escape from two things may be the same, the one of greater surface must have the lower temperature. (At the *same* temperature the wire of greater surface would obviously give out more heat.) But the wire A has a much greater surface than B, and for this reason the temperature it attains is much lower.

It is shown in more advanced text-books of the subject‡ that **for the same current the rise of temperature is proportional directly to the specific resistance and inversely to the cube of the diameter.** Hence for the same current, thin wires get much hotter than thick ones of the same material, while those of the same thickness get hotter the greater their specific resistance.

This may be easily shown by experiment.

Exp. 155. Take three wires—one of fairly thin copper, one of platinum of the same thickness, and one of much thicker platinum—and join them end to end, and join the ends of this string of wires to the terminals of a Grove's cell. The wires get hot; note that the thin platinum wire becomes much hotter than the thick one and also than the copper.

---

* *Prior* to the attainment of the final temperature, the temperature of the wire depends upon how long the current has been on; this, however, is a different question, and we do not here discuss it.

† It is not implied that they do this in the same time; as a matter of fact thin wires reach their final temperature almost instantly, while thick ones require longer; this, however, does not concern us.

‡ See *Higher Text-book of Magnetism and Electricity*, § 230.

It should be noted that it is not fair to make three *separate* experiments with the wires, using them one after the other to join the battery terminals, because the total resistance is then different in the three cases, and therefore by Ohm's Law the currents are different: by connecting the wires end to end, as described, we ensure that *the same current* goes through all of them.

If we take a wire of the same material throughout, but thicker at some parts than others, and connect its ends to the poles of a battery, the current is the same throughout, as also is the specific resistance. But the diameter is least at the thin parts, and hence by the above law *the thin parts become* hottest.

224. Effect of Rise of Temperature on the Resistance of Wires.

Exp. 156. Take a storage cell B (Fig. 215), and by means of copper wires, A, A, A, arrange it in series with a spiral, W, of rather fine platinum wire,* and a sensitive low-resistance mirror galvanometer, G. Note the deflection of the needle. Then heat W strongly by a Bunsen burner: the deflection will become slightly less. Remove the burner and allow the wire to cool; the deflection will go up to its original value.

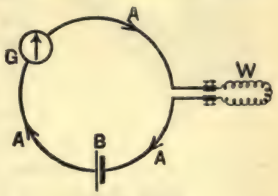

Fig. 215.

This experiment proves that the resistance of the wire W is slightly increased by a rise of temperature. The same is in general true of whatever material the wire be made *except carbon*, whose resistance is *decreased* under the same circumstances; the resistance of the filament of a glow lamp when well "alight" is about three-quarters that of the same filament when cold.

The following experiment illustrates the same thing.

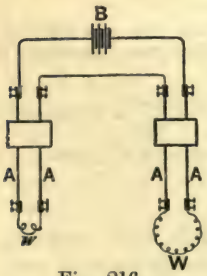

Fig. 216.

Exp. 157. Attach short and long spirals, *w* and W (Fig. 216), of thin platinum wire to lengths of stout copper wires, A, A, A, A, mounted on convenient holders (corks supported by clamps).

* It is convenient to thrust the two stout wires supporting the spiral through a cork.

Join the spirals in series, and their terminals to the poles of a storage battery of two or three cells, giving only sufficient current to make the platinum wires glow dull red. Take a beaker full of cold water and hold it up so that W is immersed. Note that *w* now becomes bright red or even white hot. The explanation is that the water cools W, thus diminishing its resistance ; by Ohm's Law the current then increases, and therefore since, other things being the same, heat produced in *w* is proportional to the square of the current, the temperature of *w* rises.

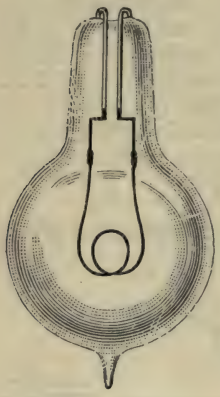

Fig. 217.

225. Electric Lamps. One of the most important applications of the heating effect of currents is the *incandescent electric lamp*. This consists of a glass globe (Fig. 217) completely exhausted of air and containing a fine filament of carbon or metal (tantalum, osmium, etc.) of high resistance attached to terminals of platinum wire fused through the glass. When a strong current is sent through the filament it gets white hot, and being in a vacuum does not oxidise and burn but glows with a steady bright light. Glow lamps are generally run in parallel from leads maintained at a constant potential difference.

EXAMPLE.—A 25-candle-power carbon filament lamp run on a 200-volt-circuit has a resistance when hot of 460 ohms. Find the current it is taking and its efficiency in watts per candle-power.

$$\text{By Ohm's law, current} = \frac{\text{potential difference in volts}}{\text{resistance}} = \frac{200}{460} = \cdot 435 \text{ ampere.}$$

Power required to "run" the lamp = 200 × ·435 ohms or 87 watts.

$$\text{Efficiency} = \frac{87}{25} = 3 \cdot 5 \text{ watts per candle-power.}$$

The filament of the new *osram* lamps is made from wolfram, a tungsten ore. These lamps when used for household purposes may be run on a 25-volt circuit, and on this circuit a 25 c.p. lamp will take about one ampere. Its efficiency is therefore one watt per candle-power.

In **Electric Arc Lamps** a strong current passes through a small gap between the ends of two carbon rods, and the incandescent ends of the rod together with the incandescent matter in

the gap is the source of light. Fig. 218 shows a rather fanciful view of the carbon points. The difference of potential between the ends of the carbon rods (about 40-60 volts) is not nearly high enough to spark across an air gap between them; but if the carbons are put in contact for an instant the heat developed at the point of contact is great enough to heat the ends of the rods to incandescence, and if the rods are then separated a little ($\frac{1}{4}$ in.) the incandescent matter in the gap conducts the current across. The ends of the rod are thus kept glowing and the *arc* persists as long as the current lasts.

Fig. 218.

In the course of running the negative carbon becomes pointed and the end of the positive one becomes hollowed, forming a *crater*. Most of the light of the "arc" comes from this crater. The positive carbon is burnt up faster than the negative one and it seems that particles of carbon are torn off the +ve carbon and deposited on the −ve carbon.

EXAMPLE.—An electric arc of 3000 mean hemispherical candle-power is maintained by a continuous current of 10 amperes from a 50-volt circuit. Find its efficiency.

Watts consumed $= 10 \times 50 = 500$.

$\therefore$ efficiency in watts per candle-power $= \frac{500}{3000} = \frac{1}{6}$.

226. Work in a Battery Circuit. Efficiency. The relations of § 220 may be applied to the several parts of a battery circuit considered in § 163.

The rate of work in the *complete* circuit is the same as the total power developed by the chemical action of the battery; hence bearing in mind that the potential difference of the extremities of the complete circuit is E., the E.M.F. of the battery, we have by equation (1), § 220,

$$\left.\begin{aligned} &\textbf{Total power} = \textbf{E C watts} \quad . \quad . \quad . \quad . \quad . \\ \text{or, as it is often written,} \\ &\textbf{Total watts} = \textbf{E.M.F.} \times \textbf{current} \quad . \quad . \quad . \end{aligned}\right\} \quad (1).$$

In like manner we have

External watts = terminal potential difference × current (2),

and

Internal watts = internal fall of potential × current . (3).

Of the last two items, (2) is the power available for external use, while (3) is the power wasted in heating the battery. Now, in any machine or other contrivance for the supply of energy, the ratio

$$\frac{\text{Power available}}{\text{Total power supplied}}$$

is called the *efficiency* of the machine; hence, from (1) and (2), we learn that

$$\text{Efficiency of a battery} = \frac{\text{Terminal potential difference}}{\text{E.M.F.}} \quad . \quad (4).$$

A rather important consequence follows from (1), *viz.* that if we increase the current supplied by a battery (by diminishing the external resistance) we increase its output of power, and therefore (cf. § 227) the rate of consumption of its material; in other words, increasing the current makes the battery *work harder*. Practical electricians usually estimate the "life" of a cell or battery as so many ampere-hours (§ 219): clearly the harder it is worked the shorter will be the time before it is exhausted.

If we take a voltameter, whose back E.M.F. is E′, and connect it in series with a battery of E.M.F., E, *greater* than E′, then the actual E.M.F. of the circuit is E − E′. If C be the current, then in one second the battery does E C joules of work, of which E′ C is spent in effecting electrolysis, and the remainder (E − E′) C is converted into heat.

227. Consumption of Zinc in a Battery. The work in any complete battery circuit is the equivalent of the "chemical potential energy" of the zinc consumed, and hence in all cases *the rate of consumption of zinc is proportional to the total power of the battery.*

Now let us apply this to a series battery of n cells each of E.M.F., E. There are n "live" parts of the circuit; and, as we follow the course of the current, there is an up-jump E in each, making a total up-jump n E, which is the E.M.F. of the battery.

Now consider *a given current.* Then in a given time a given quantity of electricity passes every section of the circuit, and this has altogether to be pumped up through a potential difference n E.* Suppose now we had only a single cell, and the

* Not quite n E, for there is a slight fall of potential above the liquid in each cell.

external resistance were adjusted so as to give the same current as before; then in the same time the same quantity of electricity would pass every section. But it would only have to be pumped up through a potential E. Hence, to produce *the same current*, n times as much work has to be done by the chemical action in a given time when a series battery of n cells is employed as when only one cell is used, and therefore *in a given time n times as much zinc is consumed.* The same argument applies, with more complexity of detail, to Daniell's or any other cells. The total up-jump of potential through n cells in series is always n E when E is the E.M.F. of each cell, and our conclusion is the same.

This conclusion is sometimes considered rather surprising, but it must be remembered that the consumption of a given amount of zinc means the production of a given quantity, not of *electricity*, but of *electrical energy*; and with n times as much zinc we get the same number of coulombs pumped up *through n times as many volts*—that is, n times as much energy. This energy, supposing the external circuit to be "dead," becomes entirely transformed into heat, of which we get n times as great a supply.

In an *arc* battery the E.M.F. is the same as that of a single cell; hence for the same current the same amount of work has to be done in a given time, and therefore the *total* amount of zinc consumed is the same.

228. Development of Electricity by Heat. Thermo-electric Currents. If a circuit be made up with two wires of different metals joined together at their ends, it is found that a current can be produced in the circuit by heating one of the junctions, or, more generally, by establishing, in any way, a difference of temperature between the two junctions. Such junctions are termed thermo-electric *elements* or *couples*. The existence of this current can be shown by including a galvanoscope in the circuit. The current is maintained so long as this difference of temperature is maintained, and varies in direction and strength as this difference varies.

Exp. 158. Take pieces of iron and copper wire of about equal lengths (say 2 feet), and join one end of each to a sensitive galvanoscope. Twist the other ends tightly together. Warm this junction with the hand or a flame. There is a deflection: thus indicating the passage of a current. Cool the junction in ice: the deflection is reversed. Find the direction of the current in the circuit; with moderate temperatures the current always flows from copper to iron across the hot junction.

Similar results are obtained with a couple formed with wire or rods of any two metals, but the direction and magnitude of the currents are different for different metals. Among the commoner metals bismuth and antimony form the most effective couple at ordinary temperatures.

If a thermo-electric couple is maintained with its junctions at different fixed temperatures, as in the case of the copper-iron couple described above, it is found that, in general, it absorbs heat at the hot junction and gives out heat at the cold junction. There is also, in general, absorption and evolution of heat in the wires of the couple; but, taking the circuit as a whole, there is always more heat absorbed than evolved, so that the circuit absorbs energy in the form of heat and transforms it into the electrical energy of the current produced in the circuit. This electrical energy is finally dissipated in the circuit as heat.

Under ordinary conditions thermo-electric currents are very weak, and thermo-electric couples cannot be economically used in practice for the development of current.

SUMMARY.—CHAPTER XV.

1. In any circuit the rate of work is V C watts (§ 220), and

$$\text{Horse power} = \frac{\text{Volts} \times \text{amperes}}{746}.$$

2. **Heat generated per second in a wire = C^2R joules** (§ 222).

3. For the same current the rise of temperature of a wire is proportional directly to the specific resistance of its material and inversely to the cube of its diameter (§ 223).

4. Application of the heating effect of currents in electric lamps, etc. (§§ 223, 225).

5. In a battery circuit—

Total watts developed = E.M.F. × current.

Watts available externally = T.P.D. × current.

Watts wasted internally = Internal drop × current.

$$\text{Efficiency} = \frac{\text{T.P.D.}}{\text{E.M.F.}} \qquad (\S\ 226).$$

6. In any battery the rate of consumption of zinc is proportional to the work per given time in the entire circuit, *i.e.* to the total power of the battery (§ 227).

7. Thermo-electric currents are formed in circuits composed of different metals when the junctions are unequally heated (§ 228).

EXERCISES XV.

1. One yard of thin, and one yard of thick copper wire are joined in series with a battery of Daniell's cells. Is there any difference between (1) the currents in, (2) the temperatures of, the two wires?

2. Two platinum spirals are included in the same circuit, and glow dull red when a current is passed through them. When one of the spirals is dipped in water the other glows more brightly. Explain these effects.

3. The poles of a battery are joined to the ends of a chain of alternate links of iron and copper. Does the chain acquire the same temperature throughout? Give reasons for your answer.

4. The current from a battery is passed at the same time through a thin wire, and through dilute sulphuric acid, connected in series. What will happen to the wire, and to the dilute acid, and what change, if any, will be produced in each case, by reversing the battery connections so as to alter the direction of the current through the wire and liquid?

5. How is it that if the poles of a battery are connected by a long thin wire, the battery does not get so hot as when a short thick wire is used?

6. A current flows through a copper wire which is thicker at one end than at the other. Is there any difference either (1) in the strength of the current at, or (2) in the temperature of, the two ends of the wire? State how they differ (if they do so at all), and why.

7. A battery of four Grove's cells in series, each cell of E.M.F. 1·8 volt and resistance ·4 ohm, has its terminals connected by a wire of resistance 2·4 ohms. Find (1) the current (in amperes), (2) the number of joules of work done per *minute* by the battery, (3) the number of calories of heat generated per minute in the wire. (1 calorie = 4·2 joules.)
Also find at what fraction of a horse-power the battery is working.

8. What experiment would you make to determine whether the current through an incandescent lamp was alternating or continuous?

9. Write down an expression for the heat generated by a given current in a wire of given resistance, and mention the units employed in measuring the quantities concerned.
In a certain incandescent lamp it is found that the current is 0·6 ampere where the E.M.F. between the terminals is 100 volts. Find the resistance of the filament.

10. A Grove's battery of E.M.F. 6 volts is put in series with a zinc-sulphate voltameter (back E.M.F. 2·27 volts). The resistance of the whole circuit being 7·46 ohms, find (1) the strength of the current, (2) the work done per second by the battery, (3) the amount of work converted into heat.

11. Solve the preceding question, everything being the same except that the voltameter is replaced by a wire of the same resistance.

12. Find the h.-p. required to run an electric light installation taking 90 amperes at 50 volts.

13. A *Kilowatt* is defined at 1,000 watts. Find the terminal potential difference of the terminals of a dynamo which is delivering to the external circuit 600 amperes and 300 kilowatts.

14. 25 storage cells, each of E.M.F. 2 volts and resistance $\frac{1}{100}$ ohm, are joined in series with an incandescent lamp installation having a resistance of 6 ohms. Find (i) the current, (ii) the terminal potential difference of the battery, (iii) the efficiency, (iv) the watts utilised by the lamps, (v) the horse-power at which the battery is working, (vi) the number of Board of Trade Units utilised per working week of 30 hours. (A watt-hour is the amount of energy supplied in one hour by a circuit delivering electrical power at the rate of one watt. A Board of Trade Unit is 1,000 watt-hours, *i.e.* a kilowatt-hour.)

15. The resistance of the filament of an incandescent lamp is 80 ohms, and it carries a current of $\frac{3}{4}$ ampere. Find how many joules of energy are generated in it per hour.

16. A platinum wire of resistance 10 ohms is immersed in a pint and a half of water at the freezing-point. Find what current must be passed through the wire in order that the water may begin to boil after six minutes. [N.B.—To raise a pint of water from freezing to boiling point requires (about) 240,000 joules of energy.]

17. Show that the efficiency of a battery is less the greater the current taken from it. [NOTE.—The student should take care not to confuse between the *efficiency* and the *actual external power*; as the current increases from zero the latter increases up to a certain point and then decreases, and it is proved in the higher part of the subjects that the external power is greatest when matters are so arranged that the T.P.D. of the battery is half its E.M.F., or, which is the same thing, when the external and internal resistances are equal, or when the current is half what it would be on short circuit.]

18. Six cells, each of E.M.F. 1·5 volts and resistance 2 ohms, are connected in series, and the terminals of the battery thus formed joined by a wire of resistance 15 ohms. Find how many watts are developed in the external circuit, and the efficiency of the installation; also find the horse-power at which the battery works.

19. An electric pocket-lamp has a resistance af 20 ohms, and is animated by a small dry battery of resistance 10 ohms and E.M.F. 6 volts. The life of the battery is estimated to be $\frac{1}{4}$ ampere-hour. Find how many flashes the lamp will give, allowing an average of 5 seconds for each flash.

20. The plates of a cell, the resistance of which is inappreciable, are connected by a platinum wire. How would (i) the rate of development of heat in the wire, (ii) the rate of consumption of zinc in the cell, be affected, if the wire were drawn out uniformly to double its length?

CHAPTER XVI.

ELECTROMAGNETIC INDUCTION.

229. Faraday's Induction Experiments. Faraday found by experiment that a current could be produced in a coil of wire by the motion of a magnet or of a current-carrying wire in its neighbourhood.

Exp. 159. *Preliminary.*—(1) Wind a length (say 10 metres) of cotton-covered copper wire to form a ring-like coil (about 2 or 3 cm. in diameter). (2) Arrange a sensitive galvanometer (preferably a moderately dead beat* low resistance reflecting instrument), adjust the index to the centre of the scale. (3) Join, say, the left terminal of the galvanometer with the negative pole of a cell. Touch the other terminal with a wire connected to the positive pole of the cell. Observe whether the index moves to the right or left.† Record the direction in which the index moves, and the terminal at which the current leaves the galvanometer. Then disconnect the cell. (4) Move a strong permanent bar magnet to-and-fro at a distance of 3 or 4 metres from the galvanometer; observe whether the galvanometer index is affected. If so, find a position further off where the magnet has no influence. (5) Clamp the coiled wire at the position thus determined, and by long wires join its ends to the galvanometer terminals.

I. (1) Quickly push nearly half the length of the magnet into the coil, note whether the index moves to the right or left. Observe that the index presently comes to rest in its original position, thus showing that the induced current of electricity that produces the deflection is only *momentary*, it only exists during the movement of the magnet. (2) Now withdraw the magnet rapidly and observe that the deflection is in the opposite direction to the previous one. (3) Repeat the operations, but move the magnet more slowly. Note that each deflection is in the same direction as before, but is smaller. (4) Next repeat the operation, using the other end of the magnet.

Finally, determine for each case, by applying the rule of § 205, the polarity, due to the induced current, of the face of the coil towards or from which the magnet is moved. If, for instance, the preliminary test

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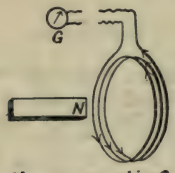
* A galvanometer is said to be dead beat when its pointer or index quickly comes to rest after being disturbed.

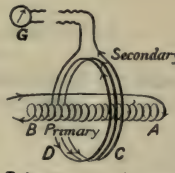
† With an indicator like that of the galvanometer in Fig. 197 describe the deflection as clockwise or counter-clockwise.

(3) shows that the index moves to the right when current leaves the galvanometer at the left-hand terminal, and the observations of I. show that the current induced when the N-end of the magnet approaches the coil (Fig. 219) produces a deflection to the right, then, on moving the finger along the wire of the coil in the direction of the current, it will be



Magnet approaching Coil

Fig. 219.



Primary current increasing

Fig. 220.

observed that it passes counter-clockwise round the end of the coil nearest the magnet and seen from the magnet. Therefore the induced polarity is north. Hence there is repulsion between the coil and the approaching N-pole. Record as in the table below.

	Operation.	Deflection.	Polarity of face of coil.
1	N-end approaching	(right or left)	(north or south)
2	N-end receding		
3	S-end approaching		
4	S-end receding		

II. Join one or two cells (Bunsen's, Grove's, or secondary) in series with a plug key (plug out) and a solenoid* (spirals, 1-2 cm. diameter, made by winding insulated copper wire on a brass or glass tube into which an iron rod can be slipped, Fig. 220). Complete the circuit through the plug key, and determine the polarity of the ends of the primary solenoid. Next push about half the length of the primary through the secondary coil, and note the direction of deflection: also on withdrawing. Repeat, using the other end of the primary. Also when the solenoid has an iron core.† In

* The solenoid joined with the battery is called the *primary*, that joined with the galvanometer is called the *secondary*.

† Use a cylindrical bar of soft iron which just fits within the tube.

each case determine the polarity of the face of the secondary coil towards or from which the primary is moved. Record as in the table below.

	Operation with primary.	Deflection.	Polarity of front face of secondary coil.
1	N-end pushed in	(right or left)	(north or south)
2	N-end pulled out		
3	S-end pushed in		
4	S-end pulled out		
5 etc.	When the primary coil has an iron core the deflections are the same in direction as before, but are of much greater magnitude.		

III. Arrange the primary (withdraw plug from key) so that the secondary lies over the end of the primary. Note the direction of the deflection when the current is (1) made, (2) broken. Determine the polarity of the end of the primary coil and that of the face of the secondary nearer to it. Repeat with the other end of the solenoid. Record as below. In Fig. 220 the end B of the primary behaves as a S-pole, and the nearer face of the secondary behaves as a N-pole.

	Operation with primary.	Deflection.	Polarity of near face of secondary.
1	Current made so that the end, A, of primary is a N-pole	(right or left)	(north or south)
2	End, A, of primary, a N-pole, current broken		
3	Current made so that the end, A, of primary is a S-pole ...		
4	End, A, of primary, a S-pole, current broken		

IV. Arrange as in III. Close the circuit through the primary. Note the direction of deflection when the soft-iron core is (1) slid into the primary, (2) withdrawn from the primary. Determine as in III. the

polarity of the end of the primary and the face of the secondary nearer to it. Repeat with the other end of the solenoid. Record as below.

	Operation with primary.	Deflection.	Polarity of face of secondary.
A	Current flowing so that the end of primary is a N-pole		
1	Iron core introduced... ...		
2	Iron core withdrawn... ...	(right or left)	(north or south)
B	End of primary a S-pole ...		
3	Iron core introduced... ...		
4	Iron core withdrawn... ...		

230. Summary of Exp. 159, I., II., III., IV. The experiments show that the induced current flows *one way* (sometimes called the *inverse induced current*) for (1) an approaching N pole,* (2) a receding S pole,* (3) a N pole increasing in strength, (4) a S pole decreasing in strength; *the reverse way* (sometimes called the *direct induced current*) for (1) a receding N pole,* (2) an approaching S pole,* (3) a N pole decreasing in strength, (4) a S pole increasing in strength.

The induced current is in each case momentary, it is called into play while the magnet is moving or the pole changing in strength.

The following summarises the observations of Exp. 159, I. and II. The induced current in the secondary always magnetises it in such a way that the force called into play between it and the inducing pole of the primary is one that opposes the relative motion of the primary and secondary; that is, if the two are being separated the force is one of attraction; if the two are being brought together the force is one of repulsion. The induced pole, for instance, is North if a North pole is approaching, it is South if a North pole is receding.

The following summarises the observations of Exp. 159, III. and IV. The induced current in the secondary always magnetises it in such a way that it opposes the change of magnetisation of

* Either of a permanent or electro magnet.

the primary;* that is, if the magnetisation of the primary is increasing in strength (by making or increasing the current or adding the iron core), then the induced polarity of the secondary is in the *opposite* direction to that of the primary, and in the *same* direction when the primary is decreasing in strength. In Fig. 220 if the connections are such that A becomes a N pole and B a S pole, then the magnetisation of the secondary is related to it as below.

	Primary Coil.	Secondary Coil.
Primary magnetisation increasing	End B a S pole and end A a N pole	End D a N pole and end C a S pole
Primary magnetisation decreasing	End B a S pole and end A a N pole	End D a S pole and end C a N pole

When unlike poles are adjacent the total magnetisation is less than the magnetisation of the primary, when like poles are adjacent the total magnetisation is greater than the magnetisation of the primary.

Now in all the cases described above the magnet or the primary coil produces a magnetic flux through the secondary, and obviously when moved, or when the magnetisation of the primary is varied, this flux alters. Moreover, we may place the secondary wire in any magnetic field whatever, and then if we do anything to alter the strength of the field, and thereby the flux through the coil, it is found that a secondary current is set up. We thus have the perfectly general law that *whenever the magnetic flux through a coil of wire is in the act of changing, an induced current flows in the coil.*

231. Amperian Currents. Consider a straight electromagnet; its magnetism is due to the current circulating round it, and by the law of polarity (§ 205) this current is going clockwise or counter-clockwise, according as the electromagnet is held so that the lines of force point away from or towards the observer.

* When a current is made or broken in a circuit it does not reach its maximum strength nor become zero instantaneously: a finite time is required to effect the change, and it is during this short interval that the induced E.M.F.'s are developed.

Now with regard to *permanent* magnets, a theory propounded by Ampère was that they owe their magnetism to electric currents circulating round their molecules and magnetising them in accordance with the law of § 205 : these are called *Amperian Currents.* Whence they are derived, or whether they even exist at all, need not concern us, but as a hypothesis they are very convenient in studying the laws of induction, and the student should acquire the habit of mentally picturing them in association with any magnet or any part of a magnetic field.* Thus, looking along an ordinary bar magnet (Fig. 221), the Amperian currents must be pictured as circulating clockwise or counter-clockwise according as the north or south pole is away from us, while taking any part of a magnetic field whether in air or in iron, and looking along its lines of force (in their positive direction), we must picture that part of the field as possessing clockwise Amperian currents.

Fig. 221.

In § 230 we mentally figured all our coils as magnets and deduced the law of induction. We may now figure all our magnets as carrying Amperian currents, and we shall find the same law.

232. Law of Electromagnetic Induction. When the secondary current flows in the same way as the primary current (either an Amperian current or an ordinary current) it is said to be a *direct* secondary current. When the secondary current flows in the opposite way to the primary current the secondary current is said to be *inverse.* In Figs. 222, 223, the primary coil P is on the left, the secondary S on the right. When the number of magnetic lines of force through the primary changes as indicated the current in the secondary flows in the direction of the dotted arrows. The secondary

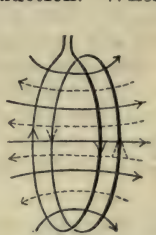


Fig. 222.

Fig. 223.

* On the most recent theory these currents are in actual existence and are due to rotating electrons (§ 90).

current in Fig. 222 is an *inverse* current, that in Fig. 223 a *direct* current.

We may therefore embody the law of the induced current in the following succinct form :—

Increased flux in primary	**I**nverse current in secondary.
Decreased „ „ „	**D**irect „ „ „

The similarity of the commencing letters of these words as indicated in thick type constitutes a convenient *memoria technica* whereby it is impossible to forget the law.

233. Case of a Rotating Coil. Let A C B (Fig. 224) be a coil of wire capable of rotation about its diameter C D, and let the ends A, B, dip into two mercury slots (not shown) from which wires are led to the terminals of a galvanometer; in this way during the rotation the same end of the coil will always be in electrical connection with the same galvanometer-terminal, and yet there will be no mechanical impediment to the motion.

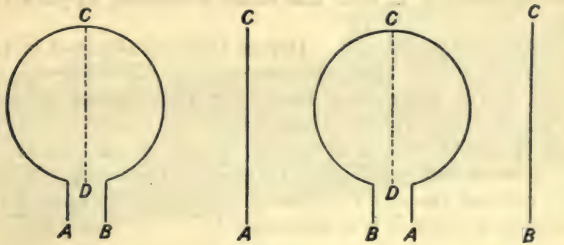


Fig. 224.

Now let the coil be placed in a magnetic field, and for clearness let us suppose the lines of force of the field perpendicular to the plane of the paper and their positive direction pointing away from us, as we view the paper in front; the Amperian currents of the field (§ 231) are therefore clockwise.

Let us start the coil in the position No. 1 (with its plane perpendicular to the lines of force), and by a series of jerks bring it into position No. 2 (with its plane along the lines of force), thence to position No. 3 (which is half a complete turn from

No. 1), and thence through position No. 4 (which is half a complete turn from No. 2) back to No. 1 again, and finally a little further, so that the coil shall have passed completely *through* all the positions, and let us observe the induced current corresponding to the several movements as indicated by the galvanometer. Then it will be noticed that the current undergoes reversal on passing positions 1 and 3, but *not* on passing 2 and 4.

To explain this let us refer to the law of § 232. From 1 to 2 the flux through the coil is *de*creasing, therefore the induced current is *di*rect, that is clockwise. From 2 to 3 the flux is *in*creasing, therefore the induced current is *in*verse, that is, counter-clockwise. It, therefore, at first looks as if there were a reversal on passing position 2. But it must be noticed that after passing position 2 *we are looking at the other side of the coil*, more or less as presented by No. 3, and a counter-clockwise current viewed from this new side is in the same direction *in the coil* as a clockwise one viewed from the original side; both, in fact, go round the coil in the direction A C B. Hence there is *no reversal on passing position* 2, and the same reasoning applies to position 4.

Now consider position 3. Before the coil attained it the flux was increasing and the current counter-clockwise, *i.e.* in the direction A C B, while after passing it the current is clockwise, *i.e.* in the direction B C A.; hence *there is a reversal on passing position* 3, and the same, of course, is true of position 1.

The general law applicable to this and all cases is that *the induced current reverses itself when the coil is in such a position that the flux through it is a maximum.**

234. Lenz's Law. We saw in § 232 that if a magnet approached a coil, N pole in front, the induced current in the coil was such as to give the face of the coil N polarity. Hence by § 218 repulsion occurs between the coil and the magnet. If the coil were suspended by fine wires this repulsion would be very noticeable. We should obtain the same result if we worked in currents. Looking from left to right in Fig. 219, the amperian currents in the magnet are clockwise, and we see the induced

* Strictly speaking, the reversal does not occur *exactly at* the position of maximum flux, but *a little beyond it*; the explanation of this is beyond our present needs.

current is counter-clockwise, hence by Ampère's first law repulsion occurs between the magnet and the coil, *i.e.* the induced current in the coil is such as to retard the approach of the magnet to it. If we consider all the other cases of induction in the same way we shall find that they all come under the following very important law:—

Lenz's Law. *When one current induces another the direction of the latter is such that mutual action between the two tends to oppose the motion which gives rise to the induced current.*

The student who remembers this law and can apply it will never have a difficulty of deducing the direction of induced currents.

235. Arago's Disc. A striking illustration of Lenz's Law is afforded by the following experiment of Arago's. A disc of

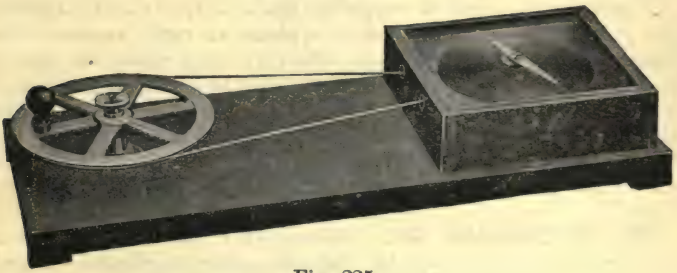


Fig. 225.

copper (Fig. 225) is made to rotate in a horizontal plane immediately below a delicately balanced magnetic needle, the axis of rotation of the disc being vertically below the pivot of the needle. As the disc rotates it is found that the needle is gradually deflected in the same direction as the rotation, and if the rate of rotation is sufficiently high, finally takes up a motion of rotation in the same sense as the disc, but at a slower rate. This result is explained by the fact that currents are induced in the copper disc by its rotation relative to the magnet, and the reaction between the disc and the needle is in accordance with

Lenz's Law such as to tend to stop the motion of the disc, but the needle being movable and not fixed, the result of this is that the needle is itself set in motion, so as to diminish the *relative* motion of the disc and needle.

The direction of the induced current in the disc is such that a current always flows along the diameter of the disc vertically below the needle in such a direction as to deflect the needle in the same direction as the rotation of the disc. The current flowing along this diameter spreads out at one end, and curving round the sides of the disc unites again at the other end; hence if the disc has a number of radial slits cut in it, the inductive effect on the needle is much less marked.

In the same way, if the copper disc is fixed, and the needle made to oscillate, the oscillations quickly die away and the needle comes to rest, as the result of the interaction of the induced currents and the oscillating needle. This *damping* effect of a copper plate under a magnetic needle is sufficiently well marked to be of practical use.

Similarly, if a disc of copper be made to spin rapidly between the poles of an electromagnet in a plane at right angles to the lines of force, it is much more difficult to spin when the current is on than when it is off; and if the disc be put in rapid motion with the current off, it is quickly brought to rest when the current is put on. The interaction of the induced currents and the poles of the magnet is such as to stop the motion of the disc. The currents in this case are often called *eddy currents*.

236. The Induction Coil. Ruhmkorff's *Induction Coil* is an arrangement of two coils for the production of induced currents of high electromotive force. The coils are arranged one inside the other, the inner, or *primary*, coil consisting of a small number of turns of thick wire, and the outer, or *secondary*, coil of a very large number of turns of thin wire. The coils are fixed in position and the induced currents in the secondary coil are obtained by arranging that a strong current in the primary coil shall be automatically made and broken a large number of times per second by an automatic "make" and "break" contact in the primary circuit. The induced currents thus obtained in the secondary coil are of very high potential, and the terminals of the coil may be made to yield sparks and "shocks" in the same way as the prime conductor of a friction or influence electrical machine.

Fig. 226 shows the exterior and Fig. 227 is a diagrammatic sketch of a common form of induction coil: the primary and

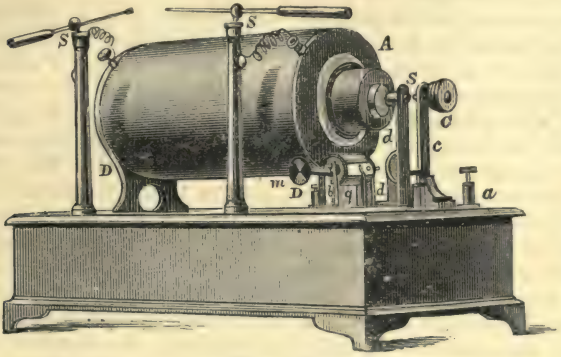


Fig. 226.

secondary coils, one inside the other, form the cylinder A B, but to strengthen the inductive action of the primary on the secondary, an iron core made up of a bundle of soft iron wires is fitted as the

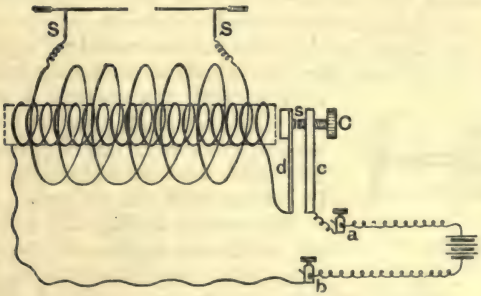


Fig. 227.

axis of the primary coil. This core increases the inductive effect by increasing along the axis of the coil the magnetic field due

to the current in the coil, so that when the current is made or broken the change in the flow of force through the coil is greater than it otherwise would be. The automatic "make" and "break" is shown to the right.

The terminals of the primary coil are seen at a, b; the current entering, say at a, flows beneath the baseboard to the base of the pillar c,* and from thence to the spring hammer d across the screw contact s between c and d. The end of the screw s is tipped with platinum, and rests on a small disc of platinum attached to the spring d, so that, as these platinum surfaces always remain clean, the contact is always good. The current then passes around the coil, and returns to the terminal b.

The action of the "make" and "break" will now be evident. When the current first passes, the iron core of the primary coil becomes an electromagnet and attracts to it the mass of soft iron which forms the head of the spring hammer d; this results in breaking the contact with s, and the current being now interrupted the iron core ceases to be an electromagnet, the hammer d now springs back into contact with s, and again makes the current. In this way the current is continuously made and broken at a very rapid rate.

The arrangement shown at D to the right of Fig. 226 is a commutator by which the current in the primary circuit may be reversed at will. The essential parts are the cylinder l, which can be rotated by the screw m, and the two upright springs p and q; the cylinder is of ebonite, and has two strips of brass parallel to its length let into its surface, and 180° apart. When it is desired to start the current these strips are brought into contact with the springs, p and q, and the direction of the current may be reversed by turning the handle m through 180° and thus interchanging the strips in contact with the springs.

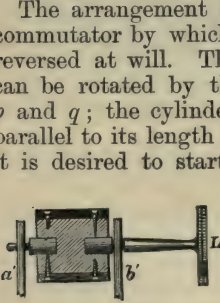


Fig. 228.

The section in Fig. 228 will show how the commutator acts—the terminals a, b (Fig. 226) are connected to the pillars a', b' (Fig. 228) which support the brass axis of the cylinder. This axis, as the figure shows, is divided, and one end is con-

* For simplicity the connections to the commutator D (described below) are not here referred to.

nected to one of the brass strips on the cylinder and the other end to the other strip. The springs p and q are directly connected to the ends of the primary coil through the make and break at s. From this description of the connections it will at once be evident how the current in the primary coil is reversed by rotating the cylinder l through 180°, so as to interchange the springs in contact with the brass strips.

The terminals of the secondary coil are shown at S S (Figs. 226 and 227) carrying discharging rods, between the ends of which sparks can be made to pass when their distance apart is not too great.

Induction coils are largely used instead of electrical machines to produce high-tension currents; they are used to actuate X-ray vacuum tubes, to make sparks in the cylinders of the engines of motor-cars, to make high-tension currents in radio-telegraphy, etc.

To set the coil working. (1) See that the commutator is *off*, so that the circuit is broken. (2) Connect two or three Bunsen's or secondary cells with the terminals, a, b. (3) Make the length of the spark gap small. (4) Adjust the head, C, so that the screw end touches the spring, d. Longer sparks may be obtained by screwing more tightly. (5) Turn the commutator so that the circuit is completed. *Remember* that the immediate neighbourhood of the spark gap is **dangerous**; do not touch the metal mountings or any part of the apparatus, except the handle of the commutator, while the coil is working.

237. Faraday's Law of Induced Currents. We have seen in the above sections that the induced current in a coil is the result of change of magnetic flux through it. Moreover, for the same coil the more rapid the change of flux the stronger will be the current, for in Exp. 159, I., we get a greater displacement of the spot of light when the magnet is moved quickly than when it is moved gradually. If we work with different coils it is found that the induced current depends not only on the rate of change of flux but also on the resistance of the coil; the induced *electromotive force*, however, depends upon the rate of change of flux *only*.

If the coil be merely a single turn of wire (not necessarily circular), it was shown by Faraday that *the induced E.M.F. is proportional to the rate of change of flux through it*, and, it being understood that the rate of change of flux is estimated in *unit tubes per second* (§ 39), the actual relation is—

$$\left.\begin{matrix}\text{Induced E.M.F.}\\ \text{in volts}\end{matrix}\right\} = \frac{\text{Rate of change of Flux}}{\text{100 million}} \quad . \quad (1).$$

If the coil be a spiral of n turns, each turn acquires the E.M.F. given by (1), and since the several turns are in series these E.M.F.'s add together like the E.M.F.'s of the cells in a series battery, so that the E.M.F. in the spiral in n times (1). Now the product of the flux through a coil and the number of turns in it is called the *effective flux*; hence the more complete statement of Faraday's Law is—

$$\left.\begin{matrix}\text{Induced E.M.F.}\\ \text{in volts}\end{matrix}\right\} = \frac{\text{Rate of change of Effective Flux}}{\text{100 million}} \quad \ldots(2).$$

(2) clearly shows that other things being the same *the induced E.M.F. is greater the greater the number of turns*, hence it is usual in induction experiments to employ coils containing *many turns of fine wire.*

An important case of (2) is that of a core wound with two equal spirals *in parallel*; electrically these correspond to two cells in parallel and (cf. § 187) *their E.M.F.'s do not add*; in reckoning the effective flux we must therefore multiply the flux through the coil by the number of turns in *one of the spirals only*, (2) then gives the E.M.F. in one of them, which (cf. § 187) is the same as that of the pair.

If the flux changes at a uniform rate the E.M.F. is, of course, constant; in general, however, the rate of change is not uniform, but taking its value *at any instant* (2) gives the E.M.F. *at that instant*, while taking its *average* value during any specified time (2) gives the corresponding *average E.M.F.*

When the E.M.F. is known the corresponding induced *current* is found by dividing it by *the total resistance of the circuit* just as in (1), § 163.

When any conductor or part of a conductor moves so as to cut magnetic lines of force a difference of potential is produced between points on it. The direction of the potential difference depends on the relative direction of motion of the conductor, and that of the magnetic lines of force. The magnitude of the potential difference depends on the number of magnetic tubes cut per unit time. The total potential difference acting along any conductor is the algebraic sum of the potential differences induced in its various parts. To produce a current the algebraic sum must therefore not be zero. In order that the resultant E.M.F. in a closed circuit may not be zero the flux through the circuit must alter.

EXAMPLES.—1. A spiral contains 500 turns of wire, and is moved in a magnetic field. At a certain instant the flux through it is 10,000 (unit tubes), and half a second later it is 4,000. Find the average E.M.F. in the coil during the half-second.

At the start the effective flux is $500 \times 10{,}000$, *i.e.* 5,000,000 unit tubes. At the end of the half-second it is $500 \times 4{,}000$, *i.e.* 2,000,000. The change in the half-second is therefore 3,000,000, *i.e.* the average rate of change is 6 *million unit tubes per second*. Hence by (2) the average E.M.F. is $\frac{6 \text{ million}}{100 \text{ million}}$, *i.e.* ·06 *volt*.

2. In the preceding question, if the resistance of the coil be 100 ohms and it be placed in circuit with a galvanometer of resistance 400 ohms, find the current through the galvanometer.

We have :—

$$\text{Current} = \frac{\text{E.M.F.}}{\text{Total resistance}} = \frac{\cdot 06}{100 + 400} = \cdot 00012 \text{ ampere or } \cdot 12 \text{ milliampere.}$$

238. Dynamos and Electromotors. Imagine a ring-shaped coil of wire like that shown in Fig. 224 placed in the strong magnetic field between the poles of a large horseshoe magnet and made to rotate rapidly round an axis at right angles to the direction of the lines of force in the field. It is evident from what was stated in § 224 that as the coil rotates the flux or number of tubes of force threading the coil is constantly changing, and, since the number threading it is a maximum and minimum every half-revolution, and also the direction of threading is reversed every half-revolution, an alternating current will be induced in the coil.

By means of mechanical contrivances wires can be placed in contact with the ends of the coil so that either alternating or continuous currents can be drawn off through the external circuit. An arrangement of this kind constitutes a simple form of *dynamo*.

Dynamos producing large currents such as are required for commercial purposes are driven by steam or by water-power where this is available, as for instance in the famous case of the Niagara Falls.

If, instead of generating an electric current by doing mechanical work on the moving parts of a dynamo, we pass through its circuit a current developed by external sources, the machinery is set in motion and is now able to do work. The dynamo is now acting as an *electro-motor*. In a dynamo mechanical energy is converted, in part at least, into electrical energy, and in an electro-motor the reverse takes place. Thus the electric current serves very well to transmit energy, and most of the Power Schemes, now so important to industrial undertakings, invoke the aid of the electric current. Some of the powerful currents produced by dynamos at the Niagara Falls are conveyed some hundreds of miles to work machinery.

239. The Telephone. If a very thin plate of soft iron D (Fig. 229) is fixed a short distance from the end of a strong bar magnet, NS, many lines of force will crowd into the intervening space, and a coil of wire, C C, may be placed so as to embrace these lines. If now the plate be made to vibrate by means of a

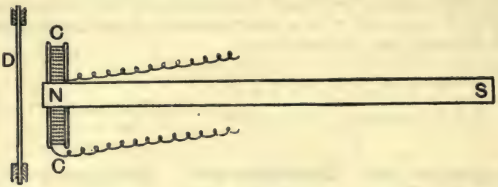


Fig. 229.

person talking to it, the distribution of the lines of force in the field is altered with each vibration, and currents are induced in the coil. If wires are led from this coil to another similarly mounted coil, the induced currents will alter the lines of force and thus make the plate of the second instrument vibrate in *exactly the same way as* the receiving plate. The words spoken are thus reproduced. Such an instrument is a simple *telephone.*

240. The Energy of Induction Currents. It remains to explain the source of the energy of induction currents. When the number of lines of force passing through a circuit in a magnetic field changes as the result of any displacement, a current starts in the circuit and, at once, a force is exerted tending to prevent the displacement to which the current is due. Hence, to effect the displacement, mechanical work must be done against this opposing force. This work is transformed into the electrical energy of the induction currents. For example, in the case of a dynamo, mechanical work is done by an engine in maintaining the rotation of the coil in the magnetic field against an opposing couple tending to prevent this rotation. This work is transformed into electrical energy in the coil and its external circuit. Hence, in the development of a current by induction, mechanical work is converted into the electrical energy of the current.

SUMMARY.—CHAPTER XVI.

1. Induction Currents and their dependence on change of magnetic flux (§ 229).

2. Amperian Currents (§ 231) and the Law of Induction in terms of them, *viz.* :—

Increased	Inverse	(§ 232).
Decreased	Direct	

3. Reversals of induced current occur at positions of maximum flux (§ 233).

4. Lenz's Law and its application to Arago's disc, etc. (§§ 234, 235).

5. The Induction Coil (§ 236.)

6. Faraday's Law of Induced Currents (§ 237).

7. Action of the dynamo, the electromotor, and the telephone (§§ 238, 239).

EXERCISES XVI.

1. You are given a source of electric current, a bar of iron, and a coil of copper wire, and are required to make the bar into a magnet. How will you do it? You are given a bar of iron, a coil of copper wire, and a magnet, and are required to generate an electric current. How will you do it?

2. Two separate coils of insulated copper wire lie one inside the other. The terminals of the outer coil are connected to a delicate galvanometer, and those of the inner to a battery which furnishes a steady current. Describe and explain the behaviour of the galvanometer (i) when a bar of unmagnetised iron is suddenly thrust out into the inner coil, (ii) when, after having been left a short time, it is suddenly withdrawn.

3. If in Fig. 219 the magnet be placed with its centre on the axis of the coil, and itself perpendicular to that axis, and then be suddenly moved parallel to itself, its centre travelling along the axis, the galvanometer needle *does not move*. Explain this.

4. In Fig. 219 a long bar magnet is employed which is pushed through the coil by a series of jerks, the induced current corresponding to each jerk being observed. Show that the current will be reversed after the middle of the magnet has passed the middle of the coil.

5. In Fig. 219, instead of passing the magnet through the coil, the following arrangement is adopted :—A soft iron core is fixed in the aperture of the bobbin, the magnet held perpendicular to it at a little distance, and then, by a series of jerks, made to approach one of its ends and drawn along that end until it leaves it by its opposite pole. Show that the induced current will be reversed at or near each end of the magnet, but not at its middle.

6. A coil of insulated copper wire is wound round a watch lying face upwards on the table, and the ends of the coil connected together. Describe the effect when the north pole of a magnet which has been resting on the face of the watch is suddenly withdrawn.

7. A coil of wire is wound on the north pole of a magnet, and the pole is then presented to a piece of iron, which is drawn towards it. Determine the direction of the current induced in the coil. In what form has energy disappeared to appear as heat developed by the current?

8. A powerful and massive bar-magnet hangs horizontally by a cord an inch above a table. Copper rings are placed flat upon the table under the poles when in its undisturbed position. Upon setting the magnet swinging, electric currents are induced in the rings. Why? Heat appears in the rings. Why, and from what source is the equivalent energy derived?

9. Two coils of insulated wire lie horizontally one inside the other. The inner is in series with a battery and an adjustable resistance, and the outer with a galvanometer. The adjustable resistance is varied so that the current in the inner coil increases at a given rate. Describe and explain in what way, if any, the indication of the galvanometer will be altered by reducing the number of turns on the outer coil, supposing the galvanometer to have (i) inappreciable resistance, (ii) very high resistance. Also what difference will it make if the inner coil is placed vertically?

10. A spiral consists of 200 turns of copper wire having a total resistance of 50 ohms, and its terminals are connected with a galvanometer of resistance 150 ohms. A magnet is moved near the spiral in such a way that in $\frac{1}{20}$ second the flux through the latter changes from 1000 to 2800 unit tubes. Find the current through the galvanometer.

11. Enunciate Lenz's Law, and hence or otherwise deduce that if there be two parallel conductors, one the "primary" in circuit with a battery, and the other the "secondary" in circuit with a galvanometer, then, when they are pushed closer together, a current will be induced in the secondary in the opposite direction to that in the primary, and *vice versâ*. What will happen if the two conductors, instead of being parallel, are at right angles?

12. Would it be possible, without electrodes, to produce a current in a hollow glass ring filled with an electrolyte? If so, how?

13. Describe the induction coil, and give some account of its actions.

14. An iron cone wound with insulated copper wire is set vertically, and a light metal ring placed loosely over its upper end. On passing a strong *alternating* current through the copper wire, the ring jumps several feet into the air. Explain this.

15. The N pole of a magnet is thrust towards a metal ring. In what direction is a current induced in the ring? How does the strength of the current depend upon the material of the ring? Why *must* the direction of the current be such as to exert a mechanical force upon the pole in a direction opposed to its motion?

16. A coil of insulated wire, of which the ends are joined, is suspended by a long fine thread from a point in its circumference. A bar magnet is moved suddenly towards the coil (which is protected from air currents) along a line perpendicular to the plane of the coil and passing through its centre. What effect is produced upon the position of the coil?

Does the effect depend upon (1) the initial position of the coil with respect to the meridian, (2) the number of turns in the coil? Give reasons.

APPENDIX I.

SOME NOTES ON MECHANICS.

241. Units. In many departments, both of science and of every-day life, we encounter things which call for estimation in numerical terms; such are length, time, velocity, electric charge, etc. In order to estimate any of these we must have a standard or *unit* of its own kind in terms of which it may be expressed—*e.g.* in assigning the length of a stick, we may take a foot as our unit and then say the stick is six feet long.

There are three special quantities calling for estimation in this way, viz. *length*, *mass*, and *time*, and the units of these are termed *fundamental*; all others, such as those of work and of electric charge and potential, being based upon them, are said to be *derived*. In England the fundamental units most commonly employed are the *foot*, *pound*, and *second*; on the Continent they are the *centimetre*, *gramme*, and *second*.* For scientific purposes the three latter are invariably adopted, and the English student, who is frequently unfamiliar with the first two, should get to know them by actual practical work. Neither of them can be *exactly* expressed in terms of the British units, but the following relations are approximately true:—

1 centimetre (cm.) = ·3937 inch, or 1 inch = 2·540 cm.

1 gramme (gm.) = ·002205 lb., or 1 lb. = 453·6 gms.

242. Work and Energy. Whenever a mechanical force is exerted in such a way as to make a body move, *work* is said to be done by the force or by the agent which exerts the force. Thus, a horse does work in pulling a cart, and a steam-engine does work in moving a train. When a stone falls, the weight of the stone (that is, the attraction of the earth upon it) does work, while if a man lifts it the man does work *against* the

* The centimetre, gramme, and second, together with all units based upon them, constitute what is termed the "C.G.S." system, which is also called the "Decimal System," because all multiples and sub-multiples of the units proceed by powers of 10 and $\frac{1}{10}$.

attraction of the earth; and in general, if a body be placed under circumstances such that a force, P, acts on it, and free play be given to the force, the latter will drive the body in a certain direction and thus do work; while if some outside force, Q, be introduced so as to drive the body in the direction opposite to that which it would take under P's influence, then Q does work against P.

Mere force apart from motion does no work: thus, when a weight rests on a table, it exerts a pressure but no work is done. Again, mere motion apart from force implies no work; when a ball is thrown along the surface of smooth ice, work has to be done to give it a start, but after that the ball will travel a great distance without any more force being applied because the ice offers very little resistance to the motion; and if we could obtain a *perfectly* smooth horizontal surface of unlimited extent, and the resistance of the air could be abolished, it would go on moving for ever without the application of any force, and therefore without any work being done.*

It is very important to note that the motion which results from the expenditure of work is not always *actual bodily* motion. If we fix a nail in a vice and file it, we exert force and do work, but we do not move the nail from its place. It will be found, however, that the nail becomes hot. Now as explained in treatises on heat, heat itself is due to rapid movement of the particles or *molecules* of a body: in this case, then, the result of the work done is to cause the molecules of the nail to move more rapidly than before, so that *the work produces heat.*

It is a very common tendency of work to produce heat, and in all machinery precautions have to be taken to prevent this. If the bearings of any machine be rough or not properly greased, then when the machine is driven, say by a steam-engine, much of the work of the engine, instead of producing the proper motion of the machine which we want, produces heat in the bearings which we do not want, and is therefore wasted. This waste is the equivalent of the work done against the resistance or friction of the bearings: by having these smooth and well greased the friction is greatly reduced, and the work done against it consequently much less.

* This is Newton's first law of motion, which is fully discussed in treatises on Mechanics.

Whenever anything has the power of doing work it is said to possess *energy*. It is usual to distinguish between two forms of energy, *potential** and *kinetic*. To understand the distinction, let us take an example of each. When a Dutch or "grandfather" clock is wound up, the weight possesses the power of driving the clock, and so doing work—that is, it possesses energy. This energy is due to the fact that *it is raised*, for after it has run down it can no longer drive the clock: it is *energy due to position*, to position in this case *relative to the earth*, and is an instance of *potential* energy.

Next consider a train approaching a station with the brakes on. The train is doing work against the friction of the brakes, which work is converted into heat: it therefore possesses energy, and this time the energy is due to the fact that the train is moving, it is *energy due to motion*, and is an instance of *kinetic* energy. By *potential energy* we mean, then, energy due to *relative position*, and by *kinetic energy* that due to *motion*. A watch spring when wound up affords a good instance of potential energy; here it is a question of the position of the coils *relative to one another*.

There is a principle fully discussed in books on Mechanics and Heat† called the principle of *Conservation of Energy*, which asserts that *energy is indestructible*; it can be transformed from one condition to another, but like matter it can neither be created nor destroyed.

Thus when a stone falls from a height it starts with a certain stock of potential energy; as it descends this gradually disappears, being transformed into kinetic; just before it strikes the ground the energy is all in the kinetic form; and *after* it has struck, this bodily kinetic energy is changed into heat, that is, molecular kinetic energy. A piece of coal possesses potential energy due to *its position relative to the oxygen of the air*—to the fact, that is, that it is separated from the oxygen but is capable of combining chemically with it. When the coal is burned it combines with the oxygen, and this potential energy is converted into heat. The mechanism of a locomotive enables us to re-convert some of

* The student to whom the subject is new must be careful not to confuse between the word *potential* as here used, and the *electric potential*: no doubt there is a relation between the electric potential and potential energy, but for the present it is best to disregard it.

† See *Senior Heat*, § 176.

this heat into bodily kinetic energy, as shown in the movement of the train; and lastly, when the brake is put on this, is again converted into heat. But throughout all these changes there is neither *creation* nor *destruction* of energy.

Energy or work, in whatever form, is capable of mathematical measurement in the same way.

243. Derived Units. The Dyne and the Erg. There are two derived units in the C.G.S. system, which, though properly belonging to Mechanics, are of great importance in Electricity and Magnetism. These are the dyne and the erg.

The **dyne** is the unit of force and is equal to the weight* of 1 gramme divided by the acceleration of gravity. The divisor is equal to 981, therefore the dyne is equal to the weight of 1·02 milligrammes.

Work is done by a force when its point of application moves in the direction of the force or has a component in this direction. The work done by the force

= force × distance through which its point of application moves (measured parallel to the line of action of the force).

Unit work is done when unit force acts through unit distance.

The unit of work in the C.G.S. system is called the **erg**. An erg is therefore the work done when a force of 1 dyne acts through a distance of 1 cm. Obviously if a constant force of F dynes acts through a distance of s cm., the work done is F s ergs.

244. The Joule and the Watt. The erg is a very small unit of work, and it is more convenient to use a larger unit. The unit chosen is called the **joule**. The joule is equal to ten million ergs, *i.e.* 1 joule $= 10^7$ ergs.

An agent or machine is said to be of unit **power** when it can work at the rate of 1 erg per second. This unit of power is very small and in practice we use a unit called the **watt**, the watt being equal to the power of an agent or machine which works at the rate of 1 joule per sec.

Work is equal to the product of power and time. Thus if an

* The meaning of weight is explained in Mechanics; the weight of a body is the force with which the earth attracts it.

agent does an amount of work W joules in time t secs. its power is $\frac{W}{t}$ watts, or if an agent of power P watts works for t secs., the work done is Pt joules (or watt-secs.). The unit of power in electrical engineering is the **kilowatt** or thousand watts, and the unit of time the hour, so that electrical energy is often spoken of in terms of **kilowatt-hours**.

245. The Parallelogram of Forces. If two forces acting on the same particle are represented by two adjacent sides of a parallelogram, drawn from their point of application, their resultant* shall be represented in magnitude and direction by the diagonal of the parallelogram drawn through that point.

Thus if two forces P and Q (Fig. 230) represented in magnitude and direction by A B and A C respectively act on a point at A, the result on A is the same if we replace P and Q by a force R represented in direction by A D and in magnitude proportional to the length of A D on the same scale as A B represents C or A C represents Q. R is called the resultant of P and Q.

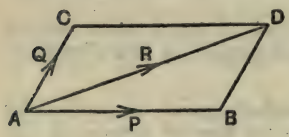


Fig. 230.

In all cases the value of R can be determined graphically by drawing the figure to scale. Thus suppose that in the above figure A B is 2 units on any scale, A C 3 units, and A D $3\frac{1}{2}$ units on that scale. Then A B represents 4 dynes, A C represents 6 dynes, and A D 7 dynes.

Thus the resultant of two forces of 4 and 6 dynes acting along A B and A C is a force of 7 dynes acting along A D.

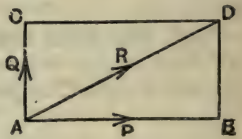


Fig. 231.

When the angle B A C is known the value of A D can be deduced by calculation.

(a) In the simple case when $\angle BAC = 90°$ (Fig. 231),

$$AD^2 = AB^2 + AC^2,$$

$$\therefore R^2 = P^2 + Q^2.$$

(Example 3, p. 196, is an instance of this.)

* That is, the single force which would have the same effect as these two combined.

(*b*) Other simple cases arise when the $\angle BAC =$ some multiple of 30° or 45°.

As an example take the case when $\angle BAC = 60°$, and let AB represent 5 dynes and AC 4 dynes (Fig. 232). Draw DE perpendicular to AB. The $\triangle DBE$ is half an equilateral $\triangle$,

$$\therefore BE = \tfrac{1}{2}BD,$$

and $$DE = \sqrt{3}\,BE;$$

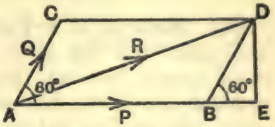


Fig. 232.

$\therefore$ AE represents $5 + \frac{4}{2} = 7$ dynes,

and DE represents $2\sqrt{3}$ dynes. Now taking the right-angled $\triangle ADE$ we see

$$AD^2 = AE^2 + DE^2 \text{ and } \therefore R^2 = 7^2 + (2\sqrt{3})^2 = 61,$$

or the resultant of forces 5 and 4 dynes inclined at an angle of 60° with each other is a force $\sqrt{61}$ dynes inclined as shown in the figure.

246. The Resolution of Forces. Just as any pair of forces, acting at a point, can be replaced by a single force that will produce the same effect, so any single force can be replaced by two forces which, acting together at the same point, will produce the same effect as the single force. Two such forces will be those represented by the adjacent sides of any parallelogram whose diagonal, passing through the same point, represents the single force.

This process of finding a pair of forces equivalent to a single force is called the *Resolution of the Force*, and the forces obtained are called the *Components of the Force*. Generally a force is resolved along two lines at right angles to each other. An instance occurs in § 48, where the magnetic field of the earth is resolved into its horizontal and vertical components.

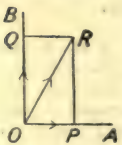


Fig. 233.

As an instance of finding components let us find the resolved parts of a force of 12 units along two axes at right angles, one axis making an angle of 30° with the line of action of the force. Then set off a line OR (Fig. 233), representing 12 units. At one end, O, draw a line, OB, making an angle of 30° with OR. Through O draw OA, at right angles to OB, on the opposite

side of O R. Through R draw R P, R Q parallel to these lines. Then O P, O Q represent the components in magnitude and direction.

247. Moment of a Force. If a body is hinged or pivoted at one point and the body is acted on by a force applied at any other point, the only possible motion of the body is one of **rotation** round the first point. It may be, therefore, possible to study the relation between what we may call the rotating effect of a force and its magnitude and position by fixing one point in the body.

DEFINITION.—The **moment of a force about a given point** is **its tendency to produce rotation** about that point regarded as fixed, and it is **measured by the product of the force and the perpendicular from the centre of rotation on its line of action.**

The Principle of Moments. When a body, acted on by several forces in one plane, is in equilibrium, the sum of the moments of forces tending to turn the body one way about any point in that plane is equal to the sum of the moments about the same point of the forces tending to turn the body the other way round.

This may be proved experimentally.

248. Couples. DEFINITION.—A **couple** consists of two equal forces acting in opposite directions along two parallel straight lines (Fig. 234). *A couple cannot by itself keep a body in equilibrium*, for it tends to rotate the body—the points of application of the two forces tending to move in opposite directions.

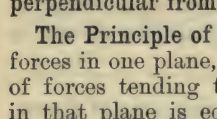


Fig. 234.

Examples of couples.—In winding a clock we apply a *couple* to the key, for we do not try to make it move to one side or the other, but simply turn it round. To spin a small top between the finger and thumb we apply a *couple* to it by moving the finger and thumb sharply in opposite directions. To unlatch a door we apply a *couple* to the handle.

DEFINITIONS.—(1) The **arm** of a couple is the perpendicular distance between the lines of action of its two components (*i.e.* the two forces forming the couple).

(2) The **moment** of a couple is the algebraical sum of the moments of its two components about any point in their plane. The two moments must be added if they tend to produce motions in the same direction, and subtracted if they tend to produce motions in opposite directions. It is easy to show that *the result will be the same* for moments taken about *any* point in the plane.

The following are equivalent definitions :—

(*a*) *The product of the measure of either force into the arm of the couple.*

(*b*) *The moment of either of the two forces about any point in the line of action of the other force.*

A couple cannot be balanced by a single force, but two couples in the same plane whose moments are equal and opposite will balance each other. This is obvious from the definition of moments.

Therefore, if a body is in equilibrium under the action of two couples, it follows that the moments of these couples must be equal and opposite.

APPENDIX II.

THE THEORY AND PRACTICE OF SOME HARDER EXPERIMENTS.

249. In this chapter we shall deal with some important instruments used in the further study of magnetism and electricity. The instruments chosen are the Magnetometer, the Quadrant Electrometer, the Sine and Ballistic Galvanometers, Shunts, the Potentiometer, and the Wheatstone Bridge. In most cases we shall commence by giving a brief outline of the theory of the principles which are involved; this will be followed by a description of the instrument, and finally by an account of the chief simple experiments in which the instrument is used.

250. Intensity of the Magnetic Field due to a Simple Bar Magnet. In § 38 we have seen how in general the intensity at any point in the magnetic field due to a simple magnet may be calcu-

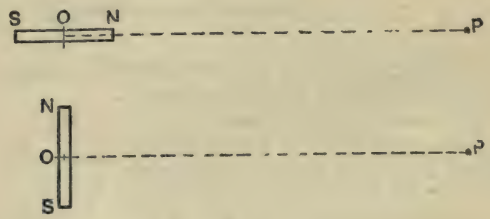


Fig. 235.

lated. There are two special cases now to be considered. These are shown in Fig. 235. In the upper figure the point P, at which the intensity is required, is on the prolongation of the magnetic axis NS; and in the lower figure on a line bisecting NS at right angles. The former is called the **end-on** position of the magnet with respect to the point considered; the latter is called the **broadside-on** position.

(1) **Field at P when the Magnet is End-on.** The direction of the field is obviously along the axis, and if the north pole of the magnet is nearer P than the south pole, the direction of the field is away from the magnet, and if the south pole is the nearer the field is directed towards the magnet. Take the former case. To find the intensity at P, suppose a unit north pole placed there, then this pole is subjected to two forces, one of repulsion due to the north pole and one of attraction due to the south pole.

Let m be the pole strength of the magnet, $2l$ its length, and M its magnetic moment, so that $M = 2ml$. Let O be its middle point and $OP = d$.

The forces on the unit pole at P are $\frac{m}{NP^2}$ away from N and $\frac{m}{SP^2}$ towards S. Therefore if F is the intensity of the field at P

$$F = \frac{m}{NP^2} - \frac{m}{SP^2} = \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2} = \frac{m(d+l)^2 - m(d-l)^2}{(d-l)^2(d+l)^2}$$

$$= \frac{4\,m\,d\,l}{(d^2-l^2)^2}$$

$$= 2\,M\,\frac{d}{(d^2-l^2)^2}. \quad \ldots\ldots \quad (1)$$

If the distance from P to the magnet is very great in comparison with the length of the magnet, *i.e.* if $\frac{l}{d}$ is very small

$$(d^2-l^2)^2 = d^4\left(1-\frac{l^2}{d^2}\right)^2 = d^4\left(1-\frac{l^2}{2d^2}+\text{negligible terms}\right) * = d^4$$

very nearly: hence

$$F = \frac{2M}{d^3}. \quad \ldots\ldots \quad (2)$$

(2) **Field at P when the Magnet is Broadside-on.** In this case a further diagram (Fig. 236) is required. Denote NP or SP by D. Then the forces on the unit pole at P are $\frac{m}{D^2}$ along NP and $\frac{m}{D^2}$ along PS. These forces may be represented by the two sides

* By the Binomial Theorem.

PQ, PR of the parallelogram PQTR, and the diagonal PT will then represent the resultant force on the same scale as PQ, PR represent the components. By symmetry it follows that PT is parallel to NS, and that the triangles PQT, NPS are similar.

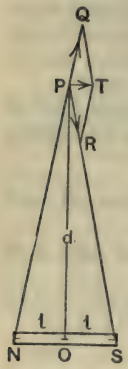


Fig. 236.

Therefore denoting as before the intensity at P by F we have

$$F : \frac{m}{D^2} :: PT : PQ$$

$$:: NS : PN$$

$$:: 2l : D$$

$$\therefore F = \frac{2ml}{D^3} = \frac{M}{D^3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Now $D^2 = d^2 + l^2$

$$\therefore F = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}}. \quad . \quad . \quad . \quad . \quad (4)$$

If $\frac{l}{d}$ is small we have

$$F = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}} = \frac{M}{d^3\left(1 + \frac{l^2}{d^2}\right)^{\frac{3}{2}}} = d^3\frac{M}{\left(1 + \frac{3}{2}\frac{l^2}{d^2} - \ldots .\right)}$$

$$= \frac{M}{d^3} \text{ approximately.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In this case F is directed through P parallel to the magnet and away from the north pole. Note that equations 1, 3, 4 are rigorously true; in practice $\frac{l}{d}$ is usually made small so that equations 2 and 5 are oftener employed. It will be seen that for the same distance from P to the magnet the intensity is twice as great for the end-on position as for the broadside-on position.

The above equations have been used in § 62.

251. The Deflection Method of Comparing Magnetic Moments and Field Strengths. It was shown in § 64 that if at any point in the earth's field there is superposed a magnetic field at right

angles to the magnetic meridian a compass needle situated at the point is deflected an angle δ from the meridian where

$$\tan \delta = \frac{F}{H},$$

H being the strength of the earth's field and F the strength of the superposed field.

Suppose this field F is due to a distant magnet in the end-on position, then $F = \frac{2M}{d^3}$ with the notation and conditions of § 249, and therefore

$$\frac{2M}{d^3} = H \tan \delta \text{ or } M = \frac{d^3 H \tan \delta}{2}. \quad . \quad . \quad . \quad (6)$$

This shows how M, the magnetic moment of a magnet, may be determined by direct measurement.

If the field is due to a magnet in the broadside-on position we have

$$\frac{M}{d^3} = H \tan \delta \text{ or } M = d^3 H \tan \delta. \quad . \quad . \quad . \quad (7)$$

Using either the end-on or the broadside-on position it follows from (6) that if with the same magnet at the same distance from a needle in different fields H, H_1, we get deflections δ, δ_1,

$$H \tan \delta = H_1 \tan \delta_1 \text{ or } \frac{H}{H_1} = \frac{\tan \delta_1}{\tan \delta}, \quad . \quad . \quad . \quad (8)$$

which shows how fields may be compared.

If with different magnets of moments M, M_1, at the same distance in the same field we get deflections δ, δ_1,

$$\frac{M}{M_1} = \frac{\tan \delta}{\tan \delta_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which shows how magnetic moments may be compared.

Instead of placing the magnets at the same distance we may alter the distances until we get the same deflection. If d, d_1 are the respective distances, then

$$\frac{M}{M_1} = \frac{d^3}{d_1^3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In doing this we place one of the magnets with its north pole to the east and the other with its north pole to the west and adjust the distances till the needle is undeviated from the meridian. It follows then that the deflections which the magnets would separately produce are equal.

252. The Vibration Magnetometer. In § 59 it was stated that when a bar magnet suspended by a torsionless fibre oscillates in a horizontal plane the square of the frequency of oscillation is proportional to the intensity of the magnetic field. The full relation, of which the above is only a portion, is contained in the equation

$$T = 2\pi\sqrt{\frac{K}{MH}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where T = the period of the vibration,

π = the ratio of the circumference of a circle to its diameter,

K = the moment of inertia of the magnet about its axis of rotation,

M = the magnetic moment of the magnet,

H = the intensity of the horizontal component of the earth's magnetic field.

The moment of inertia of a body involves its mass and dimensions and the position of the axis about which the moment is taken. For our purposes it may be looked upon as a physical constant of a body as much, say, as the volume is. For a rectangular body swinging about an axis through its centre perpendicular to the face contained by the length and breadth $K = M\frac{l^2 + b^2}{12}$ where M is the mass of the body and l and b are its length and breadth respectively. For a cylindrical body swinging about an axis through its centre perpendicular to the length of the cylinder $K = M\frac{l^2 + 6r^2}{12}$, where l is the length of the cylinder and r is the radius of cross section.

If n = frequency of the vibration

$$n = \frac{1}{T},$$

$$\therefore n = \frac{1}{2\pi}\sqrt{\frac{MH}{K}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Therefore if n, n_1 are the frequencies of vibration of the same magnet in different fields

$$\frac{H}{H_1} = \left(\frac{n}{n_1}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

This is the relation (1) of § 59.

Again, if magnets of the same mass and size (*i.e.* the same K), but of different magnetic moments, say M, M_1, vibrate with frequencies n, n_1 in the same field, then

$$\frac{M}{M_1} = \left(\frac{n}{n_1}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

253. The Horizontal Component of the Earth's Magnetic Field. The determination of H is usually carried out in two steps: (1) a Deflection experiment; (2) an Oscillation experiment.

Fig. 237.

(1) **The deflection experiment.** In this experiment a magnetometer is used. This consists of a small magnetic needle pivoted or suspended so as to move freely in a horizontal plane. When the needle is pivoted as in the form shown in Fig. 237 the deflection of the needle can be read off on a circular scale by means of a light pointer $p\ p$ attached to the needle $n\ s$, with its

length at right angles to the axis of the needle. To eliminate error due to parallax in reading the position of the pointer the circular scale should be engraved on a mirror or on a ring of paper or other substance lying on a sheet of mirror. The needle and circular scale are placed in a little box mounted on a long graduated beam SS, the length of SS being in line with the zero divisions on the scale.

When left to itself the needle *n s* of the magnetometer sets itself in the magnetic meridian, care having been taken to remove all magnets and pieces of iron from the neighbourhood. The beam is then turned until the pointer sets at zero; the beam is

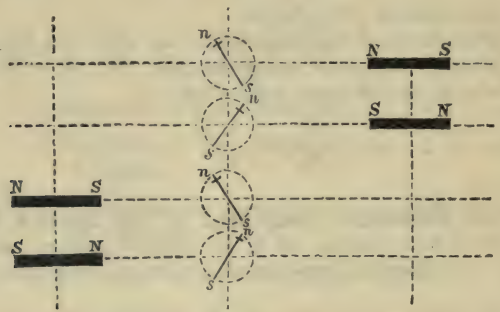


Fig. 238.

now at right angles to the magnetic meridian. A magnet NS is now placed in the groove of the beam so that it is "end-on" with respect to the needle. A reasonable deflection *d*, having been obtained the distance *d* of the centre of the magnet from the pivot of the needle is noted. Both ends of the pointer are read and the deflection noted.

For the more accurate determination of the deflection four observations for the same value of *d* are taken in the way indicated in Fig. 238, the true value of δ being the mean of the eight readings obtained.

From the readings we can by application of the formula

$$\frac{M}{H} = \frac{1}{2} d^3 \tan \delta$$

or the more exact formula

$$\frac{M}{H} = \frac{1}{2} \frac{d}{(d^2 - l^2)^2} \tan \delta$$

obtain the value of $\frac{M}{H}$.

M is usually unknown, and it is to eliminate M that the second part of the experiment is performed.

(2) **The oscillation experiment.** This experiment is performed exactly as in § 60. The application of the formula

$$T = 2\pi \sqrt{\frac{K}{MH}}$$

gives us $$MH = \frac{4\pi^2 K}{T^2}$$

(3) Denoting the values of $\frac{M}{H}$ and MH by A and B respectively we get

$$M = \sqrt{AB} \quad H = \sqrt{\frac{B}{A}}$$

whence M and H are determined in C.G.S. units.

254. Proof of the Inverse Square Law. The inverse square law was used to obtain the relation

$$\frac{M}{H} = \frac{1}{2} d^3 \tan \delta.$$

If then for the same magnet at the same place of observation we find that when placed at different distances from the magnet the product $d^3 \tan \delta$ remains constant, it follows that the inverse square law is true.

Also if for the same distance d the angles of deflection obtained in the end-on and broadside-on positions are δ and δ', then the inverse square law gives $\frac{\tan \delta}{\tan \delta'} = 2$. This may be confirmed by experiment.

Exp. 160. *Find the value of the horizontal component of the earth's magnetic field and the magnetic moment of the magnet supplied.* Use a magnetometer as figured above if one is available; if not use a well-pivoted or silk-suspended short compass needle moving over a graduated circle placed at the middle of a long graduated beam (an optical bench proves useful for this purpose). In the oscillation experiment the amplitude of swing of the magnet should be kept as small as is convenient.

EXAMPLE. In an experiment made to determine the magnitude of the horizontal component of the earth's magnetic field and the magnetic moment of the magnet, the following observations were taken :—

Details of Magnet:—Mass = 100·5 gm. Dimensions = 15·2 cm. × 1·2 cm. × ·65 cm.

Deflection Experiment. End on Position:—The values of δ quoted are the mean of 8 readings (4 positions of the magnet and both ends of needle read each time).

$d = 30$ cm. $\quad \delta = 21°$

$d = 35$ cm. $\quad \delta = 12·9°$

$d = 40$ cm. $\quad \delta = 8·6°$

Oscillation Experiment.—The magnet was suspended with its broad face vertical. The time occupied by three sets of 6 complete vibrations were 138, 142, 140 secs. respectively.

Calculation:—Using the formula $\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan \delta$ the three sets of readings in the deflection experiment give $\frac{M}{H}$ equal to 4530, 4460, 4500 respectively, whence $\frac{M}{H} = 4500$, nearly.

The Moment of Inertia of the magnet

$$= 100·5 \times \frac{(15·2)^2 + (·65)^2}{12} = 100·5 \times 19·3 = 1940.$$

Also $$T = \frac{140}{6} \text{ secs.},$$

$$\therefore MH = \frac{4\pi^2 \times 1940 \times 36}{19600} = 140,$$

$$\therefore H = \sqrt{\tfrac{140}{4500}} = ·18 \text{ dyne per unit pole, nearly,}$$

and $$M = \sqrt{140 \times 4500} = 800 \text{ C.G.S. units, nearly.}$$

The Inverse Square Law.—The constancy of the three values obtained for $\frac{M}{H}$ is evidence of the truth of the inverse square law.

Electrostatics.

255. A rough and ready comparison of electrical potential may be made with the gold leaf electroscope. The quantitative comparison of potential is made by means of instruments called electrometers. We shall give here a brief description of one of the simplest forms.

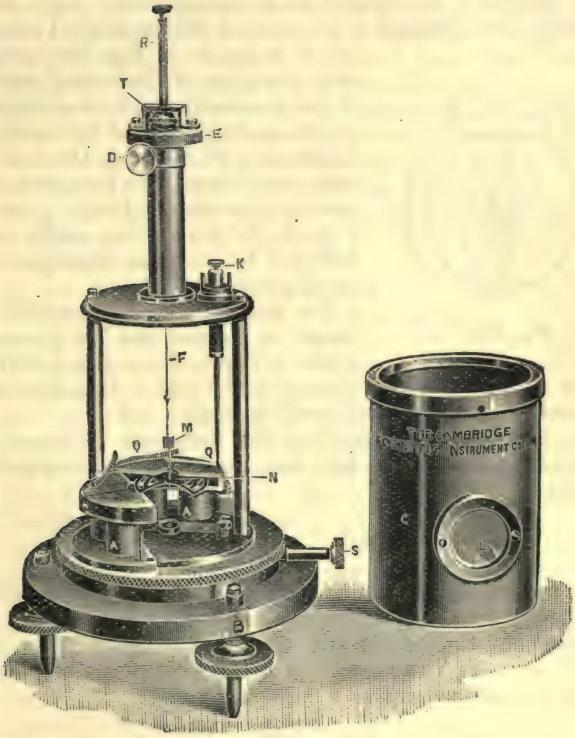


Fig. 239.

256. The Quadrant Electrometer. The quadrant electrometer consists essentially of four quadrantal boxes (such as might be obtained by cutting a shallow cylindrical box into four quadrants)

and a light paddle-shaped needle of aluminium foil or silvered paper suspended by a fine fibre so that it hangs inside these boxes. Fig. 239 shows the instrument known as the Dolezalek pattern. The four quadrants marked Q are mounted horizontally on short insulating amber stems A. These stems are fixed two by two on movable metal pieces. In the figure one of these metal pieces is moved out to show the needle N.

The needle N is charged to a comparatively high constant potential; thus charged it is attracted by the neutral quadrants, but if it lie, as in Fig. 240, symmetrically along one of the lines of separation of the quadrants, the forces of attraction are in equilibrium, and the needle is not displaced. If, however, the four quadrants are divided into two pairs by connecting opposite quadrants (*a* to *c* and *b* to *d*) by wires, the needle may be deflected by charging these pairs to unequal potentials.

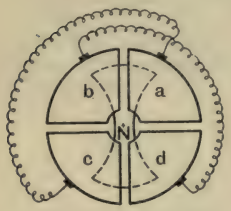


Fig. 240.

For example, if the potential of the needle is positive and the potentials of *a* and *c*, *b* and *d* are also positive, but that of *a* and *c* is higher than that of *b* and *d*, then the needle is deflected away from *a* and *c* and towards *b* and *d*.

The full relation embodying the deflection in terms of the potentials of the needle and the quadrants is contained in the formula

$$\theta = c\left(\mathrm{V} - \frac{v_1 + v_2}{2}\right)(v_1 - v_2) \quad \ldots\ldots\ldots \quad (1)$$

where θ is the deflection, c a constant depending on the fibre and the dimensions of the quadrants and needle, and V, v_1, v_2 are the potentials of the needle and the pairs of quadrants. If as is usually the case V is very great in comparison with v_1 and v_2 we get the further relation

$$\theta = c\,\mathrm{V}\,(v_1 - v_2) \quad \ldots\ldots\ldots\ldots \quad (2)$$

and if V is maintained constant for any given set of experiments, cV remains constant and we can replace cV by a new constant C, thus getting

$$\theta = \mathrm{C}\,(v_1 - v_2) \quad \ldots\ldots\ldots\ldots \quad (3)$$

If further one pair of quadrants is earthed, $v_2 = 0$ and

$$\theta = Cv_1 \quad \ldots \ldots \ldots \ldots \quad (4)$$

The needle is attached to a vertical wire of aluminium (Fig. 239), which is attached by a hook to the suspension fibre F, and this in turn is attached to a metal rod R which bears a screw so that the altitude of the needle may be adjusted by turning a milled head T. The fibre F may be either an insulator or a conductor. If the former it is made of quartz and the needle is then charged by bringing a charged conductor in contact with the screw K, this screw bearing a rod and wire Q which can be turned round to make contact with the aluminium wire supporting the needle. If the fibre is desired conducting it is made of phosphor bronze or silver, and the needle is then charged through R, R being insulated from the metal of the case by an ebonite head E. The needle may be made to set in any particular direction by turning E in its seat. E can then be clamped in position by tightening the screw D.

If one pair of quadrants is connected to a charged conductor of large capacity, the conducting system made up of the conductor and the two connected quadrants will be at a potential slightly lower than that of the conductor before connecting it to the quadrants. The other pair of quadrants being earthed it follows that the deflection of the needle is proportional to that potential. Similarly with any other charged conductor a deflection proportional to its potential may be obtained, and the ratio of the two deflections is thus a measure of the relative potentials of the two conductors.

For convenience and cleanliness the terminals attached to the two pairs of quadrants are beneath the instrument shown in Fig. 239. They pass up through amber plugs in the base of the instrument, and are connected by wires to the quadrants. In the figure the heads of the terminals are on the far side of the instrument.

As in the torsion balance the couple deflecting the needle is, in the position of equilibrium, balanced by the opposing couple due to torsion of the fibre or wire F. This, then, to secure the necessary sensitiveness must be very fine. As the deflection of the needle is in general very small the lamp and scale method of observation is employed, a mirror M being attached to the aluminium wire supporting the needle. The light reaches M through a window L in the cover C.

257. Comparison of Capacities by means of the Quadrant Electrometer. We shall assume that the capacities to be compared are so great that in comparison with them the capacity of a pair of quadrants is negligible. The capacities may be batteries of Leyden jars or parallel plate condensers made up of alternate sheets of tinfoil and paraffined paper, the 1st, 3rd, 5th, and so forth, sheets of tinfoil being joined together to form one plate of the condenser, and the 2nd, 4th, 6th, and so forth, sheets being joined to form the other plate. Batteries of primary cells may be used to charge the needle of the electrometer, or if the insulation is good a charged electrophorus may be used.

In Fig. 241 the condensers A and B are represented conventionally, their plates being p and q, r and s. The plates q and s are earthed, as are also one pair, b and d, of the quadrants.

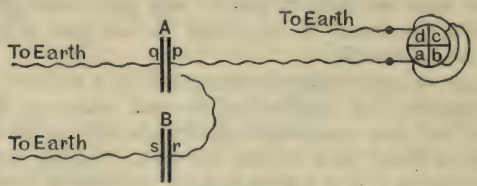


Fig. 241.

The plate p of A is connected to the insulated pair of quadrants, a and c; this plate is now charged until the deflection of the needle is considerable, say D. A wire attached to the insulated plate, r, of B is momentarily earthed to ensure that r contains no charge, and then made to touch p. The charge on A is now shared between A and B in the ratio of their capacities, and the deflection decreases to d, say, due to the fall of potential.

Let the capacities of A and B be C_1 and C_2 and V_1 and V_2 be the initial and final potentials of p. Then since the charge remains constant we have

$$C_1 V_1 = (C_1 + C_2) V_2, \qquad \text{also} \qquad \frac{V_1}{V_2} = \frac{D}{d},$$

whence $$C_1 D = (C_1 + C_2) d, \qquad \text{or} \qquad \frac{C_1}{C_2} = \frac{d}{D - d}.$$

258. Use of the Quadrant Electrometer to measure the Electromotive Forces of Voltaic Cells. If one pole of a cell is earthed and the other connected to the insulated pair of

Fig. 242.

quadrants of an electrometer, the other pair of quadrants being earthed (Fig. 242), the potential difference between the quadrants is equal to the E.M.F. of the cell, and the deflection of the needle corresponds to this potential difference. If this procedure is repeated with different cells and corresponding to their E.M.F.'s, E_1, E_2, E_3 . . . . we get deflections d_1, d_2, d_3 . . . then

$$\frac{E_1}{d_1} = \frac{E_2}{d_2} = \frac{E_3}{d_3} = \ldots\ldots$$

The advantage of this method of measuring E.M.F.'s is that the cell is on open circuit. No current is flowing, and therefore there are no polarisation effects.

Exp. 161. *Compare the E.M.F.'s of various cells :—Daniell's, Leclanché's, Grove's, Bunsen's, Storage, etc.* Taking the E.M.F. of the Daniell's cell as 1·08 volts, evaluate the E.M.F.'s of the other cells. Compare the results of your experiments with the values of the E.M.F.'s given in Ch. XII.

Current Electricity.

259. The Sine Galvanometer. One form of this instrument is shown in Fig. 243. It is exactly similar in principle and construction to the tangent galvanometer, except that the coil and needle box can be rotated round a central vertical axis, and a horizontal circular scale is provided on which the amount of this rotation can be accurately read.

For use the galvanometer is adjusted in the same way as the tangent galvanometer, but when the needle is deflected the coil is rotated after it until the needle is overtaken by it and in its deflected position lies in the plane of the coil. Fig. 244 shows

the position of the needle when in equilibrium and as in § 169, we have

$$mH \cdot 2AN = m \frac{n\pi C}{5r} \cdot 2ON,$$

$$\therefore \quad C = \frac{5Hr}{n\pi} \cdot \frac{AN}{ON} = \frac{5Hr}{n\pi} \sin \alpha \quad \ldots\ldots\ldots \quad (1)$$

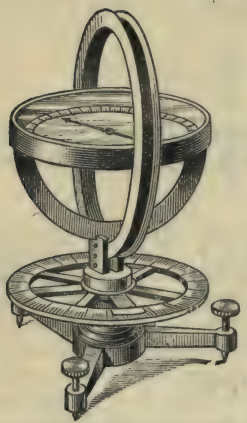


Fig. 243.

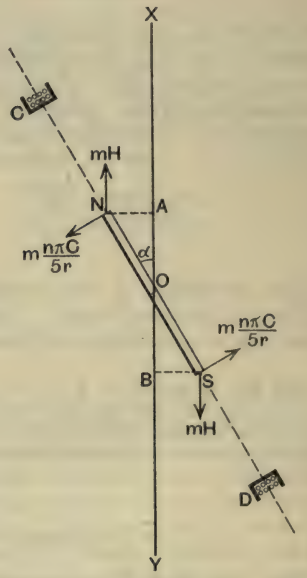


Fig. 244.

The current is therefore proportional to the sine of the deflection of the needle; for example, if one current C_1 produces a deflection of 30° and another, C_2, a deflection of 45°, then

$$\frac{C_1}{C_2} = \frac{\sin 30}{\sin 45} = \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}} = \frac{1}{1{\cdot}4}.$$

For comparative measurements the instrument may have a long needle, but for absolute measurements the needle must be short in order that the intensity of the field in which it moves may be uniform and easily determined.

The coefficient $\frac{5Hr}{n\pi}$ in (1) is called the *reduction factor* of the galvanometer; it can be calculated once for all for the same place as long as H does not sensibly vary, and marked on the instrument. Denoting it by K, equation (1) becomes

$$C = K \sin \alpha.$$

The sine galvanometer may be used for all the purposes for which a tangent galvanometer is employed. For the same current a sine galvanometer will give a larger deflection than a tangent galvanometer of exactly the same dimensions and winding.

260. The Ballistic Galvanometer. An ordinary mirror galvanometer whose needle has a long period of vibration (say about 10 secs. or over) is called a ballistic galvanometer.

A light needle may be made ballistic by loading it with a piece of lead (this increases its moment of inertia) or weakening the magnetic field in which it swings (by altering the control magnet). Either method increases the period of vibration of the needle (cf. § 252).

This form of galvanometer is of value in measuring the quantity of electricity discharged from a condenser or set in motion around a circuit by an induced current. Such a needle will practically not move from its initial position during the short time the current is flowing, and hence it receives the total impulse due to the momentary current.

The electric impulse around the circuit gives the needle a kick, and if the magnitude of the first deflection is not too large it is proportional to the quantity of electricity flowing through the galvanometer.

Of course the deflection is not maintained, and the needle returns after a few oscillations to its position of rest; some care therefore is required to read the first deflection accurately. It is of little use watching the spot of light as it moves outwards. The discharge should be repeated several times so that the place on the scale to which the spot moves may be carefully watched and so an exact reading taken.

261. Shunts. It sometimes happens that the current to be passed through a galvanometer turns out to be too strong for the instrument. When this occurs, instead of changing the galvanometer for another, it is more convenient to join the terminals of the instrument by a wire of known resistance, thus allowing a known portion of the current to be shunted through the wire. The remainder of the current passes through the galvanometer, and the ratio of this portion to the whole current evidently depends on the relative values of the resistance of the galvanometer and the wire laid across its terminals. This wire is called a **shunt**, and when a galvanometer has a shunt across its terminals it is said to be shunted.

Fig. 245.

Fig. 245 represents a shunted galvanometer, the two halves of the shunt being separated by a plug key P, as illustrated in Fig. 150.

Let G and S be the resistances of the galvanometer and shunt respectively and C the current passing along the circuit. Then if the key is out of the plug all this current goes through the galvanometer. If, however, the plug is inserted the current divides and, as shown in § 171, the portion going through the galvanometer $= \frac{S}{S+G} \cdot C$, and when S and G are known the measure of this current given by the galvanometer reading evidently serves to measure C. If $S = \frac{1}{9}G$ the current through the galvanometer is equal to one-tenth of the main current. Similarly, if $S = \frac{1}{99}G$ or $\frac{1}{999}G$ the current in the galvanometer is $\frac{1}{100}$ or $\frac{1}{1000}$ the main current.

Shunts of this kind are often provided with galvanometers, and are made up in boxes, like small resistance boxes, with convenient arrangements for using any one of the three. For rough general purposes, however, a piece of wire joining the terminals acts as an efficient shunt.

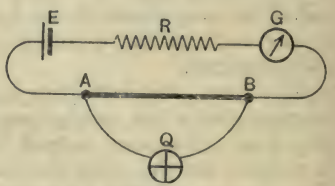


Fig. 246.

262. Proof of Ohm's Law. Ohm's Law (§ 163) says that the potential difference between the extremities of a conductor bears a constant ratio to the current through the conductor. To prove this law an electric circuit may be set up as illustrated in Fig. 246. E is a voltaic cell or battery of several cells which drives a current through a circuit made up of a conductor AB, a tangent galvanometer G, and an adjustable resistance R (§ 172). The conductor AB may be any piece of wire, and its terminals should be joined to the terminals of a quadrant electrometer Q. The tangent galvanometer measures the current through AB, and the quadrant electrometer the difference of potentials between A and B. The current is altered by varying R, and also, if necessary, by adding cells to the battery. Readings of G and Q are taken for each value of R.

It will be found that the potential difference between A and B is proportional to the current through AB for all values of the current. The ratio of the potential difference between A and B to the current through AB is called the **resistance** of AB. The experiment therefore proves that the resistance of AB is constant for all values of the current.*

263. The Resistances of different portions of a uniform wire are proportional to the lengths of the portions. To prove this it is necessary to replace the conductor AB by a uniform wire (Fig. 247), and the wires attached to the electrometer should then be attached in succession to pairs of points, PT, TR, RS ... distant, say l_1, l_2, l_3 ... cm. apart, the current being maintained steady during the time. The potential differences between these pairs of points will be found to be proportional to the lengths of wire between them, thus proving that the resistances of portions of a uniform wire are proportional to their lengths.

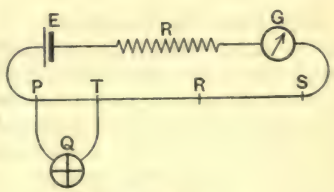


Fig. 247.

* This is only true if the other conditions of AB, such as its temperature, state of strain, etc., are kept constant.

264. The Resistances of equal lengths of wires of the same material but of different cross-section are inversely proportional to the areas of cross-section of the wires. The easiest way of getting wires of different cross-section is to join pieces from the same piece of wire in parallel; thus two pieces of the same length and of equal lengths joined in parallel form practically one piece of the same length, but of twice the cross-section (Fig. 248).

If the current is kept constant, and A and B are first joined to the electrometer and then B and C, it will be found that the potential difference between A and B is twice that between B and C, thus proving that the resistance of AB is twice that

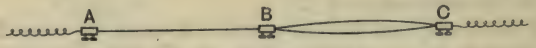


Fig. 248.

of the resistance between B and C. If n wires are used between B and C it will be found that the resistance of AB is n times that between B and C.

In this way the formula $R = k\frac{l}{a}$ of § 162 may be proved, and also the formulae of § 171 for the resistances of combinations of wires.

265. Potential-measurers. It is necessary to use an electrometer rather than a galvanometer in proving Ohm's law, because until Ohm's law is proved we have no right to assume that the current through a galvanometer is proportional to the potential difference applied to its terminals. Once, however, Ohm's law is proved, we may use a galvanometer to measure and compare potential differences, provided that the potential difference to be measured is not altered by the attachment of the galvanometer. This will practically be the case when the galvanometer has a very high resistance,* compared with that of the portion of the circuit to which it is attached.

Such a galvanometer may be graduated to read the voltage directly. It is then called a voltmeter (§ 217).

* If the galvanometer has a low resistance coil a high resistance may be put in series with it.

Exp. 162. *Measure the current in a circuit by means of a voltmeter and known resistance.* Join the known resistance in series with the rest of the circuit and the voltmeter in parallel with the known resistance. Switch on the current and note the indication of the voltmeter. Then

$$\text{current (in amperes)} = \frac{\text{voltage (in volts)}}{\text{resistance (in ohms)}}.$$

The resistance should be small and capable of carrying a considerable current. The voltmeter should read to fractions of a volt.

Exp. 163. *Find the resistance of an incandescent lamp when glowing.*

NOTE AND CAUTION.—A wired lamp-holder and a wired wall plug or lamp plug are required for this experiment. Before testing, the connections, etc., must be examined by a responsible demonstrator.

Join an ammeter A in series with the lamp, and arrange to connect the two with the electric lighting mains. Join a voltmeter V in parallel with the ammeter and lamp (as in Fig. 249). Close the circuit; read the voltmeter and the ammeter, and apply Ohm's law as above. The value of resistance obtained is that of (lamp + ammeter). The resistance of an ammeter is generally negligible. Hence the value is practically the resistance of the lamp.

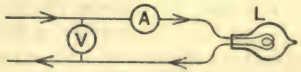


Fig. 249.

266. The Potentiometer.

A potentiometer is an instrument for the comparison and measurement of potentials. A simple form may be made as follows:—Paste a piece of drawing paper about 20 × 3½ in. in

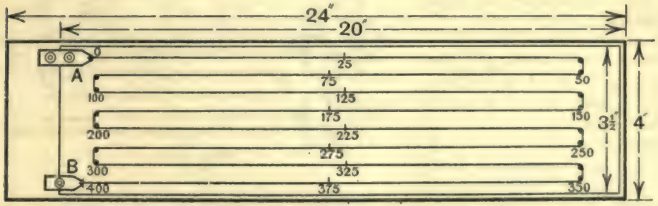


Fig. 250.

size to a board about 24 × 4 × ½ in. in size (Fig. 250). Rule 8 parallel lines down the board 49 cm. long and 1 cm. apart. Intersect these by perpendicular lines drawn 1 cm. apart. Number the divisions 0, 5, 10, 15, 20 . . . . up to 400, as indicated in the figure, the last cm. being obtained by producing the lowest

line 1 cm. to the left. Drive brass pins halfway into the wood at the ends of the long lines. Take a piece of thin uncovered German silver wire over 400 cm. long, solder one end to a piece of copper strip A, already provided with two binding screws, and fasten A to the wood so that its end comes up to the O mark. Then wind the wire around the nails and finally solder the wire to another piece of copper strip B, the length of exposed wire being 400 cm.

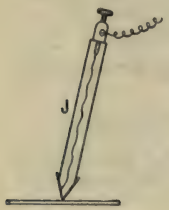


Fig. 251.

The potential at any point of the wire can be "tapped" by a jockey J (Fig. 251). This is a piece of wood $6'' \times \frac{1}{2}'' \times 1\frac{1}{2}''$, one end of which is sharpened to an edge over which a piece of copper foil is placed; the other end carries a terminal binding screw connected to the foil by means of a wire run down through the wood.

Exp. 164. *On a wire carrying a current find points such that the difference of potential between adjacent points is constant; in other words, graduate a wire into parts of equal resistance.* Join in series a constant cell E [use a Daniell (after a moment's short circuiting) or a secondary cell], a plug key P, and the uniform wire (represented diagrammatically in Fig. 252). It is advisable, though not essential, to introduce a galvanoscope S, and rheostat R. Adjust the latter to keep the deflection of S constant during the experiment. To one of the terminals of a sensitive galvanometer join a wire and a jockey J, and to the other a resistance box RB and a jockey K. Make the box resistance as high as possible.

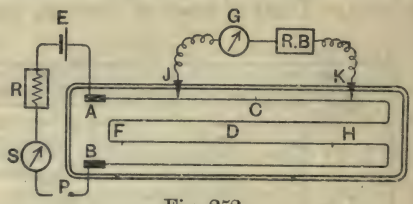


Fig. 252.

Complete the stretched wire circuit through the plug key. Put J in contact with A and K at a point distant about one-fifth of the length of the wire. Adjust the box resistance until an adequate deflection of the galvanometer needle (about one-half or two-thirds the scale) is produced. Note the position of the index of the needle or spot of light; also of the points A, C, where J and K are pressed.

Now press J at C and K at different points until one, D, is found such that the deflection is the same as before. Note the reading of D.

Press J at D, with K find the point F such that the deflection is the same as before. Note the reading of F. Similarly find other points H, etc. Finally break the circuit. Note the distances AC, CD, DF, FH . . .

If these lengths are equal it follows that the wire is uniform throughout.

267. Comparison of electromotive forces by direct application of Ohm's Law.—(1) If a cell or battery of E.M.F. E_1, resistance, B_1, is joined in series with resistances equal to R_1 and a galvanometer of resistance, G, then the current (C_1) round the circuit is such that

$$E_1 = C_1 (B_1 + G + R_1).$$

(2) If a second cell of E.M.F. E_2, resistance, B_2, is now substituted for the first and the resistance altered to another value, R_2, then the current (C_2) round the circuit is such that

$$E_2 = C_2 (B_2 + G + R_2).$$

Therefore
$$\frac{E_1}{E_2} = \frac{C_1 (B_1 + G + R_1)}{C_2 (B_2 + G + R_2)}.$$

I. *General method.* If the resistance of the external circuit is, in each case, so high that the resistances of the batteries may be neglected, then

$$\frac{E_1}{E_2} = \frac{C_1}{C_2} \cdot \frac{G + R_1}{G + R_2}.$$

II. *Constant resistance method.* If in the second case the resistance is not altered from the value R_1, then

$$E_1/E_2 = C_1/C_2.$$

III. *Equal deflections method.* When the value of R_2 is adjusted so that the same deflection of the galvanometer needle is obtained in the two cases, then $C_2 = C_1$ and

$$\frac{E_1}{E_2} = \frac{R_1 + G}{R_2 + G}.$$

Exp. 165. *Compare the E.M.F.'s of Bunsen's and Daniell's cells by the constant resistance method.* (a) Connect the cells firstly in conjunction and secondly in opposition in series with a galvanometer and a key and compare the currents produced. If a tangent galvanometer is used the method is identical with Exp. 146, p. 340. If a high resistance galvanometer is used, then with the notation above

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{C_1}{C_2} = \frac{d_1}{d_2}$$ where d_1, d_2 are the

deflection obtained with the cells in conjunction and opposition respectively. This method can only be used for "constant" cells.

(*b*) Connect the cells in turn to a high resistance mirror galvanometer, a resistance being placed in series to reduce the deflection if this is inconveniently large.

$$\text{Then} \qquad \frac{E_1}{E_2} = \frac{d_1}{d_2}$$

where d_1, d_2 are the deflections obtained.

Exp. 166. *Compare the E.M.F.'s of Daniell's and Leclanché's cells by the equal deflection method.* Connect up the Daniell's cell to a high resistance galvanometer (of known resistance G) and a resistance box. Unplug a resistance, R_1 ohms, till the deflection is of a convenient magnitude. Replace the Daniell by the Leclanché cell and unplug a resistance, R_2, until the deflection is the same as before.

$$\text{Then} \qquad \frac{E_1}{E_2} = \frac{R_1 + G}{R_2 + G}.$$

268. Comparison of the Electromotive Forces of Cells by the Potentiometer (Poggendorff's Null or Compensation Method). The principle of this method consists in balancing the E.M.F. of the cells to be compared against the difference of potential between two points in the external circuit of a battery of considerably higher E.M.F. than that of either of the cells. Let C (Fig. 253) represent the battery, AB the potentiometer wire, and R and S an adjustable rheostat and galvanoscope both of low resistance introduced for the regulation of the current.

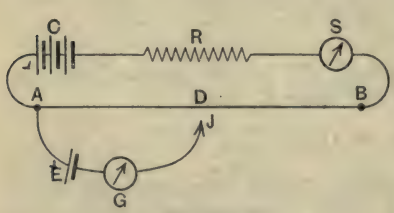


Fig. 253.

Suppose that the positive pole of C is connected to A and that to A there is also attached a branch circuit consisting of a cell, E (positive pole joined to A), a sensitive galvanometer, G, of high resistance, and a jockey, J. Before J touches the potentiometer wire there is no current through E and G. The potential of the positive pole of E is therefore equal to the potential of $A = V_A$, say. The potential of the negative pole of $E = V_A - E$ where E is the E.M.F. of the cell. This is also the potential of the jockey, J.

Take a point D on the wire, let its potential be V_D. Then if $V_D = V_A - E$ no current will flow between J and D when J touches D, *i.e.* the needle of G will not be deflected. If V_D is not equal to $V_A - E$ then there will be a current through E and G. Suppose the position of J adjusted so that the needle of G is undeflected, and that D is the point of contact. Then

$$V_D = V_A - E,$$

i.e. $$E = V_A - V_D$$

$$= \text{Current in A B} \times \text{Resistance of A D}. \quad . \quad . \quad (1)$$

If E is now replaced by a cell, E_1, and the same adjustment made the point of contact being F, then

$$E_1 = V_A - V_F$$

$$= \text{Current in A B} \times \text{Resistance of A F}. \quad . \quad . \quad (2)$$

From (1) and (2) we get

$$\frac{E}{E_1} = \frac{\text{Resistance of A D}}{\text{Resistance of A F}} = \frac{\text{Length of A D}}{\text{Length of A F}}$$

since the wire is uniform.

The advantages of this method of comparing E.M.F.'s are (1) it is a null method (a very sensitive galvanometer may therefore be used); (2) the relation between the deflection of the galvanometer and the current is not required; (3) the comparison is realised when no current is passing, hence the cells are practically on open circuit, and the relations obtained are independent of their resistances.

Exp. 167. *Compare the E.M.F.'s of a Leclanché's and a Daniell's cell.* The battery attached to the ends of the potentiometer wire must be of practically constant E.M.F., and the potential difference between the ends of the wire must be greater than the E.M.F. of any cell to be tested. For this purpose two Daniell's cells will suffice; they may be short-circuited for a few moments before use, this makes their action more constant.

EXAMPLE. In a potentiometer experiment to compare the E.M.Fs. of a Daniell's and a Leclanché's cell the point of balance for the Daniell's cell was 231 cm. from the positive end of the potentiometer, and the point of balance for the Leclanché's cell was 294 cm. away. Given that the E.M.F. of the Daniell is 1·1 volts, find that of the Leclanché cell.

We have $$\frac{\text{E.M.F. of Leclanché}}{\text{E.M.F. of Daniell}} = \frac{294}{231}.$$

$$\therefore \text{ E.M.F. of Leclanché} = \tfrac{294}{231} \times 1{\cdot}1 = 1{\cdot}4 \text{ volts}.$$

269. The Principle of the Wheatstone Bridge. This is an arrangement for the ready comparison of resistances. Imagine a portion of an electric current, as shown in Fig. 254, where a current C arriving at A is split up into two parts, one part C_1 going along A X B, the other part C_2 going along A Y B, the two parts rejoining at B. The difference of potential between A and B is of course the same whichever arm we go along, hence points can be found in each branch at which the potential is the same at corresponding points in the other branch. Let X and Y be two such points so that the potential of X is equal to the

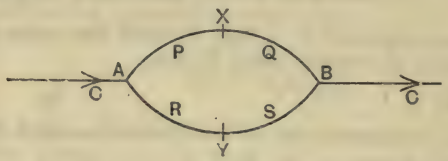


Fig. 254.

potential of Y. Denote the potentials of A, B, X, Y by V_A, V_B, V_X, V_Y and the resistances of the portions A X, X B, A Y, Y B by P, Q, R, S. Now since, by Ohm's law

$$\text{fall of potential} = \text{current} \times \text{resistance},$$

we have $V_A - V_X = C_1 P$ and $V_X - V_B = C_1 Q$

also $V_A - V_Y = C_2 R$ and $V_Y - V_B = C_2 S$

and since $V_X = V_Y$

we have $C_1 P = C_2 R$

and $C_1 Q = C_2 S$

whence by division $$\frac{P}{Q} = \frac{R}{S},$$

from which it is evident that if the ratio of P to Q is known the ratio of R to S can be found, and if three out of the four quantities P, Q, R, S are known the fourth can be calculated.

The equality of the potentials of X and Y may be tested by joining a galvanometer in series with X and Y. If the galvanometer needle is not deflected then we know that $V_X = V_Y$.

It is left as an exercise for the student to prove that if $\frac{P}{Q} = \frac{R}{S}$ and the current terminals are removed from A and B and joined to X and Y, A and B are at the same potential, so that if a galvanometer were joined to A and B its needle would not be deflected.

The loop AXBY made up of the four arms AX, XB, AY, YB is known as the Wheatstone Bridge or Wheatstone Net arrangement. It may be applied to the measurement of resistance in the following ways :—

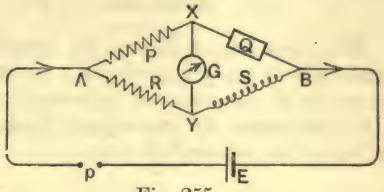


Fig. 255.

(1) Let two known resistances P and R (their resistances need not be known if their ratio is known) constitute the two arms AX and AY (Fig. 255), and let the arm XB contain a resistance box and the arm YB the resistance to be measured. A and B are joined to the terminals of a cell E which supplies the current. Then if the terminals of a sensitive reflecting galvanometer, G, are connected to the points X and Y it will be possible to adjust the resistance of Q so that the galvanometer needle is not deflected. We know then that $V_x = V_y$

and therefore $$\frac{P}{R} = \frac{Q}{S},$$

whence $$S = \frac{Q\,R}{P}.$$

In this method Q can be only altered by a definite number of ohms or fractions of a ohm depending on the box used. Also as P and R are fixed it may happen that no adjustment of Q will procure an exact balance of the galvanometer. This leads us to modify the arrangement as in the next method.

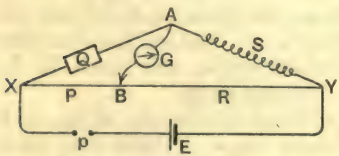


Fig. 256.

(2) The resistances of portions of a uniform wire are proportional to their lengths, therefore if P and R are replaced by a

uniform wire XY (Fig. 256), the battery connected to X and Y and the galvanometer to A and a jockey J, J may be slid up and down XY until a point of balance is found. Let B be the point of balance when Q and S are the resistances in the other arms.

$$\text{Then} \quad \frac{Q}{S} = \frac{\text{Resistance of XB}}{\text{Resistance of BY}} = \frac{\text{Length of XB}}{\text{Length of BY}}.$$

The position of B may be found with considerable accuracy and thus Q and S accurately compared. The method is most accurate if the point of balance is near the middle of the wire. The resistance Q unplugged from the resistance box may be adjusted to accomplish this.

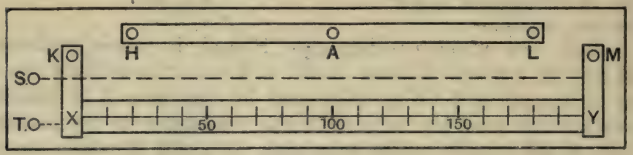


Fig. 257.

A simple form of Wheatstone Bridge may be made as follows: From some copper strip $\frac{1}{2}$ in. wide and $\frac{1}{32}$ in. thick cut three pieces, two of them 3 in. long and the third 18 in. Nail the two short strips to the table exactly 20 in. apart, and over them stretch tightly between nails a piece of carefully straightened, uncovered, uniform German-silver wire about 1 mm. in diameter. Solder the wire to the strips, making good joints, taking care that the junction runs up to the edge of the strip, but not on the wire beyond. Also solder pieces of covered copper wire to the strips. Remove the strips from the table and cut off the projecting ends of the German-silver wire. Solder two binding screws, K, M, to the ends of the short strips, and three, H, L, and A, to the ends and middle of the long strip.

Make a paper scale 20 in. long, divided into tenths of an inch and numbered to read 0 to 200 both ways. Paste this on a board 24 × 4 × $\frac{1}{2}$ in. Nail the short strips to the board so that the German-silver wire, X Y, lies over the scale and the edges of the strips come to the ends of the scale. The copper wires should be pushed through the board to the back. Solder two

pieces of copper wire 3 in. and 24 in. long to two other binding screws, S and T, and fasten the screws to the board at the left-hand end, the attached wires also being pushed through to the back. Solder together the short wire from one binding screw to the wire from the near strip, and the long wire from the other binding screw to the wire from the far strip. Fasten the long copper strip to the middle of the long side of the board.

The bridge is now complete. The resistances to be compared are inserted in the gaps K to H and L to M (Fig. 257). The battery is attached to S and T, *i.e.* virtually to X and Y, and the galvanometer to A and the jockey which taps the wire. As a variation the battery and galvanometer may be interchanged. The resistances of the strips, binding screws, and soldered joints are negligible.

270. Measurements of Resistance by means of the Wheatstone's Bridge. Wires of manganin, German silver, platinoid, brass, copper, iron may be used. It is convenient to cut definite lengths of these (50 in., 100 cm.) and clamp to a board. Leave some straight portions exposed so that the diameter may be measured.

In the following experiments a Leclanché cell may be used to supply the current. The galvanometer should be sensitive; a single needle or astatic pointer galvanometer is usually sufficient.

Exp. 168. Insert pieces, Q, S, of the same wire, but of different lengths, in the gaps and find the point B on the stretched wire at which no deflection occurs in the galvanometer when the jockey makes contact. Note the lengths of the portions XB, BY into which the stretched wire is divided and show that

$$\frac{\text{Length of Q}}{\text{Length of S}} = \frac{\text{XB}}{\text{BY}}.$$

Interchange Q and S and now find a new point of balance B′. Show that

$$\frac{\text{Length of Q}}{\text{Length of S}} = \frac{\text{B′Y}}{\text{XB′}}.$$

If the bridge is badly made $\frac{\text{XB}}{\text{BY}}$ and $\frac{\text{B′Y}}{\text{XB′}}$ will not be equal to $\frac{\text{Q}}{\text{S}}$, hence this experiment is a test of the accuracy of the construction.

Exp. 169. Intercharge battery and galvanometer and repeat the last experiment. The same result should be obtained.

Exp. 170. Compare the resistances of wires of the same material of equal lengths but of unequal diameters. Measure the diameters with a screw gauge. Prove thus that the resistances are inversely proportional to the squares of the diameters.

Exp. 171. Compare the resistances of two wires of different materials. Apply the formula $R = k\frac{l}{a}$ and thus compare the specific resistances of the materials of the two wires. Thus if

$$R_1 = k_1\frac{l_1}{a_1} \text{ and } R_2 = k_2\frac{l_2}{a_2}$$

$$\frac{R_1}{R_2} = \frac{k_1}{k_2}\cdot\frac{l_1a_2}{l_2a_1} \text{ and } \frac{k_1}{k_2} = \frac{R_1}{R_2}\cdot\frac{l_2a_1}{l_1a_2}.$$

Enter up the results as in the following table :—

Wire	Balance at point from	$\frac{R_1}{R_2}$	Length of wire	Diameter of wire	$\frac{l_2a_1}{l_1a_2}$	$\frac{k_1}{k_2}$
(1) Platinoid	93·1 cm.	·87	36·5 cm.	·0627 cm	1·77	1·54
(2) German Silver ...	106·9 cm.		29·9 cm.	·0425 cm.		

Exp. 172. Measure the resistance of several pieces of wire in terms of a piece of wire taken as a standard. Then put the several wires in parallel and measure the joint resistance. Show that this confirms the relation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \qquad \text{(Art. 171)}$$

Then join the wires in series (solder the joints) and measure the resistance. Thus confirm the relation

$$R = R_1 + R_2 + \ldots \qquad \text{(Art. 171)}$$

Exp. 173. *Measure the coefficient of increase of resistance of copper with temperature.* Take a long piece of thin covered copper wire, coil it about a rod of wood and solder the ends to thick leads which screw into one of the gaps of the bridge. Insert the coil into a steam heater as used in specific heat work. Find the resistance at the room temperature $t°$ C., in terms of a coil of wire inserted in the other gap. Now pass steam through the heater until the coil attains a constant temperature, T, and again find the point of balance. It will be found that the resistance of the copper coil has greatly increased. Evaluate R_t and R_{100} and so find the coefficient of increase of resistance from temperature from the formula

$$\frac{R_t}{R_T} = \frac{1 + at}{1 + aT}.$$

It is convenient if the resistance taken as standard is of such magnitude that the balance crosses over from one side of the middle of the bridge wire to the other side during the heating.

EXAMPLE. An experiment w. s made with the bridge described above to find the coefficient of increase of resistance of copper with temperature. Denote the copper coil by A and the standard coil by C. At 15° C., balance was obtained at a point 88·4 divisions from A. At 96·5° C., balance occurred at a point 101·6 divisions from A. Find α.

$$\text{Resistance of A at } 15°\text{C.} = \frac{88{\cdot}4}{111{\cdot}6} \times \text{Resistance of C.}$$

$$\text{Resistance of A at } 96{\cdot}5°\text{C.} = \frac{101{\cdot}6}{98{\cdot}4} \times \text{Resistance of C.}$$

$$\therefore \frac{R_{15}}{R_{96{\cdot}5}} = \frac{88{\cdot}4/111{\cdot}6}{101{\cdot}6/98{\cdot}4} = \frac{1}{1{\cdot}30}$$

$$\therefore 1{\cdot}30(1 + 15\alpha) = (1 + 96{\cdot}5\alpha)$$

$$\therefore {\cdot}30 = (96{\cdot}5 - 19{\cdot}5)\alpha$$

$$\text{whence } \alpha = \frac{{\cdot}30}{77}$$

$$= {\cdot}0039$$

271. Measurement of Resistance of a Cell by a Tangent Galvanometer. Owing to the changes which occur in temperature and concentration of a voltaic cell its resistance does not remain constant for any considerable length of time and an accurate measurement of it is not possible.

When a cell of E.M.F. E and resistance B is in series with a tangent galvanometer of resistance G the current

$$C = \frac{E}{B + G} = K \tan \alpha \qquad \ldots \ldots \quad (1)$$

where K is the constant of the galvanometer and α is the deflection. If a resistance R is now inserted in series with the cell and galvanometer the deflection decreases to α' and we have

$$\frac{E}{B + G + R} = K \tan \alpha'. \qquad \ldots \ldots \quad (2)$$

From (1) and (2)
$$\frac{B + G + R}{B + G} = \frac{\tan \alpha}{\tan \alpha'}$$

$$\text{or } 1 + \frac{R}{B + G} = \frac{\tan \alpha}{\tan \alpha'}.$$

If R is known and G is known or negligible then B may be calculated.

Exp. 174. Find the resistance of a Daniell's cell. Use the thick wire of the galvanometer so that G = O. Use a resistance box and insert firstly the one ohm coil in the circuit, and secondly the two ohm coil.

EXAMPLE. A Daniell's cell attached to the thick coil of a tangent galvanometer gave a deflection of $45\frac{1}{2}°$. The insertion of a one ohm coil caused the deflection to drop to $23\frac{1}{3}°$, and the insertion of another one ohm coil caused a further decrease to $15\frac{1}{2}°$. Find the resistance of the cell.

We have from the 1st and 2nd readings $1 + \frac{1}{B} = \frac{\tan 45\frac{1}{2}}{\tan 23\frac{1}{3}} = \frac{1{\cdot}02}{{\cdot}43} = 1 + \frac{59}{43}$

$$\therefore B = \tfrac{43}{59} \text{ ohm} = {\cdot}73 \text{ ohm}.$$

From 1st and 3rd readings we get $1 + \frac{2}{B} = \frac{\tan 45\frac{1}{2}}{\tan 15\frac{1}{2}} = \frac{1{\cdot}02}{{\cdot}28} = 1 + \frac{74}{28}$

$$\therefore B = \tfrac{28}{37} \text{ ohm} = {\cdot}75 \text{ ohm}.$$

$\therefore$ Resistance of cell = ·74 ohm approx.

272. Quantity of electricity set in motion during a change of magnetic flux through a circuit. We saw in § 237 that the induced E.M.F. in a circuit is proportional to the change of magnetic flux through the circuit. If C.G.S. units are used the electromotive force of the induced current at any instant is equal to the change of flux during a short interval of time divided by that interval of time. Thus if the flux changes from N_1 to N_2 in a short time t the average E.M.F. during the change is given by

$$E = \frac{N_1 - N_2}{t}.$$

If R is the resistance of the circuit the average value of the current is given by

$$C = \frac{E}{R} = \frac{N_1 - N_2}{t} \text{ whence } Ct = \frac{N_1 - N_2}{R}.$$

But Ct is the total quantity of electricity that has traversed the current due to the inductive change whence, we get the result

$$Q = \frac{N_1 - N_2}{R}.$$

The quantity Q may be measured with a ballistic galvanometer (§ 258).

273. Determination of the angle of dip by the Earth Inductor. If a vertical coil of wire forming a closed circuit is placed with its plane perpendicular to the magnetic meridian and it is quickly rotated through a right angle a current will be induced in it due to the change of the number of the earth's lines of force threading the coil.

If A is the effective area of the coil and H the horizontal component of the earth's magnetic field the flux through the coil is AH when the coil is perpendicular to the meridian and zero when in the meridian, hence during the rotation a quantity of electricity $\frac{AH}{R}$ is set in motion where R is the resistance of the circuit. If the rotation is continued quickly for another right angle the flux further changes to $-AH$, for the flux is travelling through the coil in the opposite direction to what it was at first, hence the total change of flux for the 180° of rotation is 2AH and the total quantity of electricity set in motion is $\frac{2AH}{R}$. If the coil is in series with a ballistic galvanometer the first deflection α of the needle is proportional to $\frac{2AH}{R}$.

If now the coil is placed with its axis of rotation horizontal and then rotated quickly through 180° from one horizontal position to the other the quantity of electricity sent round the circuit is $\frac{2AV}{R}$ where V is the vertical component of the earth's magnetic field. Suppose that the first deflection now is β. Then

$$\frac{\beta}{\alpha} = \frac{V}{H}.$$

But by § 48 if θ is the angle of dip

$$\tan\theta = \frac{V}{H}.$$

$$\text{Therefore } \tan\theta = \frac{\beta}{\alpha}.$$

Hence from the value of the two deflections the angle of dip may be found.

A coil mounted in a manner suitable for the carrying out of this experiment is called an **earth inductor.**

EXERCISES XVII.

1. What do you understand by magnetic moment? Describe experiments to show how the moments of two magnets may be compared.

2. Describe the earth as a magnet.
How would you find the resultant magnetic intensity due to the earth at any point?

3. Show that the magnetic intensity, due to a short magnet, at a point on the prolongation of its axis is twice that at a point equally distant from the centre of the magnet on the line at right angles to the axis of the magnet and passing through its centre.
How would you arrange an experiment to demonstrate this result?

4. Give a description of the apparatus required, and of the method employed, in determining the law of force between two magnetic poles.

5. How would you determine experimentally the ratio of the vertical and horizontal components of the earth's magnetic field?

6. Draw diagrams to illustrate the directions of the forces acting on the needle of a tangent and a sine galvanometer respectively when a current is passing through the coil. Upon what do the magnitudes of the forces depend?

7. Explain the action of a shunt.
A galvanometer has a resistance of 171 ohms. With what resistance must it be shunted in order that only one-tenth of the current in the main circuit may pass through it?

8. State Ohm's law. Point out how the method of comparing E.M.F.'s by the potentiometer depends on Ohm's law.

9. How would you compare the electromotive forces of two batteries?
A cell of E.M.F. 2 volts has internal resistance 1 ohm. What will be the difference of potential between its terminals when these are connected by a wire of resistance (i) one ohm, (ii) two ohms?

10. Explain how you would find the specific resistance of the material of a wire.

11. Describe experiments to illustrate the induction of a current in a circuit by the motion of another circuit conveying a constant current.
What special features should the galvanometer have for use in these experiments?

ANSWERS TO THE EXERCISES.

NOTE.—A few answers are omitted as being either sufficiently obvious or too lengthy. In most instances the *conclusions* only are given, but in a few of the more salient the reasoning is briefly added. In the case of numerical answers the nearest approximation is given.

PART I.—MAGNETISM.

EXERCISES I. (*Page* 13.)

1. In a horseshoe magnet the poles are at N and S; the simplest way is to suspend the magnet by a thread tied to the middle of the bend when it hangs with its poles downwards and comes to rest with N towards the north.

3. A temporary magnet, its ends of opposite polarity to those of the bar magnet respectively below them.

4. The ball falls off; the inductive action of the south pole on the ball counteracts that of the north pole.

5. (1) Falls off; cf. preceding question. (2) The inductive action of the south pole assists that of the north, and the latter holds the ball tighter. But at the same time the south pole attracts it, and if near enough may pull it off, its weight of course assisting.

7. No, it will be less.

8. It will move towards the iron.

9. The deflection becomes less.

10. Hold a compass-needle near the rod, and bring it gradually downwards until a point is found which attracts indifferently either pole of the needle; this point is the middle of the rod.

EXERCISES II. (*Page* 28.)

5. Test the ends *and also intermediate points* by a compass-needle.

6. If at the middle the two fractured faces will have *the same* polarity, if elsewhere *opposite*.

8. The steel rod will become magnetised, but the iron will, as soon as the magnet is removed, become neutral.

10. This is practically a case of circumferential magnetisation.

16. No; the molecular rigidity of the steel would prevent its molecular chains opening.

EXERCISES III. (*Page* 54.)

1. See Fig. 258. **2.** See Fig. 259. **3.** See Fig. 260.

4. Both its ends will acquire southern and its central portion northern polarity.

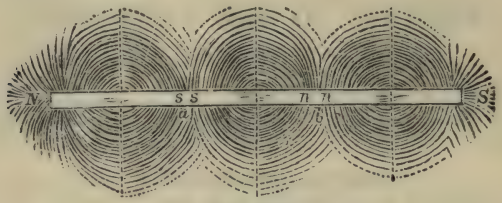


Fig. 258.

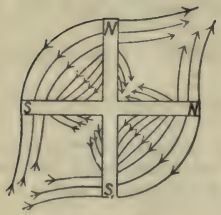


Fig. 259.

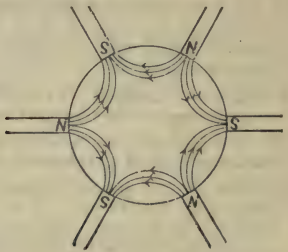


Fig. 260.

5. 7 : 12. **6.** $x = 32$. **7.** 2 in. **8.** 54 dynes, 55·08 mgr.-wt.

9. 60 and 120 unit poles. **10.** 5000 dynes.

11. 2 dynes per unit pole (nearly). **12.** 1800 $\sqrt{2}$ dynes.

13. (i) 90 dynes, (ii) ·46 dyne nearly, (iii) 460 dynes nearly.

15. 100 dynes per unit pole. **16.** $\frac{1}{40}$ sq. cm.

17. (i) 50, (ii) $2\frac{1}{2}$, (iii) $1\frac{7}{13}$. **18.** 60. **20.** 20.

22. The deflection is less with the iron. We may regard it from two points of view ; we may say (1) that by induction each end of the bar acquires polarity opposite to that of the end of the magnet which it overlaps, so that the two oppose each other's action in the external field ; or (2) that the lines of force that issue from the north pole of the magnet have to work their way round into its south pole, and, according to the principle of least reluctance, make a sharp turn round through the iron bar rather than taking a journey through the air-field.

23. The aperture of the pipe is screened from the magnetic action, and very few filings collect over it.

24. Diminished.

26. (1) Soft iron being highly permeable draws practically all the flux into itself, so that the filings over the air-space beyond the poles will cease to arrange themselves in definite curves. (2) The permeability of ordinary steel is fairly high, but distinctly less than that of soft iron, so that more of the flux will be left in the air-space and there will still be some curves over it, though fewer than before. (3) The permeability of copper being practically the same as that of air, the copper will have no effect.

27. (i) The needle will be screened and be practically unaffected by the magnet. (ii) Here again the needle will be screened. This is easily explained by the principle of least reluctance; the lines of force of the magnet have to get from its north to its south pole, and it will be "less trouble" for them to work their way round through the iron than through the external air.

28. The field will be gradually strengthened until when the magnets touch it becomes equal to the internal field; compare the effect of gradually approaching the opposite poles of two bar magnets.

Exercises IV. (*Page* 87.)

1. (1) $I = \cdot 47$ (about). (2) $V = \cdot 43$ (about).

2. (1) The needle will not move. (2) Everything depends upon the relative strengths of the earth's horizontal field and the field due to the magnet at the place where the needle is located. If the earth's field is the strongest the needle will not move, but if the magnet's field is the strongest it will make a complete half-turn and set with its north pole pointing magnetic south.

10. (1) Perpendicular to the meridian. (2) Along a line of dip.

12. (1) The north pole of the compass is deflected, more or less towards the west. (2) The needle is either unaffected or turns with its north pole towards the south, according to the strength of the polarity acquired by the mast.

13. (i) 25, (ii) 24·5.

15. $M_A : M_B :: 13 : 25$.

16. $m_A : m_B :: 4 : 5$.

17. Compare their magnetic moments, and note that the magnetic moment of each is equal to its pole strength multiplied by its *length*.

18. The vertical component of the earth's magnetism tends to make the needle dip, and the non-centric pivoting is arranged to compensate for this. Nearest the north pole.

19. The N poles will point (magnetic) north-west and north-east.

20. Dip decreased in both cases in either hemisphere.

21. (1) Suspend so that it can turn horizontally, and note the direction along which it sets. (2) Suspend so that it can turn *freely*; then, if you are in the northern hemisphere, the north lies in the direction of the end that slopes downwards If you don't know which hemisphere you are in, you must make preliminary inquiries.

24. The south pole would be twice as strong as either north pole; it would set indifferently.

25. The bow of the ship is a N.-seeking pole, the stern a S.-seeking pole, hence when sailing eastward the bow of the ship will repel the N. pole of the compass and attract the S. pole, the stern will attract the N. pole and repel the S. pole. Therefore the N. end of the needle will point W. of the magnetic north. When sailing southward the ship's magnetism acts on the needle in the same way as the Earth's, and the compass points magnetic N. and S.

27. 370 unit-poles.

PART II.—ELECTROSTATICS.

Exercises V. (*Page* 113.)

1. The pith ball will be attracted by the glass rod, and if the electrification of the latter is fairly strong, will, after touching it, be repelled.
2. Repulsion.
3. Attraction.
4. The india-rubber attracts the glass.
5. The silk repels the brass.
6. (1) Nothing. (2) Attraction. (3) Attraction.
7. It will be attracted.
9. Copper, iron, water, wood, silk, fur, glass, air.
10. Hold the rod near the ball, and see if it is repelled after touching.
12. The pair suspended by silk threads will first be attracted upwards to the conductor, and will afterwards be repelled from it, and from each other; the other pair will be repelled from each other without first being attracted to the conductor.

Exercises VI. (*Page* 131.)

1. (i) Nothing. (ii) Nothing. (iii) Potential becomes zero. (iv) Charge shared in such a way as to produce the same potential in both conductors. (v) Same as (iv). (vi) Flow of electricity from the one of higher potential. (vii) Nothing.
3. Beat an insulated metal ball with india-rubber and hold it over the electroscope; if the leaves diverge more, they are at positive potential. But if they diverge less, discharge the metal ball, beat it with fur, and again hold it over; if now they diverge more, they are at negative potential.

4. (*a*) If B were removed first the negative charge previously induced on A would escape to earth ; for if it did not it would maintain A at free negative potential, which is impossible, since the potential of an earthed conductor is zero. (*b*) No. (*c*) No.

5. (1) Diverge at positive potential. (2) Collapse. (3) Diverge at negative potential.

6. (1) Positive. (2) No; it was weaker (less strongly negative), on account of the positive charge acquired by B.

9. (1) Diverge. (2) Collapse. (3) Diverge again.

10. First give B a negative induced charge by means of A, then set A aside and give the electroscope a positive induced charge by means of B.

11. (1) They will both diverge to the same extent because by Poisson's Principle they both have the same (induced) potential. (2) When touched by the finger both acquire potential zero and their leaves collapse, but when the finger is removed and then the sphere both acquire the same (free) negative potential and again their leaves diverge, both to the same extent.

Exercises VII. (*Page* 159.)

1. Up-gradient as we near the metal; at the metal itself we are at the top of the gradient, and remain on a level all through the interior until we approach the level of the mouth ; we then begin to go down, until at a distance of, say, a couple of feet the potential is practically zero.

2. (i) Diverge. (ii) Diverge still more.

3. Yes ; opposite to the cage.

4. The charge of the shot will be completely emptied into the cylinder, so that it will acquire a positive charge, while the vessel will receive no charge at all.

5. No difference in either case.

7. (i) Do not diverge. (ii) Diverge at positive potential. (iii) Diverge at negative potential.

9. Diverge to same extent as when outside the cage.

10. (i) Divergence very slightly decreased. (ii) Divergence considerably decreased. (iii) Divergence becomes the same as it was to begin with.

11. The hand, being an earthed conductor, weakens the potential of the leaves.

12. (1) Very slight decrease of divergence. (2) Increase. (3) Decrease ; if A is earthed still greater decrease.

14. (1) No divergence. (2) Divergence.

16. They remain at rest.

21. From the symmetry of the arrangement it follows that B and C are at the same Potential, and therefore, by Poisson's Principle, when connected by a wire *nothing whatever happens.*

22. In the first case, yes; in the second, no. In the first case the metal wall acquires a certain potential and the leaves a *weaker* induced potential; the second case is practically discussed in the text-book.

23. (i) The third sphere receives equal and opposite impressed potentials from the other two, and its actual potential is therefore zero. (ii) By inductive displacement the side of the third sphere nearest the positively charged sphere becomes negatively charged, and the other side positively.

24. (1) Leaves act inductively on pot; divergence increased. (2) See § 112; divergence same as in (1).

Exercises VIII. (*Page* 170.)

1. Charge keeps to the outside. No.

2. (1) It is quite small at the pointed parts. (2) It is now very big at the pointed parts.

3. They will remain permanently diverged.

4. The charge must be free, and the sphere a long way from other conductors, and surrounded by nothing but dry air. There *are* other conditions under which it is true, but these are what is commonly intended.

5. (1) Orange acquires a charge of the same kind as that of the body. (2) Of the opposite kind.

6. (1) B partly discharged. (2) B almost completely discharged.

9. No change until the needle gets outside the pail, then fairly rapid discharge.

Exercises IX. (*Page* 191.)

1. Greater in the second case.

2. The divergence will diminish, because the approach of B weaken's A's potential.

4. After merely insulating B they will remain close together; when A is moved a little farther they will diverge, and the ebonite rod will make them diverge still more.

5. Connect the prime conductor to the inner coat and the rubbers to the outer.

11. The capacity of the small jar is less than that of the big one. Hence by the formula $Q = CV$ the potential of the small jar is (supposing the charge positive) *higher* than that of the big one; the spark then passes in accordance with Poisson's principle, or more strictly the principle of § 114.

12. No effect. The machine gives a +ve charge on the prime conductor, and a −ve charge on the inside of the chamber; but the total charge is nil. Also the potential of the chamber is the same as if this total charge belonged to it "freely."

13. The two would have the same potential, and therefore the sparks would have the same *length*. But A has a greater capacity than B, therefore A's spark is *brighter*.

EXERCISES X. (*Page* 223.)

1. $2\sqrt{6}$ in. **2.** 4 cm. **3.** About 2·88 dynes.

4. Decreased. $\frac{\text{Force in air}}{\text{Force in oil}}$ = K of oil. **5.** 20 dynes.

7. (i) 20 dynes per electrostatic unit. (ii) 10 ditto.

8. (1) 2·0 dynes per unit charge. (2) Ditto, but direction reversed. (3) 3·0 (4) Ditto, but direction reversed. **9.** 16 dynes

10. 93 dynes. **11.** (1) 500 ergs; (2) − 500 ergs.

13 2400 ergs; 600 electrostatic units of potential.

15. 24 electrostatic units of potential.

16. (1) 42 cm. beyond B. (2) There are two such points, one 18 cm. beyond B, the other 5·04 cm. from B and between B and A.

17. 5 cm. **18.** $6\frac{2}{3}$ electrostatic units of potential.

20. (1) $\frac{Q_1 + Q_2}{C_1 + C_2}$. (2) $\frac{C_1(Q_1 + Q_2)}{C_1 + C_2}$, $\frac{C_2(Q_1 + Q_2)}{C_1 + C_2}$. (3) $\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$.

21. Charge on suspended sphere before touching = 19·8 *es.u.q.* very nearly **22.** 5 units.

23. Second part (*anywhere* between them) = ·05 dyne.

PART III.—ELECTRODYNAMICS.

EXERCISES XI. (*Page* 265.)

2. Precisely the same action as in Qn. 1, except that the current, instead of going through the wire, will pass direct from the copper to the zinc at the place where they touch.

6. B has three times the resistance of A. **7.** 3 amperes.

8. $66\frac{2}{3}$ ohms. **9.** $2\frac{1}{2}$ volts.

10. (1) $\frac{1}{2}$ ampere, 9 volts. (2) $1\frac{1}{2}$ amperes, 3 volts.

11. (i) $\frac{2}{3}$ volt, (ii) $1\frac{1}{3}$ volts.

13. (i) $\frac{1}{2}$ ampere, (ii) $12\frac{1}{2}$ volts, (iii) $6\frac{1}{4}$ volts, (iv) $6\frac{1}{4}$ volts. **15.** No.

16. 299 ohms. **17.** 108 volts. **20.** About 0·7 ampere.

21. The resistance of the coil is $2\frac{1}{3}$ times that of the rest of the circuit. Proceed thus: Let E be the E.M.F. of the cell, x the resistance of the coil, and R that of the rest of the circuit including the cell and galvanometer. Then by Ohm's Law the current without the coil is $\frac{E}{R}$, and with the coil $\frac{E}{R+x}$. Hence by the terms of the question

$$\frac{E}{R} \div \frac{E}{R+x} = \frac{10}{3}\ ;\ i.e.\ \frac{R+x}{R} = \frac{10}{3}\ ;$$

$$\therefore\ 3R + 3x = 10R\ ;\ \therefore\ 3x = 7R\ ;\ \therefore\ x = 2\tfrac{1}{3}R.$$

22. In smaller galvanometer ; 30°. **23.** With few.

24. 4 725 microhms ; about ·54 ohm. **25.** About 58 metres.

26. (i) 2 : 20, (ii) 1 : 1. **27.** $\frac{1}{3}$ ampere ; it becomes $\frac{1}{4}$ ampere. **29.** 6 ohms.

Exercises XII. (*Page* 290.)

1. 28 volts.

3. $2e$; the cell put in the wrong way is said to be in "opposition" to the others, and its E.M.F. subtracts instead of adding.

5. It will be less, because the terminal potential difference on closed circuit is less than the E.M.F.

6. $\frac{1}{2}$ an ampere ; 6 volts. **7.** One-third as great.

8. Compare the E.M.F.'s by the condensing electroscope. **9.** 40.

11. 1·75 amperes, ·35 ampere, 1·75 volts. **12.** 10.

16. With electroscope no difference ; with low-resistance galvanometer the big cells give a decidedly greater deflection ; with high-resistance galvanometer practically no difference.

17. (1) ·88 ampere. (2) 10·9 volts. **18.** No difference whatever.

19. The north pole will be deflected westward ; the reason is that the zinc plate is replaced by platinum, the *direct* E.M.F. of the cell is taken away, and there remains nothing but the back E.M.F., which sends a current the opposite way round the galvanometer coils.

22. Yes ; the potential of the insulated plate just before lifting the earth plate will be that of the terminal *last touched*, hence the order B A will give the greater divergence.

23. Use a larger number, say 50 to 100 cells in series, so as to get the requisite E.M.F.

24. (i) 10·5 volts, (ii) − 10·5 volts, (iii) 6·3 and − 4·2 volts, (iv) indefinite, but their potential-*difference* is 10·5 volts.

Exercises XIII. (*Page* 315.)

1. $H_2SO_4 + H_2O = H_2 + H_2SO_4 + O$.

2. The end of the rod which the current strikes will become coated with a film of finely divided copper and the other end with a film of copper oxide. **4.** About 0·3. **5.** 10·2 amperes nearly.

6. Ag : Cu : H : : 27 : 23·6 : 1.

7. Deflection becomes 60 ; rate of deposition 3 times as great.

8. Yes. A current would flow from the platinum to the copper, thence through the Daniell to the zinc, thence to the Grove zinc, and thence through the Grove back to the platinum.

9. The Daniells give their natural and the Grove its reversed action : in the latter hydrogen or zinc is deposited on the zinc plate, while oxygen is liberated on the platinum plate. ·25 ampere.

10 ·2 and ·1 ohm respectively.

17. Hydrogen will be given off from both rods, but after the $CuSO_4$ is poured in it will appear on the zinc rod only.

18. Four-fifths as great.

19. The two together would have no E.M.F., and therefore if *they alone* constitute the circuit there would be no current. If there were other cells in the circuit they would merely offer dead resistance.

20. $\frac{1}{2}$ ampere, $\frac{1}{10}$ ampere.

Exercises XIV. (*Page* 351.)

1. The north pole will move eastwards.

2. (i) At whichever end of the spiral the current enters, the north pole of the needle points away from the middle of the tube. (ii) Ditto.

3. North pole moves westward.

4. (1) Very feeble side polarity. (2) Magnetised lengthways.

11. Magnetised circumferentially.

12. Place a Kelvin Astatic Galvanometer in the circuit. The reason the current is only momentary is that there is no conducting path from one pole of the battery to the other ; a current merely flows between each pole and the plate thereto connected until the pole and plate acquire the same potential, and then ceases.

13. Hold the wire horizontally parallel to the magnets and midway between them ; if they are of equal strength they will not move.

14. Yes, in all localities the north pole will move upwards.

15. No effect whether the rod be of iron or brass.

24. (1) Moves towards the fixed one ; (2) Makes a half-turn (probably first moving a little away) and then moves towards it ; (3) No action but the very least displacement will cause it to make a quarter-turn and then move towards the fixed spiral.

Exercises XV. (*Page* 367.)

1. (1) No. (2) The thin wire gets hotter.

3. The iron has the greater specific resistance, and therefore gets hotter.

6. (1) No. (2) The thin end gets hotter.

7. (1) 1·8. (2) 778. (3) 111, very nearly; nearly ·017 H.P.

8. An alternating current is one whose direction undergoes continual rapid reversals; consequently it will not produce electrolysis since every beat reverses the effect of the preceding beat. Any voltameter will therefore serve as a test. **9.** $166\frac{2}{3}$ ohms.

10. (1) $\frac{1}{2}$ ampere. (2) 3 joules. (3) 1·86 joules.

11. (1) ·8 ampere. (2) 4·8 joules (3) All of it.

12. 6 H.P. very nearly. **13.** 500 volts.

14. (1) 8 amperes. (2) 48 volts. (3) 96 %. (4) About ·54 H.P. (5) 11·5 B.O.T.U. **15.** 162000. **16.** 10 amperes.

18. $1\frac{2}{3}$ watts, $\frac{3}{746}$ H.P. **19.** 900 flashes. **20.** (1) Quartered, (2) quartered.

Exercises XVI. (*Page* 385.)

3. In this case there is at no part of the movement any flux *through* the coil but only *across* it, and increase or decrease of this produces no induced current.

7. Imagine the magnet placed so that its north pole is away from you and therefore its amperian currents *clockwise*; when the iron approaches the magnet the latter becomes by induction more strongly magnetised, and therefore the flux along it is *increased*, hence the induced current is *counter-clockwise*. Potential energy due to the separation of the magnet and the iron.

9. (i) Deflection unaltered, (ii) deflection decreased. If inner coil vertical no deflection in either case. **10.** 18 *micro*amperes.

12. Yes; one way is to push a magnet through the ring.

14. Apply Lenz's Law and Ampère's First Law.

16. Apply the law of Electromagnetic Introduction and Ampère's First Law; the coil is repelled. (1) The action *of the magnet* does not, but the *resultant* action does; (2) the action *of the magnet* does.

Exercises XVII. (*Page* 428.)

7. 19 ohms.

9. (i) 1 volt, (ii) $1\frac{1}{3}$ volts.

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